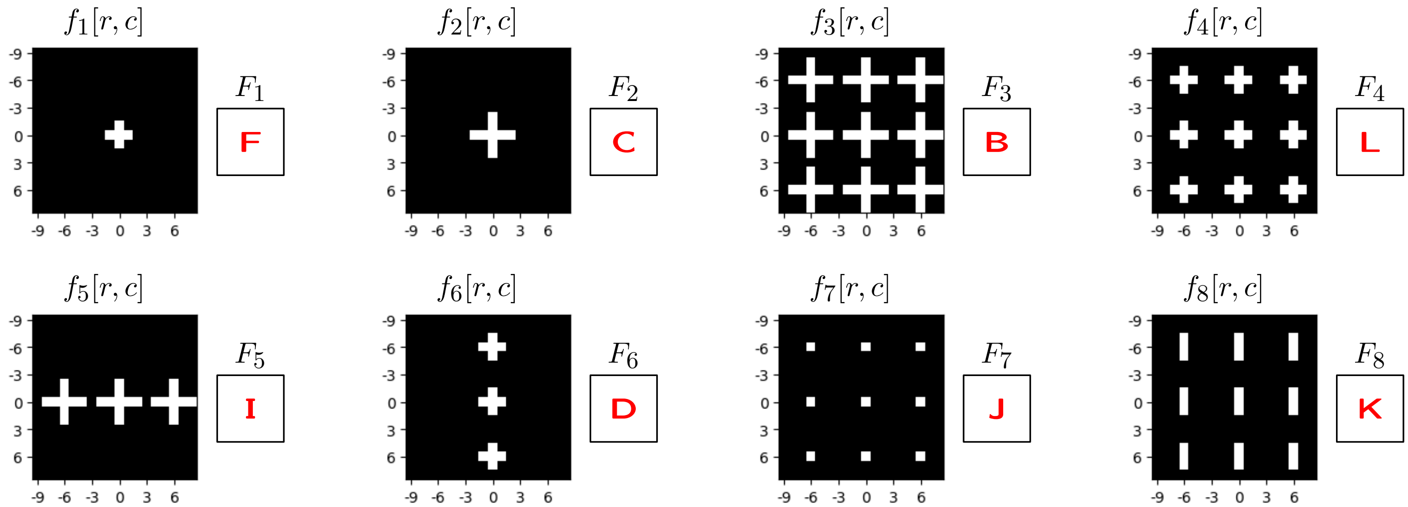
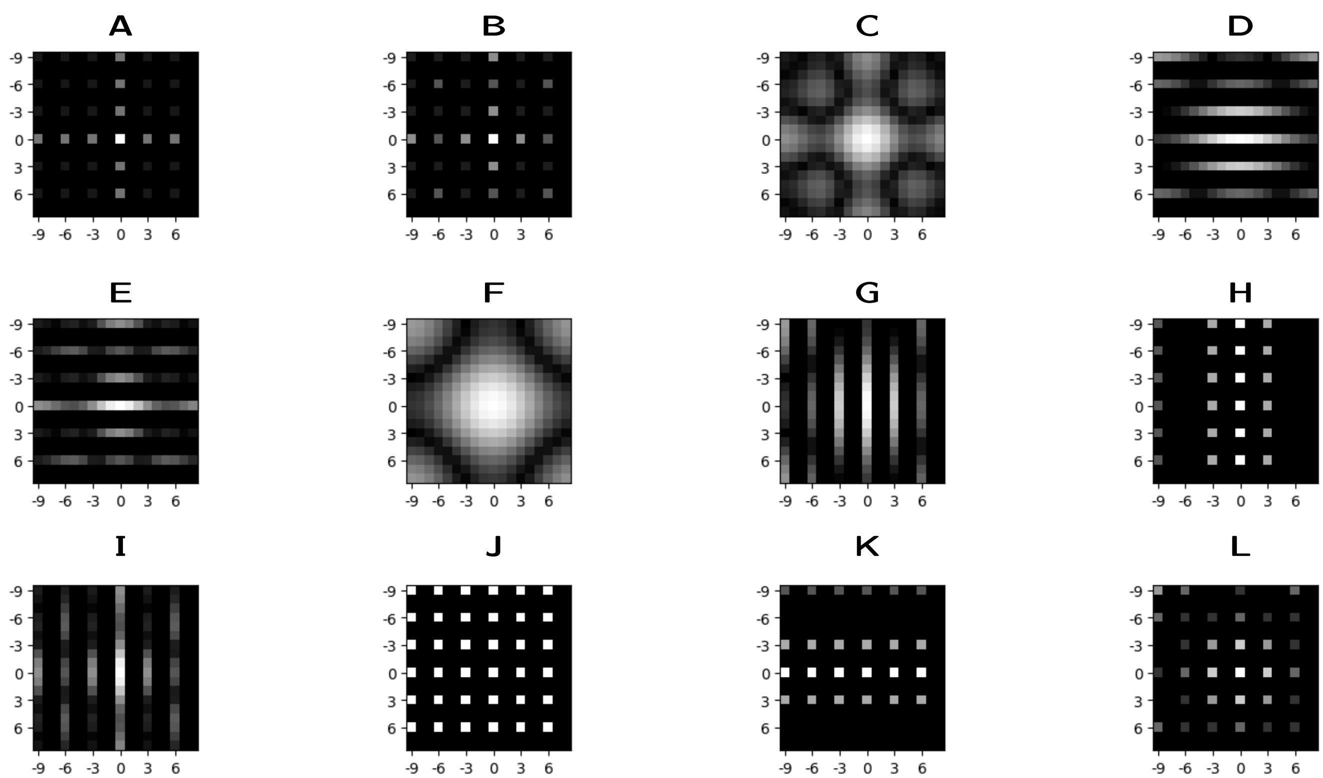
Pluses and Minuses

The panels below show eight images ($f_1[r,c]$ to $f_8[r,c]$ with black (0) or white (1) pixels that are indexed by row number r and column number c where $-9 \le r < 9$ and $-9 \le c < 9$.



Each of the following panels shows the magnitude of a 2D DFT. For each panel, black represents a value of 0 and white represents the largest magnitude in that panel (which may be different for each panel). Determine which of the panels shows the magnitude of the 2D DFT of each of the images above, and enter the corresponding letter (A-L) in the appropriate box.



$f_1[r, c]$

This signal contains a DC component (due to the white square at $r = c = 0$, as well as a horizontal fundamental component (due to white squares at $c = \pm 1$, and a vertical fundamental component (due to white squares at $r = \pm 1$. The periods of the fundamental components are equal to the image width $C = 18$ and height $R = 18$. Thus the 2D DFT is panel **F**.

$f_2[r, c]$

This signal includes not only fundamental components (as in f_1 but also second harmonic terms (due to the which squares at $c = \pm 2$ and $r = \pm 2$). Thus the 2D DFT is panel **C**.

$f_7[r, c]$

This image is periodic in r and c, so its 2D DFT will be composed of isolated dots. Since there are 3 dots across the field in both the r and c directions, the lowesst, non-zero harmonic in each direction will be the third. Thus the 2D DFT is panel **J**.

$f_3[r, c]$

The f_3 image is the convolution of f_2 with f_7. Therefore F_3 will be the product of F_2 (C) with F_7 (J). Thus the 2D DFT is panel **B**.

$f_4[r, c]$

The f_4 image is the convolution of f_1 with f_7. Therefore F_4 will be the product of F_1 (F) with F_7 (J). Thus the 2D DFT is panel **L**.

$f_8[r, c]$

The f_8 image is the convolution of a large, rotated minus sign with f_7. The rotated minus sign is skinny and tall. Therefore, its 2D DFT would be short and fat. The only pattern of isolated dots that fits this description is panel **K**.

$f_5[r, c]$

This image is periodic in c but not in r. Therefore its 2D DFT is dots in k_c continuous in k_r. This description could fit with I or G. The big plus signs produces the shorter vertical bars in panel **I**.

$f_6[r, c]$

This image is a rotated version of f_5 with small plus signs, which produces the longer horizontal bars in panel **D**.