

## Tetra

Let  $f[r, c]$  represent the following image as an array of pixel brightnesses in the range  $[-\frac{1}{2}, \frac{1}{2}]$  where black corresponds to  $-1/2$ , white corresponds to  $+1/2$ , and the background grey corresponds to 0. The pixels are indexed by their row number  $r$  (with  $-\frac{1}{2}R \leq r < \frac{1}{2}R$ ) and column number  $c$  (with  $-\frac{1}{2}C \leq c < \frac{1}{2}C$ ), and  $R = C = 200$ .



Consider eight images that are derived from  $f[r, c]$  – four of which are defined by spatial functions  $h_1[r, c]$  and  $h_2[r, c]$ :

$$h_1[r, c] = \sin\left(\frac{40\pi r}{R}\right)$$

$$h_2[r, c] = \begin{cases} 1 & \text{if } r \text{ is even} \\ 0 & \text{otherwise} \end{cases}$$

and four of which are defined by frequency functions  $H_3[k_r, k_c]$  and  $H_4[k_r, k_c]$ :

$$H_3[k_r, k_c] = j \sin\left(\frac{40\pi k_r}{R}\right)$$

$$H_4[k_r, k_c] = \begin{cases} 1 & \text{if } k_r \text{ is even} \\ 0 & \text{otherwise} \end{cases}$$

Determine which of the images on the next page corresponds to each of the following image combinations, and enter the appropriate label (A-L) in each of the boxes below.

$(f \times h_1)[r, c]$ 

E
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$(f \circledast h_1)[r, c]$ 

A
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$(f \times h_2)[r, c]$ 

B
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$(f \circledast h_2)[r, c]$ 

F
---

$(f \times h_3)[r, c]$ 

G
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$(f \circledast h_3)[r, c]$ 

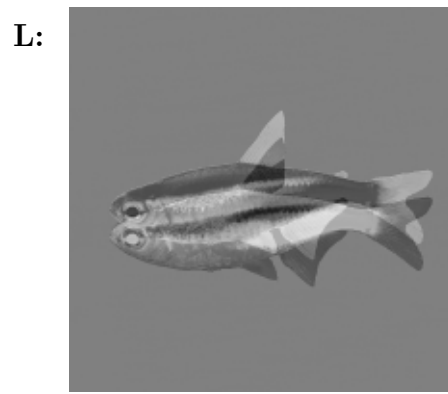
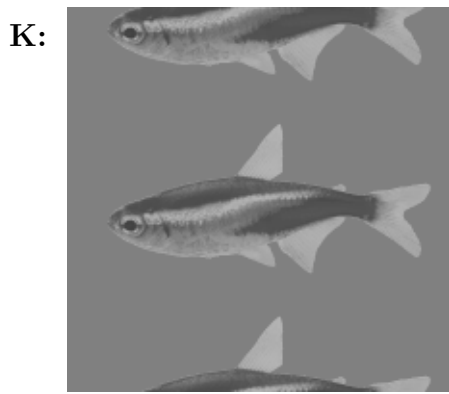
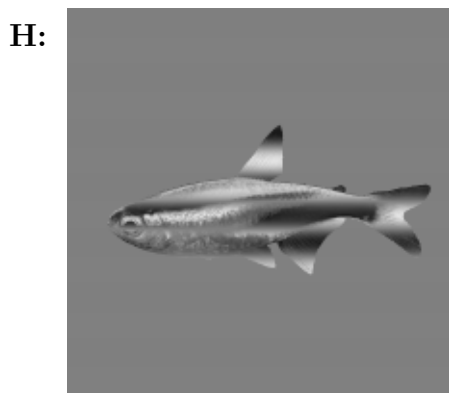
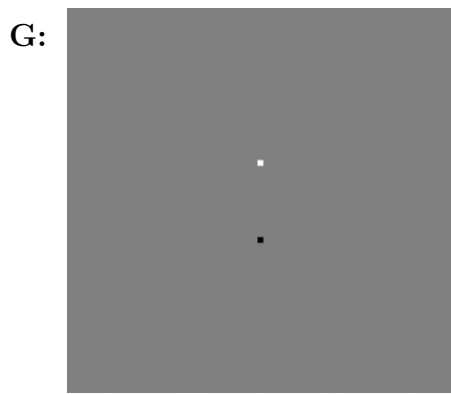
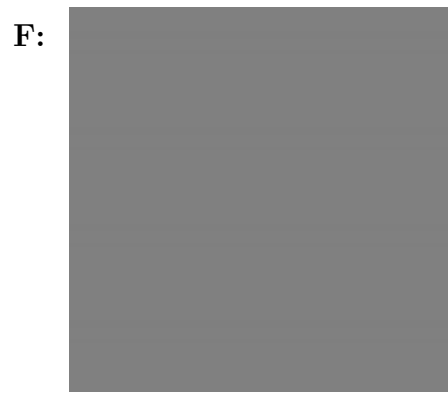
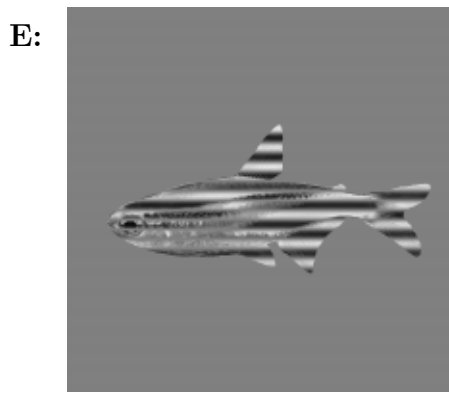
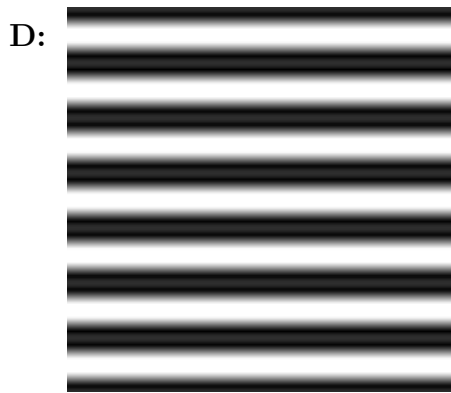
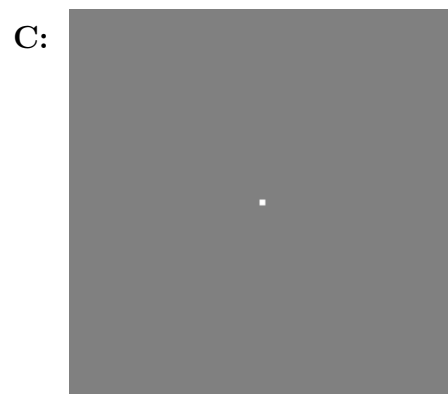
I
---

$(f \times h_4)[r, c]$ 

C
---

$(f \circledast h_4)[r, c]$ 

K
---



$$h_1[r, c] = \sin\left(\frac{40\pi r}{R}\right)$$

This image consists of 20 horizontal stripes with brightnesses changing sinusoidally along the vertical dimension. Multiplying the fish image by this signal puts horizontal stripes on the fish but does not affect the background since its brightness is zero.

Thus the answer is **E**.

Convolving the fish image with  $h_1$  is equivalent to multiplying their 2D DFTs. The 2D DFT of  $h_1$  is a positive dot at  $r = -20, c = 0$  and a negative dot at  $r = 20, c = 0$ . Multiplying these dots times the 2D DFT of the fish selects only the two basis functions at the dots location.

The result is a striped image shown in **A**.

$$h_2[r, c] = \begin{cases} 1 & \text{if } r \text{ is even} \\ 0 & \text{otherwise} \end{cases}$$

This image consists of 100 alternating horizontal stripes of white and black. Multiplying the fish image by this signal puts finely spaced horizontal stripes on the fish but does not affect the background since its brightness is zero.

Thus the answer is **B**.

Convolving the image with  $h_2$  is equivalent to multiplying their 2D DFTs. The 2D DFT of  $h_2$  is a positive dot at  $r = -1, c = 0$  and a negative dot at  $r = 1, c = 0$ . Multiplying these dots times the 2D DFT of the fish selects only the two basis functions, corresponding to the dots' locations. However, the amplitudes of these components in the fish image are small. Thus the resulting stripes are not visible at the output.

The result is shown in **F**.

$$H_3[k_r, k_c] = j \sin\left(\frac{40\pi k_r}{R}\right)$$

This 2D DFT consists of 20 horizontal stripes with amplitudes changing sinusoidally along the vertical dimension. The inverse DFT of this pattern is two dots, much like image G. Multiplying the fish image by this signal creates an image that has the background gray color (which represents 0) everywhere except for the two dots in G.

Thus the answer is **G**.

Convolving the fish image with the two dots that comprise  $h_3$  produces a positive image of the fish that is shifted up 20 pixels plus a negative image of the fish that is shifted down 20 pixels.

The result is a striped image shown in **I**.

$$H_4[k_r, k_c] = \begin{cases} 1 & \text{if } k_r \text{ is even} \\ 0 & \text{otherwise} \end{cases}$$

This 2D DFT consists of 100 horizontal stripes that alternate between white and black along the vertical dimension. The inverse DFT of this pattern is two dots: one at the origin and one in the center of the top row of the image. Multiplying the fish image times this signal sets the top dot to zero, making it blend in with the background. Only the bottom dot remains.

Thus the answer is **C**.

Convolving the fish image with the two dots that comprise  $h_4$  produces two images of the fish. The first is at the original location of the fish. The second is shifted up by half the frame, with the upper part of this shifted fish circularly shifted into the bottom part of the new image.

The result is **K**.