

Fourier Series Representations of Modulation

The frequency content of a signal can be (reversibly) altered by a process called **modulation**. Let $g(t)$ represent a **modulated** version of the signal $f(t)$, where

$$f(t) = \cos(2\pi 10t) + 2 \sin(2\pi 30t)$$

and

$$g(t) = f(t) \cos(2\pi 1000t).$$

As we will see later in the semester, modulation is widely used in communications systems based on electromagnetic (radio) waves, including cellular systems, wifi, bluetooth, GPS, and navigation systems.

Part a. Is the function $g(t)$ periodic? If yes, determine the fundamental period of $g(t)$. If not, explain how you know that it is not periodic.

Yes. The function $g(t)$ is periodic.

The fundamental period of $g(t)$ is 0.1 seconds, as can be seen by substituting $t+0.1$ for t in the expression for $g(t)$:

$$\begin{aligned} g(t+0.1) &= \left(\cos(2\pi 10(t+0.1)) + 2 \sin(2\pi 30(t+0.1)) \right) \cos(2\pi 1000(t+0.1)) \\ &= \left(\cos(2\pi 10t + 2\pi) + 2 \sin(2\pi 30t + 6\pi) \right) \cos(2\pi 1000t + 200\pi) \\ &= \left(\cos(2\pi 10t) + 2 \sin(2\pi 30t) \right) \cos(2\pi 1000t) \\ &= g(t) \end{aligned}$$

This construction shows that $g(t)$ is periodic in time t with period $T = 0.1$ seconds. By similar reasoning, $g(t)$ is also periodic in $T = 0.2$ seconds, but $g(t)$ is not periodic in $T = 0.05$ seconds. Thus $T = 0.1$ seconds is the smallest interval for which $g(t)$ is periodic. Therefore $T = 0.1$ seconds is the fundamental period of $g(t)$.

Part b. Find a Fourier series representation of $f(t)$ with the following form:

$$f(t) = c_0 + \sum_{k=1}^{\infty} \left(c_k \cos(k\omega_o t) + d_k \sin(k\omega_o t) \right)$$

where ω_o represents the fundamental frequency of the Fourier series.

Determine the value of ω_o :

$\omega_o =$

20π radians/second

Determine the nonzero values of c_k and d_k and enter them in the table below. Use a separate line for each value of k for which either c_k or d_k or both are nonzero. Leave unused rows of the table empty.

k	c_k	d_k
1	1	
3		2

Part c. Find a Fourier series representation of $g(t)$ with the following form:

$$g(t) = c_0 + \sum_{k=1}^{\infty} \left(c_k \cos(k\omega_o t) + d_k \sin(k\omega_o t) \right)$$

where ω_o represents the fundamental frequency of the Fourier series.

Determine the value of ω_o :

$\omega_o =$

20π radians/second

Determine the nonzero values of c_k and d_k and enter them in the table below. Use a separate line for each value of k for which either c_k or d_k or both are nonzero. Leave unused rows of the table empty.

Substituting $f(t)$ into the expression for $g(t)$ yields

$$\begin{aligned} g(t) &= \left(\cos(2\pi 10t) + 2 \sin(2\pi 30t) \right) \cos(2\pi 1000t) \\ &= \frac{1}{2} \cos(2\pi 1010t) + \frac{1}{2} \cos(2\pi 990t) + \sin(2\pi 1030t) - \sin(2\pi 970t) \end{aligned}$$

k	c_k	d_k
97		-1
99	$\frac{1}{2}$	
101	$\frac{1}{2}$	
103		1