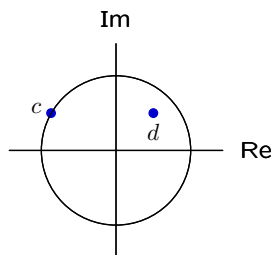
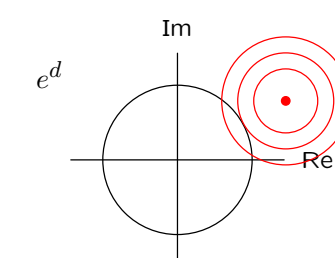
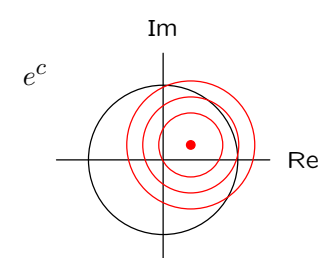
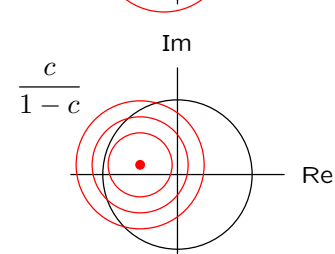
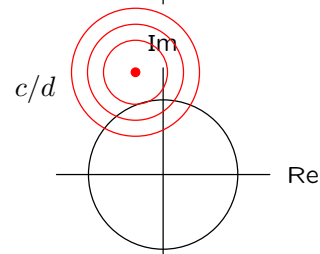
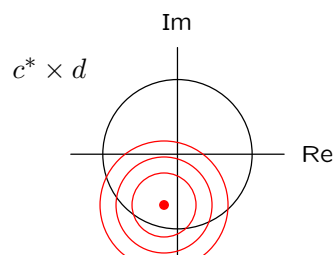
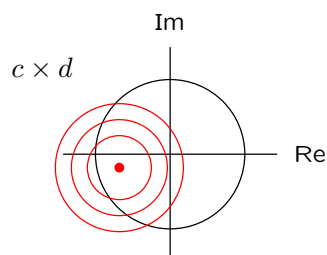
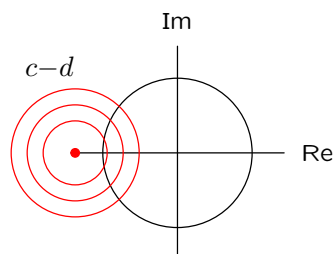
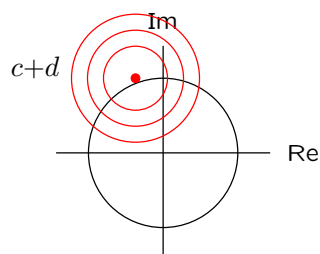


Complex Pairs

Let c and d represent the complex numbers shown by filled dots in the following diagram, where the real and imaginary parts of the complex numbers are shown on the horizontal and vertical axes, respectively, and the circle has a radius of 1.

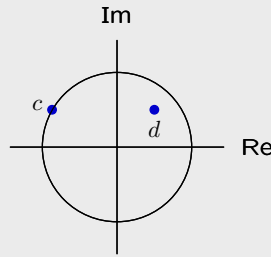


Below are eight complex-valued functions of c and d , each paired with a depiction of the complex plane demarked by the unit circle. Evaluate each expression and mark its value on the complex plane with a dot. Note that e represents Euler's number (2.71828...) and c^* represents the complex conjugate of c .



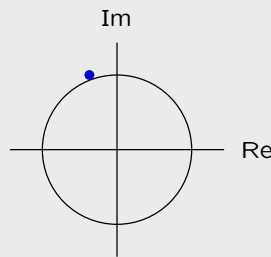
Complex Pairs

Start by estimating the numbers c and d . The number c has a magnitude very close to 1 and an angle that is about 180° minus 30° . Both the real and imaginary parts of d are approximately 0.5.



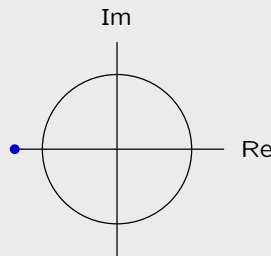
Part a: $c+d$

It is usually easiest to add complex numbers in their cartesian form. We can approximate the cartesian form of c as $-0.87 + 0.5j$ and the cartesian form of d as $0.5 + j0.5$. Then the cartesian form of $c+d$ is approximately $-0.37 + j$.



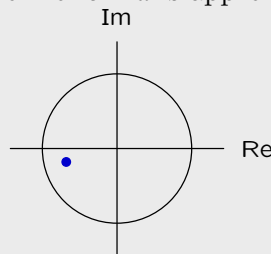
Part b: $c-d$

Using the same approximations as in part a, $c-d$ is approximately -1.4 .



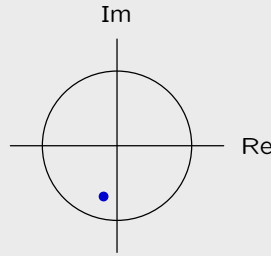
Part c: cd

It is usually easiest to multiply complex numbers in their polar form. If we approximate the polar form of c as $e^{j5\pi/6}$ and d as $0.7e^{j\pi/4}$, then the polar form of $c \times d$ is approximately $0.7e^{j13\pi/12}$.

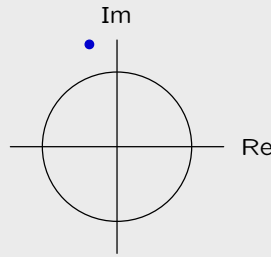


Part d: c^*d

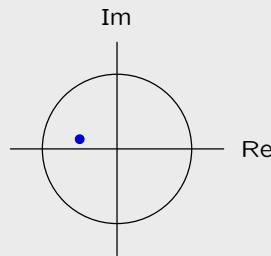
Using the same approximations as in part c, the number c^* would be $e^{-j5\pi/6}$ and the product of c^* and d would be $0.7e^{-7\pi/12}$.

**Part e: c/d**

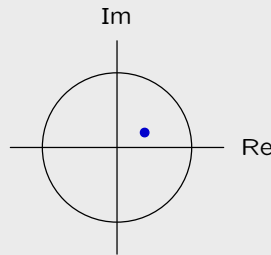
Division is also easiest in polar form. Using the approximations in part c, the number c/d is approximately $1/0.7 e^{j7\pi/12}$.

**Part f: $\frac{c}{1-c}$**

The magnitude of c is approximately 1 and the angle of c is approximately $5\pi/6$. The magnitude of $1-c$ is approximately 2 and the angle of $1-c$ is approximately $-\pi/12$. The magnitude of $c/(1-c)$ is therefore approximately 0.5 and the angle of $c/(1-c)$ is approximately $\frac{5\pi}{6} + \frac{\pi}{12} \approx \frac{11\pi}{12}$.

**Part g: e^c**

Express c in cartesian form, which is approximately $-0.87 + 0.5j$. Then e^c is $e^{-0.87+0.5j} = e^{-0.87}e^{0.5j}$. $e^{-0.87}$ is approximately the reciprocal of e , which is about $1/3$. $e^{0.5j}$ is approximately $e^{j\pi/6}$, which is a small positive angle.



Part h: e^d

The constant $e^d = e^{0.5+0.5j}$. $e^{0.5}$ is the square root of e , which is about 1.6. $e^{0.5j}$ is a small positive angle (as shown in the previous part).

