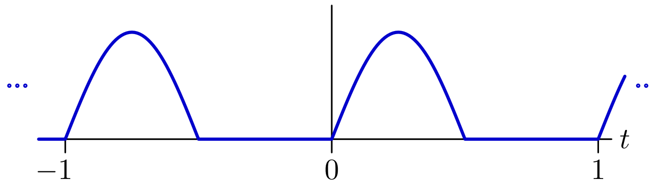
Fourier Series Transformations

Part a. Let $f_1(t)$ represent the following periodic, continuous-time signal, with period $T = 1$:

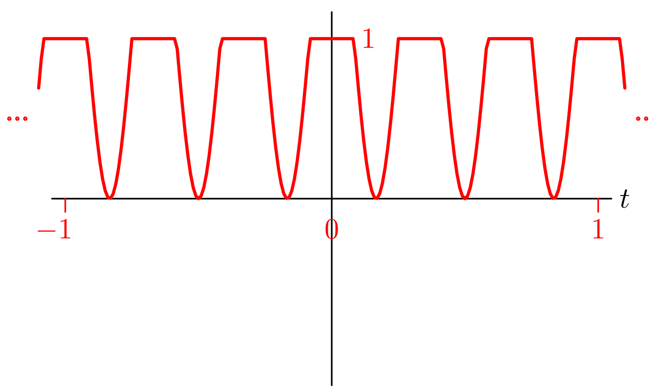
$f_1(t) = \max(0, \sin(2\pi t))$



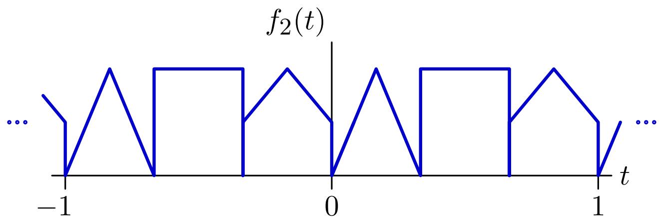
Let $g_1(t) = 1 - f_1\left(3t - \frac{1}{4}\right)$.

Sketch $g_1(t)$ on the following axes. Label the important parameters of your plot.

$g_1(t) = 1 - f_1\left(3t - \frac{1}{4}\right)$



Part b. Let $f_2(t)$ represent the following periodic, continuous-time signal with period $T = 1$:



Let $g_2(t) = 1 - f_2\big(3t - \frac{1}{4}\big)$.

Let $F_2[k]$ and $G_2[k]$ represent the Fourier series coefficients for $f_2(t)$ and $g_2(t)$, respectively, where both series are computed **with the same period** $T = 1$. Determine expressions for each of $G_2[0]$ through $G_2[15]$ in terms of the Fourier coeeficients $F_2[k]$. Your table entries can contain real and/or imaginary numbers and constants such as e and π. Your entries should not contain integrals or infinite sums.

k	$G_2[k]$
0	$1 - F[0]$
1	0
2	0
3	$j\,F[1]$
4	0
5	0
6	$F[2]$
7	0

k	$G_2[k]$
8	0
9	$-j\,F[3]$
10	0
11	0
12	$-F[4]$
13	0
14	0
15	$j\,F[5]$

$$F_2[k] = \frac{1}{T}\int_T f(t)e^{-j\frac{2\pi k}{T}t}dt$$

$$G_2[k] = \frac{1}{T}\int_T g(t)e^{-j\frac{2\pi k}{T}t}dt = \frac{1}{T}\int_T \left(1 - f\left(3t - \frac{1}{4}\right)\right)e^{-j\frac{2\pi k}{T}t}dt$$

$$= \frac{1}{T}\int_T e^{-j\frac{2\pi k}{T}}\,dt - \frac{1}{T}\int_T f\left(3t - \frac{1}{4}\right)e^{-j\frac{2\pi k}{T}t}dt$$

Let $\tau = 3t - 1/4$. Then $d\tau = 3dt$.

$$G_2[k] = \delta[k] - \frac{1}{T}\int_{3T} f(\tau)e^{-j\frac{2\pi k}{T}(\frac{\tau}{3}+\frac{1}{12})}\frac{1}{3}d\tau$$

$$= \delta[k] - e^{-j\frac{2\pi k}{12T}}\frac{1}{3T}\int_{3T} f(\tau)e^{-j\frac{2\pi k}{T}(\frac{\tau}{3})}d\tau$$

$$= \delta[k] - e^{-j\frac{2\pi k}{12T}}\,F_2[k/3]$$

Notice that the $\delta[k]$ term contributes 1 if $k = 0$ and 0 otherwise. Also notice that $G_2[k] = 0$ unless $k \bmod 3 = 0$.

One way to think about this is that the period of $f(t)$ is 1 second, and therefore $f(t)$ can be expressed as a sum of harmonics that are integer multiples of 1 Hz. The $g(t)$ signal is a compressed version of $f(t)$, so the harmonics of $g(t)$ are spread out by a factor of three.

A second way to think about this is by looking at the $g(t)$ function itself. Since $g(t)$ is periodic in $1/3$ second, we should be expecting that the Fourier series for $g(t)$ should only contain integer multiples of 3 Hz.