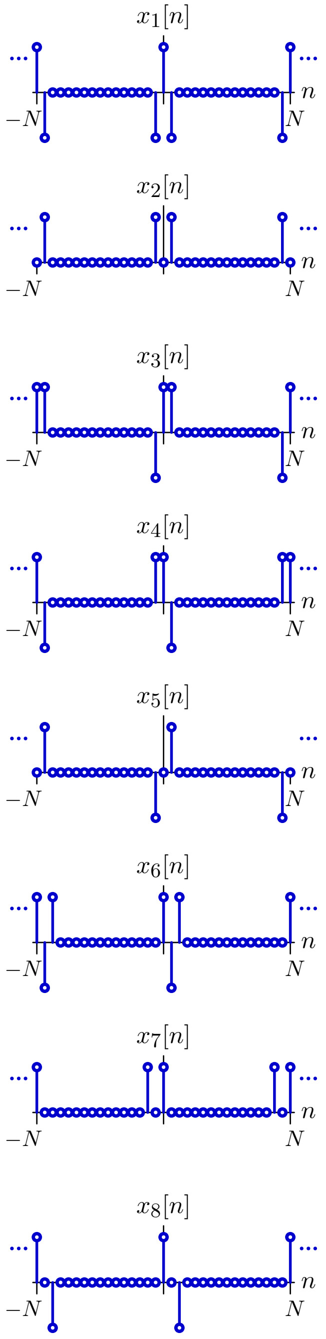
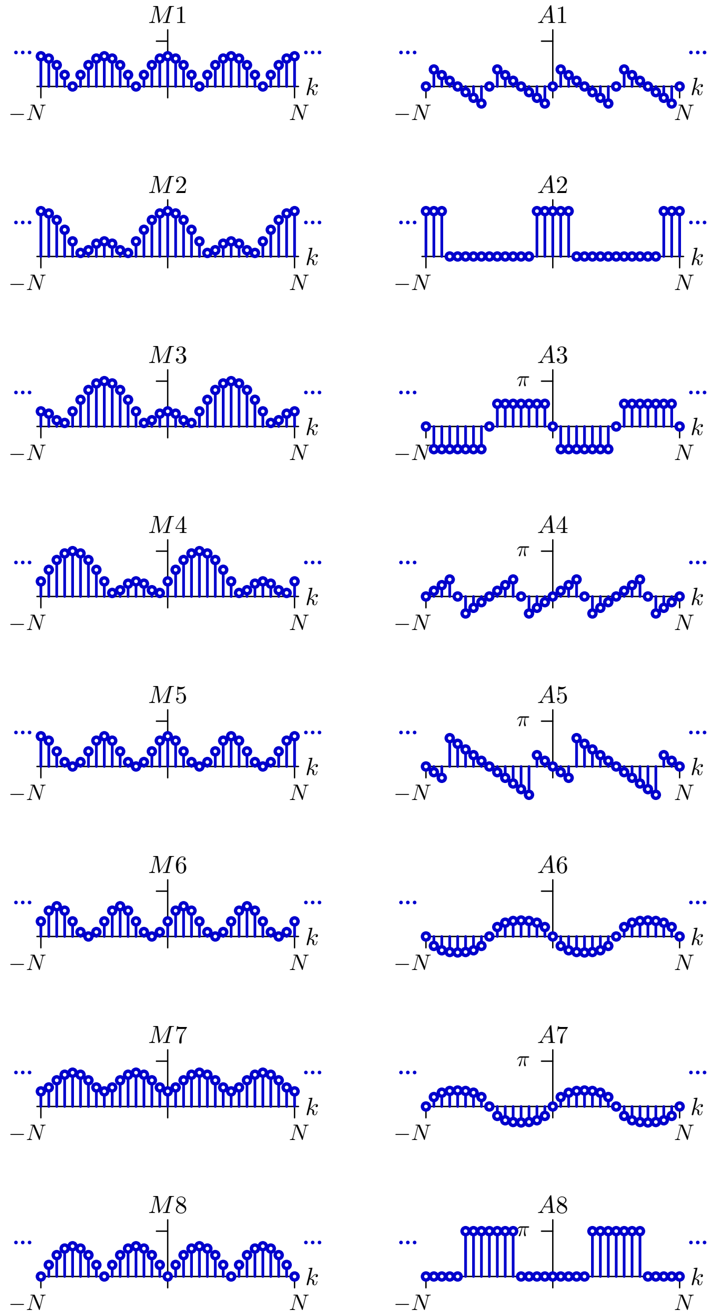
Fourier Series Matching

Each of the signals $x_i[n]$ in the left column below is periodic with period $N = 16$. Find the Fourier series coefficients $X_i[k]$ for each signal and then identify which of plots $M1 - M8$ shows the magnitude of $X_i[k]$ and which of plots $A1 - A8$ shows the angle of $X_i[k]$ as functions of k. Enter your answers in the boxes provided.



	M	A
$x_1[n]$	M3	A2
$x_2[n]$	M1	A8
$x_3[n]$	M7	A6
$x_4[n]$	M7	A7
$x_5[n]$	M8	A3
$x_6[n]$	M3	A5
$x_7[n]$	M1	A4
$x_8[n]$	M8	A1



Worksheet (intentionally blank)

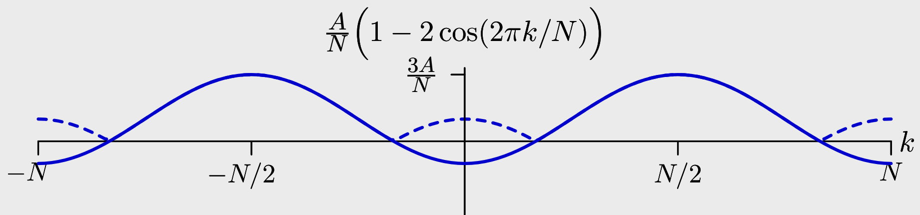
Find the Fourier series representation $X[k]$ for each of the given $x[n]$ using the analysis equation.

$$X[k] = \frac{1}{N} \sum_{n=\langle N \rangle} x[n] e^{-j\frac{2\pi}{N}kn}$$

We will assume (arbitrarily) that the values of $x[n]$ are $-A$, 0, or A.

Since the signals are all real-valued, the corresponding magnitudes will be symmetric about $k = 0$. This eliminates M_4 and M_6.

Part 1. $X_1[k] = \frac{A}{N}(-e^{j\frac{2\pi}{N}k} + 1 - e^{-j\frac{2\pi}{N}k}) = \frac{A}{N}(1 - 2\cos(2\pi k/N))$



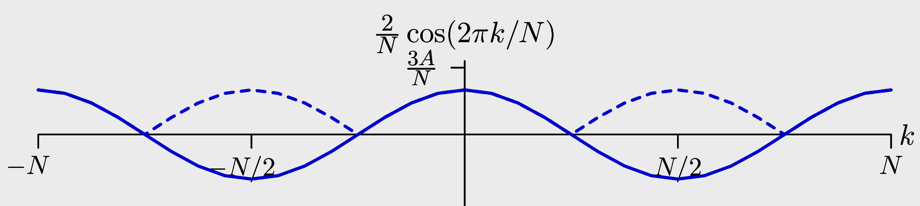
The magnitude (dashed) has a small peak near $k = 0$ and larger peaks at $k = \pm N/2$.

Answer = M3.

The angle is 0 for the range of k near $N/2$ (where there is no dashed line) and π for the range of k near 0, where the solid and dashed lines differ in sign).

Answer = A2.

Part 2. $X_2[k] = \frac{1}{N}(e^{j\frac{2\pi}{N}k} + e^{-j\frac{2\pi}{N}k}) = \frac{2}{N}\cos(2\pi k/N)$



The magnitude (dashed) has equally large peaks at $k = 0$ and $k = N/2$ and sharp nulls between.

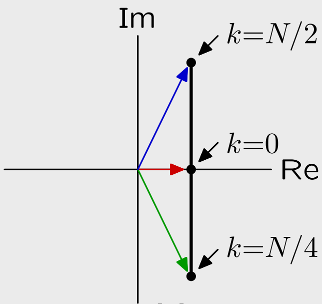
Answer = M1.

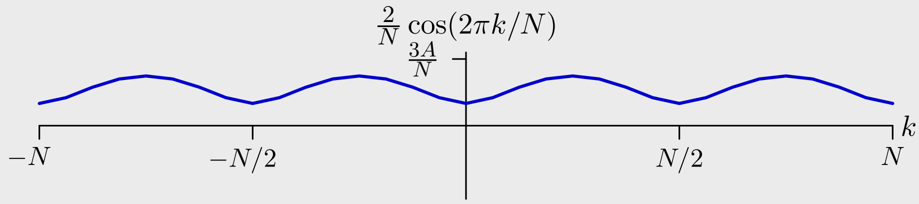
The angle is 0 for the range of k near $k = 0$ and π for the range of k near $N/2$ (where the dashed and solid curves differ in sign).

Answer = A8.

Part 3. $X_3[k] = \frac{1}{N}(-e^{j\frac{2\pi}{N}k} + 1 + e^{-j\frac{2\pi}{N}k}) = \frac{1}{N}\Big(1 - 2j\sin(2\pi k/N)\Big)$

This part is a bit trickier to plot since (unlike parts 1 and 2) this one has both real and imaginary parts.





When $k = 0$, $X_3[k]$ is 1. As k increases, the imaginary part of $X_3[k]$ gets increasingly negative – from 0 at $k = 0$ to -2 at $k = N/4$. Correspondingly, the magnitude increases from 1 at $k = 0$ to $\sqrt{5}$ at $k = N/4$.

As k increases from $N/4$ to $N/2$, the imaginary part of $X_3(k]$ change from -2 to 0 and the magnitude drops from $\sqrt{5}$ back to 1.

The plot of magnitude is not exactly sinusoidal, but it is smooth and does not have sharp notches.

Answer = M7.

The angle of $X_3[k]$ start at 0 for $k = 0$ and gradually decreases (going negative) for k between 0 and $N/4$. As k increases from $N/4$ to $N/2$, the angle decreases back to zero. The pattern from $N/2$ to N is similar to the pattern from 0 to $N/2$ except that the sign is now flipped to positive.

Answer = A6.

Part 4. $X_4[k] = \frac{1}{N}(e^{j\frac{2\pi}{N}k} + 1 - e^{-j\frac{2\pi}{N}k}) = 1 + 2j\sin(2\pi k/N)$

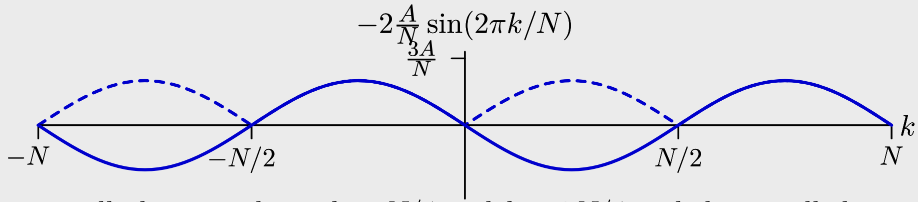
This part is similar to part 3, except that the imaginary part of $X_4[k]$ is the negative of that of $X_3[k]$.

Thus the magnitude is the same as part 3, i.e., M7.

The angle is the negative of that in part 3, i.e., A7.

Part 5. $X_5[k] = \frac{1}{N}(-e^{j\frac{2\pi}{N}k} + e^{-j\frac{2\pi}{N}k}) = -2j\sin(2\pi k/N)$

This part is purely imaginary.



The magnitude has equally large peaks at $k = N/4$ and $k = 3N/4$ and sharp nulls between.

Answer = M8.

The angle is $-\pi/2$ for $0 < k < N/2$ and $\pi/2$ for $N/2 < k < N$.

Answer = A3.

Part 6. $X_6[k] = \frac{1}{N}(1 - e^{-j\frac{2\pi}{N}k} + e^{-j\frac{2\pi}{N}2k}) = e^{-j2\pi k/N}(-1 + 2\cos(2\pi k/N))$

Notice that $x_6[n] = -x_1[n-1]$. Neither the delay nor the negation will affect the magnitude. Therefore the magnitude is given by M1.

To find the angle of $X_6[k]$, start with the angle of $X_1[k]$ (plot A2). Negating $x_1[n]$ adds π to all of the angles. As a result, the angle is 0 for k close to 0 and π otherwise.

Next consider the effect of the delay, which multiplies the Fourier transform by $e^{-j2\pi k/N}$. This delay adds an angle of $-2\pi k/N$ to each frequency point k. The resulting angle is shown in A5.

Part 7. $X_7[k] = \frac{1}{N}(e^{j\frac{2\pi}{N}2k} + 1) = e^{j2\pi k/N}(2\cos(2\pi k/N))$

$x_7[n]$ is a version of $x_2[n]$ that is shifted backwards in time by 1 sample. The time shift does not affect the magnitude. Therefore the magnitude is the same as part 2 – i.e., M1.

Without the delay, the angle would have been A8. The shift adds an angle of $2\pi k/N$ to each frequency point k, resulting in A4.

Part 8. $X_8[k] = \frac{1}{N}(1 - e^{-j\frac{2\pi}{N}2k}) = 2je^{-j2\pi k/N}\sin(2\pi k/N)$

This is a negated and delayed version of part 5. Neither the negation nor the delay affect the magnitude, which is therefore given by M8.

Without the delay, the angle would have been $\pi/2$ for $0 \le k \le N/2$ and $-\pi/2$ for $N/2 \le k \le N$. The delay adds a downward sloping phase, resulting in plot A1.