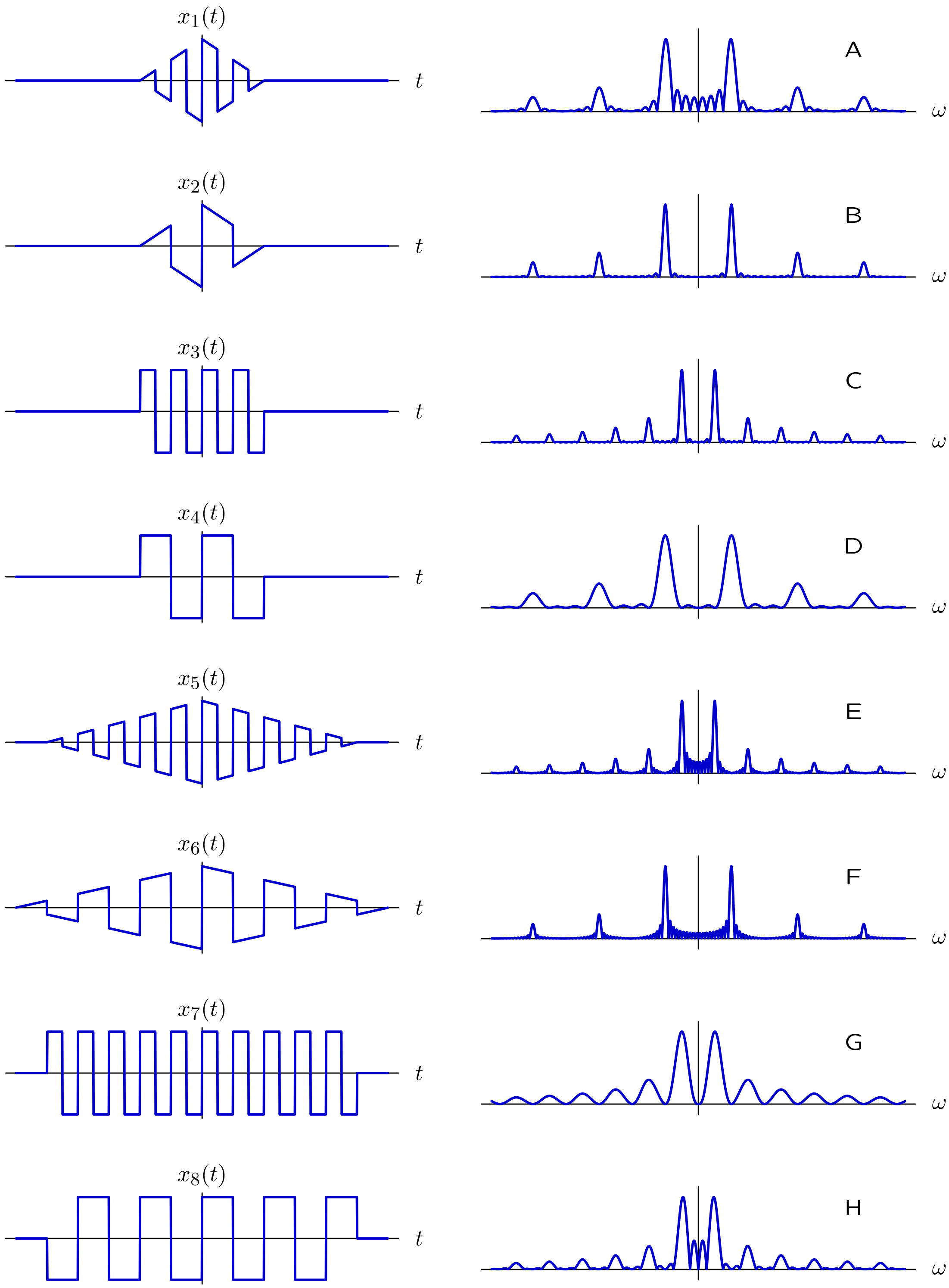
Time-Frequency Patterns

Eight time waveforms are shown in the left panels below. The corresponding Fourier transform magnitudes are shown in the right panels, however, the order has been shuffled. For each signal on the left, find the corresponding CTFT magnitude plot on the right.



The time waveforms can be classified as having three important parameters:
• period: The periods of x_1, x_3, x_5, and x_7 are half as long as those of the others.
• shape: The overall shape of the waveform is either triangular or square, and can be thought of as multiplying an underlying periodic time signal.
• overall length: x_1, x_2, x_3, and x_4 are short, the others are long.

These parameters affect the magnitudes of the Fourier transforms in distinct ways. Let A-H represent the waveforms in the right column.
• The period in time is inversely related to the period in frequency. Thus A, B, D, and F (which have longer periods in frequency) correspond to x_1, x_3, x_5, and x_7.
• Since the shape multiplies the time waveform, it convolves with the frequency waveform. The shape is long compared to the periodicity of the time waveform, therefore the convolution affects the short timescale (i.e., between the periods). The square (in time) has more high frequencies than the triangle, so the square in frequency has larger overshoot. Thus A, E, F, and H correspond to squares (x_3, x_4, x_7, and x_8), and the others correspond to triangles.
• The overall length is long compared to the periodicity of the time waveform, therefore the convolution affects the short timescale (i.e., between the periods). The longer the shape, the shorter the spread around each lobe in the frequency domain. Therefore, the broad lobes (A, D, G, and H) correspond to the short overall lengths (x_1, x_2, x_3, and x_4).

This combination of constraints leads to the following solution:

$x_1(t) \rightarrow$ D
$x_2(t) \rightarrow$ G
$x_3(t) \rightarrow$ A
$x_4(t) \rightarrow$ H
$x_5(t) \rightarrow$ B
$x_6(t) \rightarrow$ C
$x_7(t) \rightarrow$ F
$x_8(t) \rightarrow$ E