Cats

Ben Bitdiddle took a photograph of his cat, but he only saved the associated DFT coefficients $X[k_r, k_c]$, rather than saving the original image. However, he knows the original 77×51 image looked like this:

Ben tries several different methods of recovering the original image based on $X[k_r, k_c]$. For each of the methods described below, indicate which of the images on the next page (labeled A-T) would have resulted from that approach. In these images, grey colors represent positive values (black represents 0, white represents 1), and red represents negative values (black represents 0, bright red represents -1). For all parts, assume that $r = 0, c = 0$ corresponds to the upper-left corner of the image (rows increase downward, and columns increase to the right).

Approaches

1. Applying the inverse DFT to the real part of X.
2. Applying the inverse DFT to the imaginary part of X.
3. Applying the inverse DFT to j times the imaginary part of X.
4. Applying the inverse DFT to X after setting $X[0, 0] = 0$.
5. Applying the inverse DFT to X after setting $X[25, 38] = 0$.
6. Applying the inverse DFT to X after subtracting $\frac{1}{51 \times 77}$ from every value.
7. Applying the inverse DFT to X after multiplying every value by $e^{j\pi}$.
8. Applying the inverse DFT to X after multiplying every value except $X[0, 0]$ by $e^{j\pi}$.
9. Applying the inverse DFT to X after negating the phase of every value.

Question

Which image matches each process?

If an approach would have led to an image with nonzero imaginary components (and thus would have resulted in a Python error when trying to save the image), enter ‘X‘ for that approach.

Here is the definition of a 2D DFT:

$$X[k_r,k_c] = \frac{1}{RC}\sum_{r=0}^{R-1}\sum_{c=0}^{C-1} x[r,c]\left(e^{-\frac{j2\pi k_r r}{R}}\right)\left(e^{-\frac{j2\pi k_c c}{C}}\right)$$

1. The real part of a sum is the sum of the real parts. Also, the real part of a complex number is half the sum of that number with its complex conjugate. So the real part of the DFT is

$$\begin{aligned}\mathrm{Re}\left(X[k_r,k_c]\right) &= \frac{1}{2}\frac{1}{RC}\sum_{r=0}^{R-1}\sum_{c=0}^{C-1} x[r,c]\left[\left(e^{-\frac{j2\pi k_r r}{R}}\right)\left(e^{-\frac{j2\pi k_c c}{C}}\right)+\left(e^{\frac{j2\pi k_r r}{R}}\right)\left(e^{\frac{j2\pi k_c c}{C}}\right)\right]\\ &= \frac{1}{2}\left(X[k_r,k_c]+X[-k_r,-k_c]\right)\end{aligned}$$

   The inverse DFT of the real part of the original DFT is equal to the sum of the original image plus a version of the original image that is flipped across both $r=0$ and $c=0$. Thus the answer is M.

2. By similar reasoning, the inverse DFT of the imaginary part of the original DFT is equal to $\frac{1}{2j}$ times difference between the original image and a version of the original image that is flipped across both $r=0$ and $c=0$. Notice that the resulting image is purely imaginary. Therefore the answer is X.

3. Multiplying j times the answer to the previous part results in a purely real image that is half the difference of the original image minus a version of the original image that is flipped across both $r=0$ and $c=0$. Therefore the answer is I.

4. Setting $X[0,0]=0$ will shift the pixel values in the original image so that the darker pixels will become negative (i.e., red) while the brighter pixels will become darker gray. Therefore the answer is S.

5. Setting $X[25,38]=0$ will introduce imaginary components to the image. In the original image, the imaginary part of $X[-25,-38]$ is cancelled by the equal and opposite imaginary part of $X[25,38]$. Zeroing $X[25,38]$ therefore introduces an imaginary part to the resulting image. The answer is X.

6. Subtraction $\frac{1}{51\times 77}$ from every value of $X[k_r,k_c]$ will reduce the brightness of the resulting image at $r=c=0$. The resulting image is P.

7. Multiplying $X[k_r,k_c]$ by $e^{j\pi}=-1$ will make the brightest pixels into the deepest red while black pixels will become just faintly red (i.e., mostly black). The answer is Q.

8. Multiplying $X[k_r,k_c]$ by $e^{j\pi}=-1$ for all k_r,k_c except $k_c=k_c=0$ will change black pixels to white and vice versa. The answer is B.

9. Negating the phase of $X[k_r,k_c]$ for all k_r and k_c will flip the image about both $r=0$ and $c=0$. The answer is G.

Images

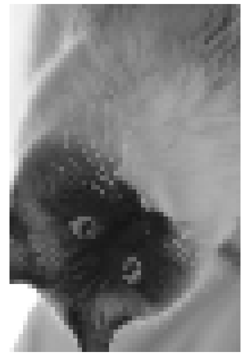
A

B

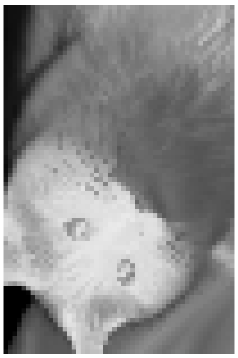
C

D

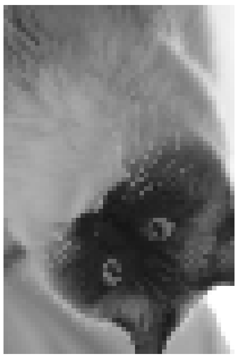
E



F

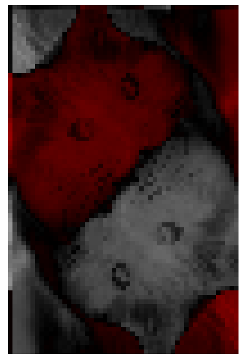


G

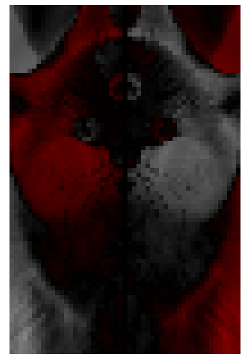


H

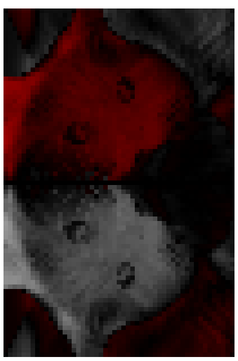
I



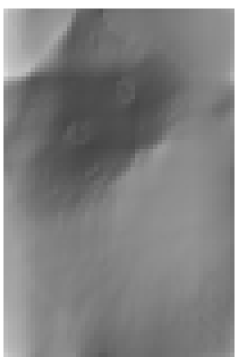
J



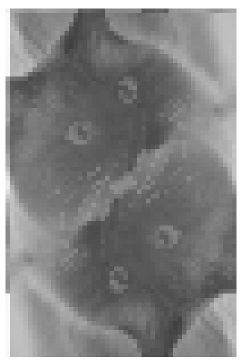
K



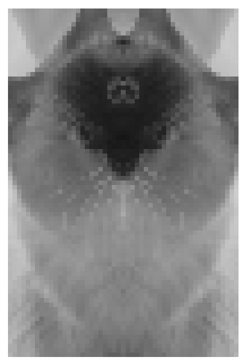
L



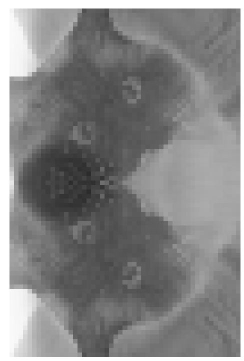
M



N

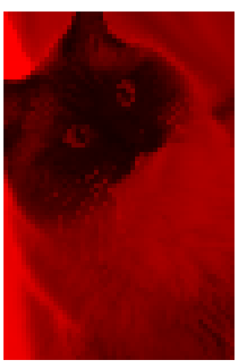


O

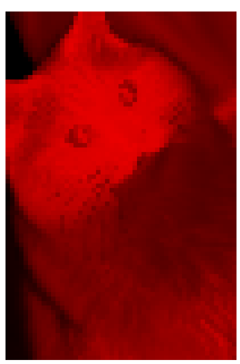


P

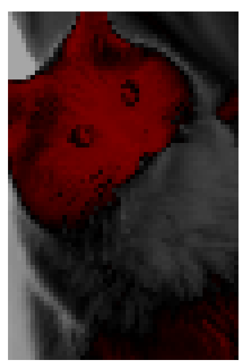
Q



R



S



T