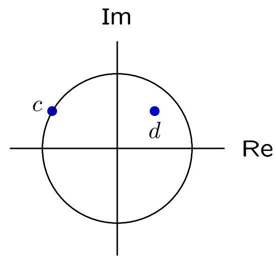
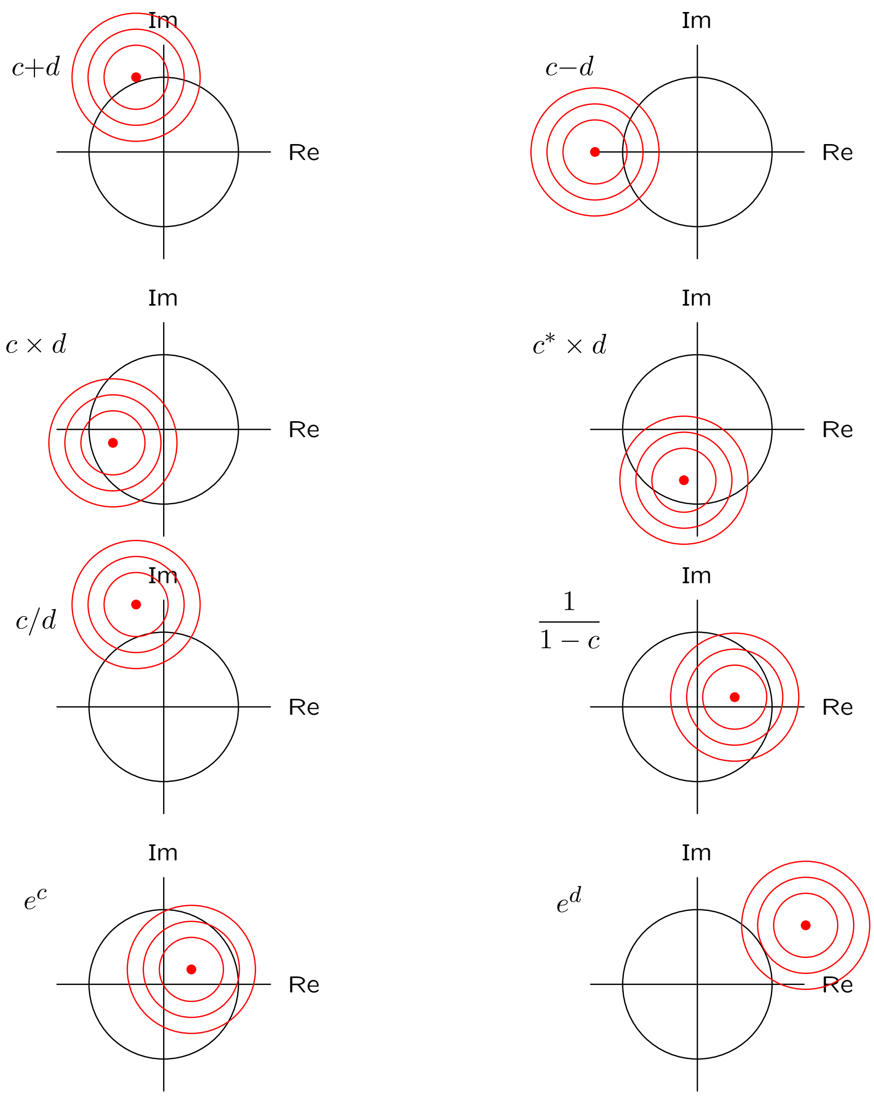
Complex Pairs

Let c and d represent the complex numbers shown by filled dots in the following diagram, where the real and imaginary parts of the complex numbers are shown on the horizontal and vertical axes, respectively, and the circle has a radius of 1.

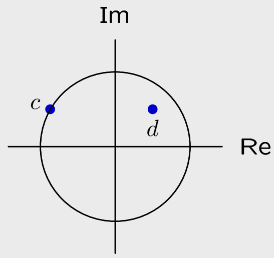


Below are eight complex-valued functions of c and d, each paired with a depiction of the complex plane demarked by the unit circle. Evaluate each expression and mark its value on the complex plane with a dot. Note that e represents Euler's number (2.71828...) and c^* represents the complex conjugate of c.



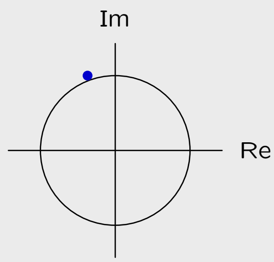
Complex Pairs

Start by estimating the numbers c and d. The number c has a magnitude very close to 1 and an angle that is about 30° less 180°. Both the real and imaginary parts of d are approximately 0.5.



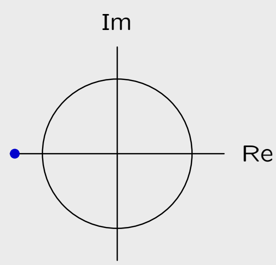
Part a: $c+d$

It is usually easiest to add complex numbers in their cartesian form. We can approximate the cartesian form of c as $-0.87+0.5j$ and the cartesian form of d as $0.5+j0.5$. Then the cartesion from of $c+d$ is approximately $-0.37+j$.



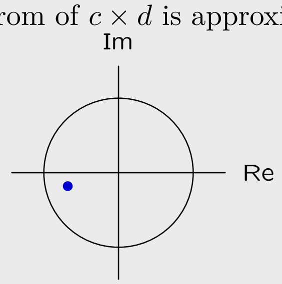
Part b: $c-d$

Using the same approximations as in part a, $c-d$ is approximately -1.4.



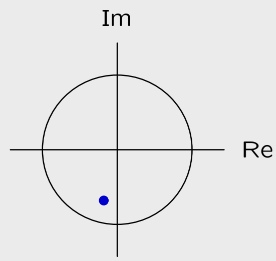
Part c: cd

It is usually easiest to multiply complex numbers in their polar form. If we approximate the polar form of c as $e^{j5\pi/6}$ and d as $0.7e^{j\pi/4}$, then the polar from of $c\times d$ is approximately $0.7e^{j13\pi/12}$.



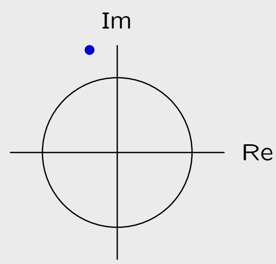
Part d: c^*d

Using the same approximations as in part c, the number c^* would be $e^{-j5\pi/6}$ and the product of c^* and d would be $0.7e^{-7\pi/12}$.



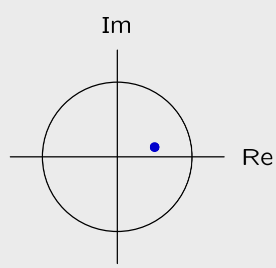
Part e: c/d

Division is also easiest in polar form. Using the approximations in part c, the number c/d is approximately $1/0.7\, e^{j7\pi/12}$.



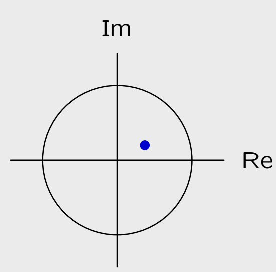
Part f: $\dfrac{1}{1-c}$

Start by computing the denominator using the cartesian form of c. Then $1-c$ is approximately $1.87-0.5j$. The magnitude of $1-c$ is then approximately 2, and the angle is slightly negative. The magnitude of the reciprocal of c is then approximately 0.5 and the angle is slightly positive.



Part g: e^c

Express c in cartesian form, which is approximately $-0.87+0.5j$. Then e^c is $e^{-0.87+0.5j} = e^{-0.87}e^{0.5j}$. $e^{-0.87}$ is approximately the reciprocal of e, which is about $1/3$. $e^{0.5j}$ is approximately $e^{j\pi/6}$, which is a small positive angle.



Part h: e^d

The constant $e^d = e^{0.5+0.5j}$. $e^{0.5}$ is the square root of e, which is about 1.6. $e^{0.5j}$ is a small positive angle (as shown in the previous part).