More Peaks and Valleys

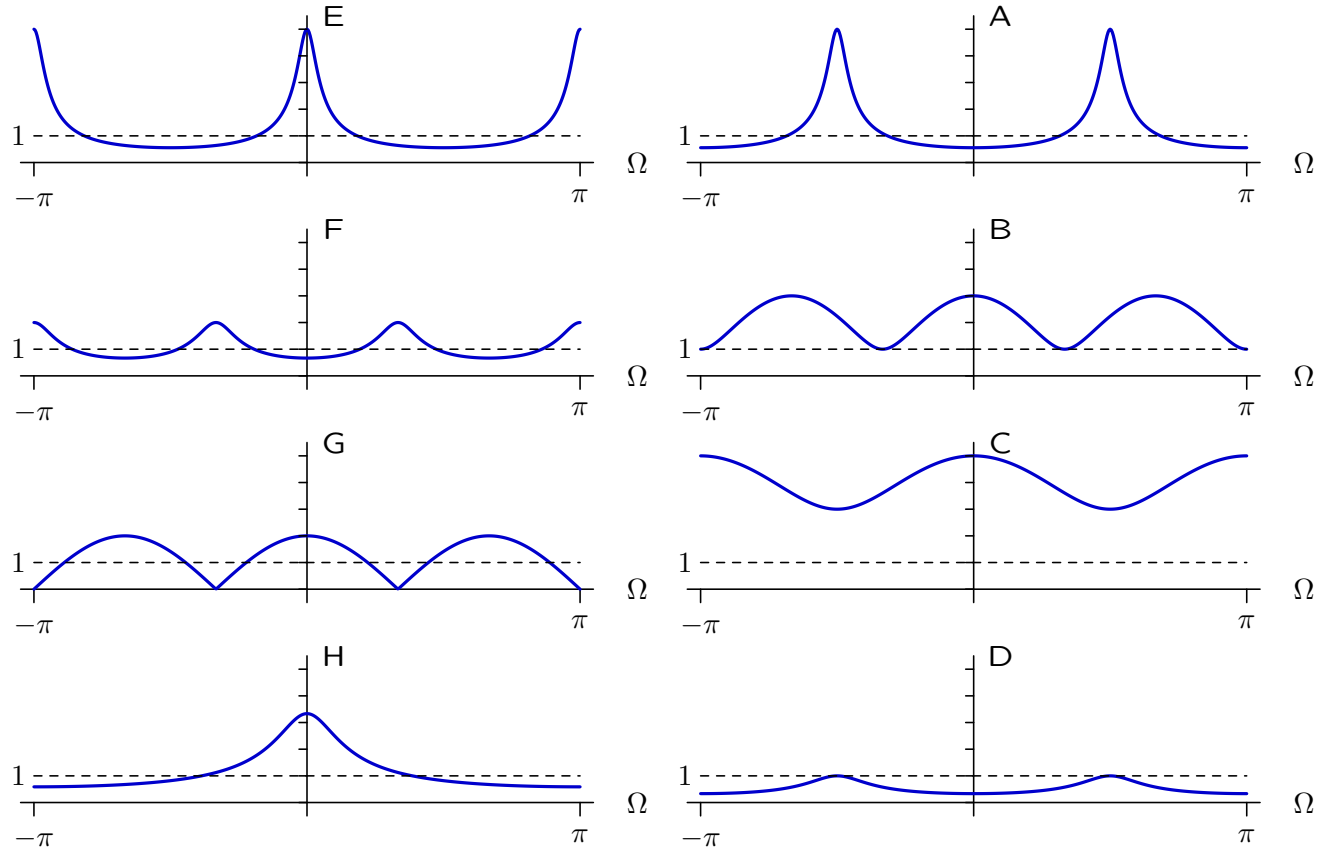
Each of the plots below shows the magnitude of the frequency response of discrete-time system that can be described by one of the following difference equations, where α and m are parameters.

Form A: $y[n] = x[n] + \alpha x[n-m]$
Form B: $y[n] = x[n] + \alpha y[n-m]$

Each row in the following table gives the parameters for one of the plots.
Write the letter of the corresponding plot in the right column of the table.

form	α	m	Frequency Response Enter A–H
A	1	3	**G**
A	4	2	**C**
A	2	3	**B**
B	0.7	1	**H**
B	-0.5	3	**F**
B	-0.8	2	**A**
B	-2	2	**D**
B	0.8	2	**E**

Note that the dashed line marks where the magnitude would be equal to 1.



Worksheet (intentionally blank)

Find the frequency responses from the difference equations.

Form A:

$$Y(\Omega) = X(\Omega) + \alpha e^{-j\Omega m} X(\Omega)$$

$$H_A(\Omega) = \frac{Y(\Omega)}{X(\Omega)} = 1 + \alpha e^{-j\Omega m}$$

Form B:

$$Y(\Omega) = X(\Omega) + \alpha e^{-j\Omega m} Y(\Omega)$$

$$H_B(\Omega) = \frac{Y(\Omega)}{X(\Omega)} = \frac{1}{1 - \alpha e^{-j\Omega m}}$$

Both of these frequency responses are periodic in Ω with period $2\pi/m$. Thus m tells us the number of periods of the frequency responses shown in the plots:

plot A: 2
plot B: 3
plot C: 2
plot D: 2
plot E: 2
plot F: 3
plot G: 3
plot H: 1

The magnitudes of the frequency responses are

$$\left|H_A(\Omega)\right| = \left|1 + \alpha\cos(\Omega m) - j\alpha\sin(\Omega_m)\right| = \sqrt{(1 + \alpha\cos(\Omega m))^2 + (\alpha\sin(\Omega_m))^2} = \sqrt{1 - 2\alpha\cos(\Omega m) + \alpha^2}$$

$$\left|H_B(\Omega)\right| = \frac{1}{\left|1 + \alpha\cos(\Omega m) + j\alpha\sin(\Omega_m)\right|} = \frac{1}{\sqrt{(1 + \alpha\cos(\Omega m))^2 + (\alpha\sin(\Omega_m))^2}} = \frac{1}{\sqrt{1 + 2\alpha\cos(\Omega m) + \alpha^2}}$$

The maximum magnitudes of both forms occur when $2\alpha\cos = -2\alpha$ and the minimum occurs when $2\alpha\cos = 2\alpha$.

	α	max	min
form A	α	$\left\|1 + \|\alpha\|\right\|$	$\left\|1 - \|\alpha\|\right\|$
form B	α	$\left\|\frac{1}{1 - \|\alpha\|}\right\|$	$\left\|\frac{1}{1 + \|\alpha\|}\right\|$
line 1	1	2	0
line 2	4	5	3.3
line 3	2	3	1
line 4	0.7	3.3	0.6
line 5	−0.5	2.0	0.7
line 6	−0.8	5	0.6
line 7	−2	1.0	0.3
line 8	0.8	5	0.6

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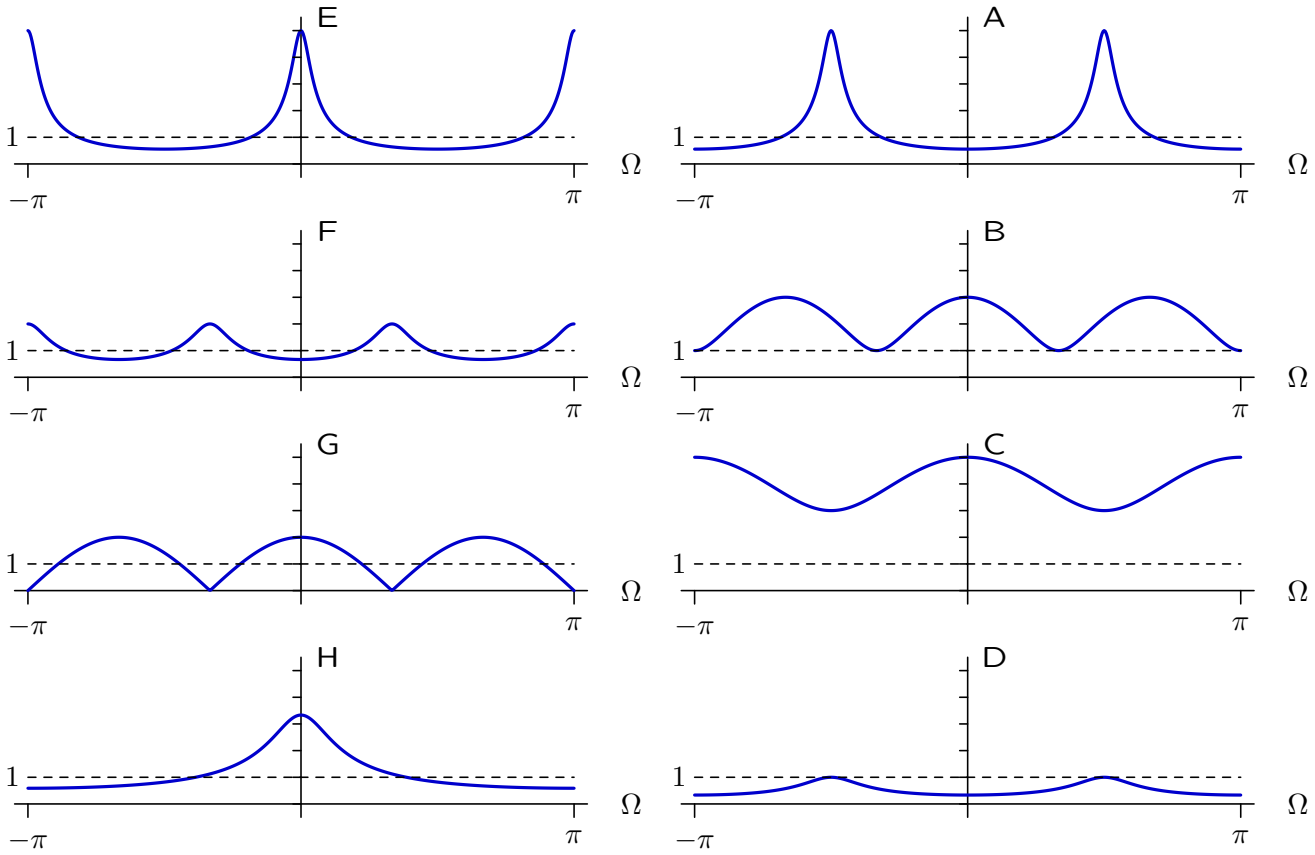

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