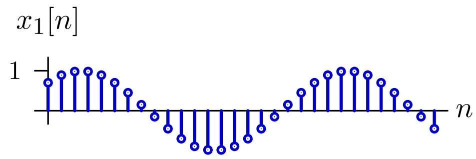
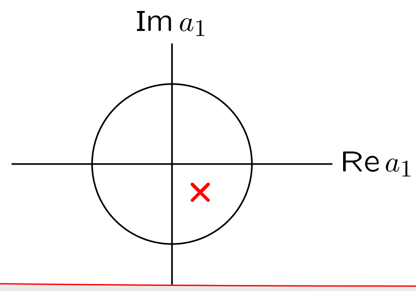
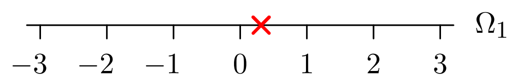
Describing Sinusoids

Part a. Let $x_1[n] = a_1 e^{j\Omega_1 n} + a_1^* e^{-j\Omega_1 n}$ as shown in the following figure.



Estimate a_1 and Ω_1, where Ω_1 is real-valued and a_1 may be complex. Place an "$\times$" on the complex plane shown below (where the circle has a radius of 1) to indicate the value of a_1. Also, place an "$\times$" on the number line shown below to indicate the value of Ω_1.





This signal can be written as

$$x_1[n] = a_1 e^{j\Omega_1 n} + a_1^* e^{-j\Omega_1 n} = a_1 e^{j\Omega_1 n} + (a_1 e^{j\Omega_1 n})^* = 2\,\mathrm{Re}\left(a_1 e^{j\Omega_1 n}\right) = \mathrm{Re}\left(2a_1 e^{j\Omega_1 n}\right) .$$

Thus this signal is the real part of a vector $2a_1$ in the complex plane whose angle increases by Ω_1 radians per sample n. Since the period of $x_1[n]$ is 20,

$$e^{j\Omega_1 n} = e^{j\Omega_1 (n+20)} = e^{j\Omega_1 n} e^{j20\Omega_1}$$

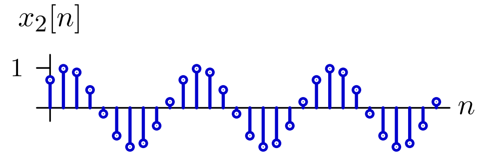
and the period of the complex exponential is 2π, it follows that $20\Omega_1 = 2\pi$, and

$$\Omega_1 = \frac{2\pi}{20} \approx 0.314 .$$

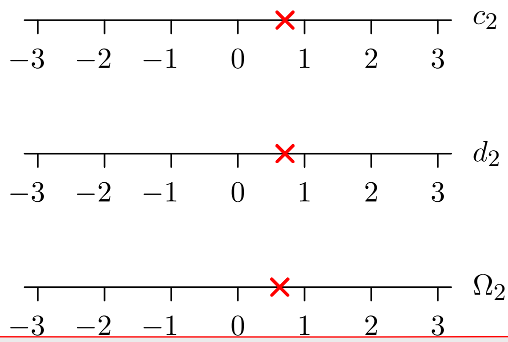
The peak amplitude is approximately 1, so $|2a_1| = 1$, and $|a_1| = 0.5$. The first peak of $x_1[n]$ occurs about 2.5 samples after $n = 0$, which corresponds to approximately $2.5/20 = 1/8$ of a cycle. So the angle of a_1 must start at approximately $-\pi/4$ and therefore $\angle a_1 \approx -\frac{\pi}{4}$. Thus

$$a_1 \approx \frac{1}{2} e^{-j\pi/4} .$$

Part b. Let $x_2[n] = c_2 \cos(\Omega_2 n) + d_2 \sin(\Omega_2 n)$ as shown in the following figure.



Estimate the real-valued constants c_2, d_2, and Ω_2. Place an "$\times$" on each of the number lines shown below to indicate these values.



We can express $x_2[n] = c_2 \cos \Omega_2 n + d_2 \sin \Omega_2 n$ as

$$\begin{aligned} x_2[n] &= c_2 \left(\frac{e^{j\Omega_2 n} + e^{-j\Omega_2 n}}{2} \right) + d_2 \left(\frac{e^{j\Omega_2 n} - e^{-j\Omega_2 n}}{2j} \right) \\ &= \frac{1}{2}(c_2 - jd_2)e^{j\Omega_2 n} + \frac{1}{2}(c_2 + jd_2)e^{-j\Omega_2 n} \\ &= \mathrm{Re}\left((c_2 - jd_2)e^{j\Omega_2 n} \right) \end{aligned}$$

Since the period of $x_2[n]$ is 10,

$$e^{j\Omega_2 n} = e^{j\Omega_2 (n+10)} = e^{j\Omega_2 n} e^{j10\Omega_2}$$

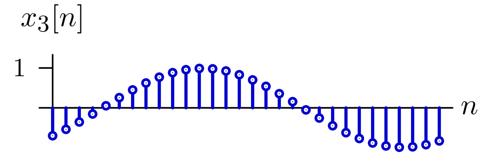
and the period of the complex exponential is 2π, it follows that $10\Omega_2 = 2\pi$, and

$$\Omega_2 = \frac{2\pi}{10} \approx 0.628 \,.$$

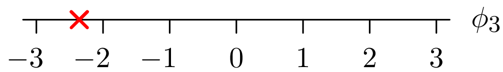
The peak amplitude of $x_2[n]$ is approximately 1, so $|c_2 - jd_2| = \sqrt{c_2^2 + d_2^2} = 1$. The first peak of $x_2[n]$ occurs at about 1/8 of a cycle. So the angle of $c_2 - jd_2$ must be approximately $-\pi/4$. Therefore

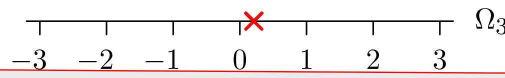
$$c_2 \approx d_2 \approx \frac{1}{\sqrt{2}} \approx 0.7 \,.$$

Part c. Let $x_3[n] = \cos(\Omega_3 n + \phi_3)$ as shown in the following figure.



Estimate the real-valued constants ϕ_3 and Ω_3. Place an "$\times$" on each of the number lines shown below to indicate these values.





The period of $x_3[n]$ is approximately 30, so

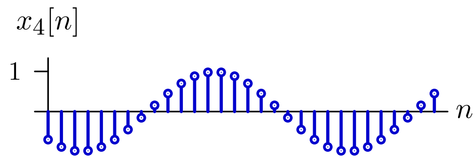
$$\Omega_3 \approx \frac{2\pi}{30} \approx 0.2\,.$$

The signal peaks at $n = 11$, so

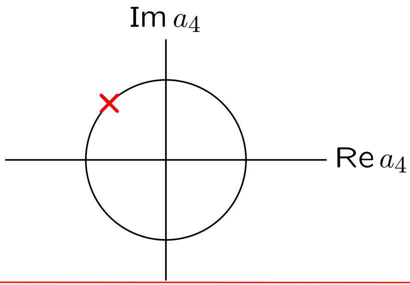
$$x_3[n_0] = \cos(\Omega_3 n_0 + \phi_3) \approx 1$$

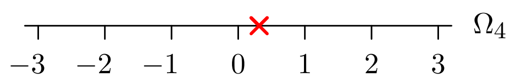
so $\phi_3 \approx -11\Omega_3 = 22\pi/30 \approx -2.3$.

Part d. Let $x_4[n] = \text{Re}\left(a_4 e^{j\Omega_4 n}\right)$ as shown in the following figure.



Estimate a_4 and Ω_4, where Ω_4 is real-valued and a_4 may be complex. Place an "×" on the complex plane shown below (where the circle has a radius of 1) to indicate the value of a_4. Also, place an "×" on the number line shown below to indicate the value of Ω_4.





The period of $x_4[n]$ is 20, so

$$\Omega_4 = \frac{2\pi}{20} \approx 0.314\,.$$

The peak amplitude is approximately 1, so $|a_4| \approx 1$. The first peak of $x_4[n]$ occurs near $n = 12.5$, which corresponds to approximately $12.5/20 = 5/8$ of a cycle, so that the angle of a_4 must be approximately $\angle a_4 \approx -\frac{5}{4}\pi$.