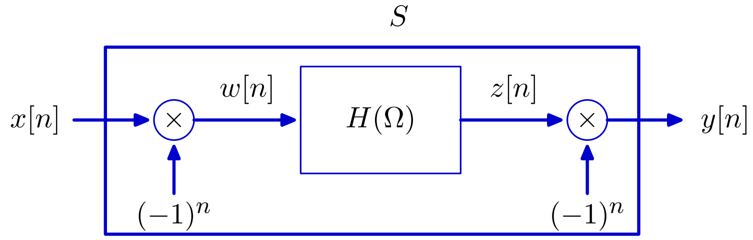
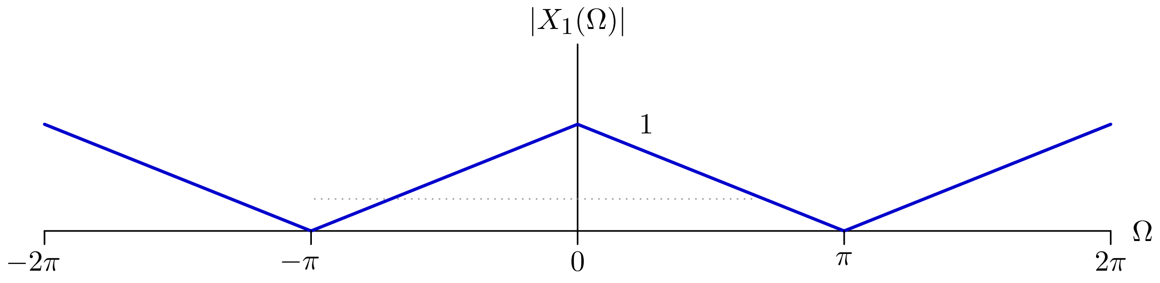
Filtering Consider a system whose input $x[n]$ and output $y[n]$ are related as shown in the box labeled S below



where $H(\Omega)$ represents a linear, time-invariant system with the following frequency response.

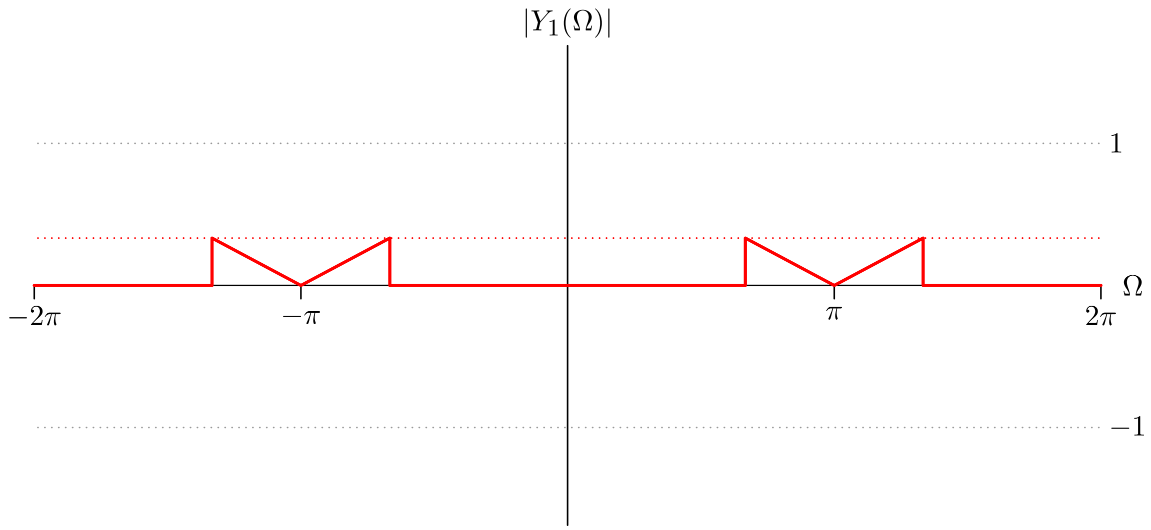
$$H(\Omega) = \begin{cases} 1 & \text{if } 0 \le |\Omega| \le \frac{\pi}{3} \\ 0 & \text{if } \frac{\pi}{3} < |\Omega| < \pi \\ H(\Omega \bmod 2\pi) & \text{otherwise} \end{cases}$$

Part a. Let $Y_1(\Omega)$ represent the DTFT of the output $y_1[n]$ that results when the DTFT of the input $x_1[n]$ has the following form:



Note that $X_1(\Omega)$ is periodic in Ω with period 2π.

Sketch $|Y_1(\Omega)|$ on the axes below. Label the important points of your sketch.



Part b. Is the system S linear?

Briefly explain.

The transformation from $x[n]$ to $y[n]$ can be expressed as a convolution with $g[n] = (-1)^n h[n]$ (see below). Convolution is a linear operator in the sense that $(ax_1) * h + (bx_2) * h = (ax_1 + bx_2) * h$. Therefore the system S is linear.

$$\begin{aligned}
y[n] &= (-1)^n z[n] \\
&= (-1)^n (w * h)[n] \\
&= (-1)^n \sum_{m=-\infty}^{\infty} w[m]h[n-m] \\
&= (-1)^n \sum_{m=-\infty}^{\infty} (-1)^m x[m]h[n-m] \\
&= \sum_{m=-\infty}^{\infty} x[m](-1)^{n+m}h[n-m] \\
&= \sum_{m=-\infty}^{\infty} x[m](-1)^{n-m}h[n-m] \\
&= \sum_{m=-\infty}^{\infty} x[m]g[n-m]
\end{aligned}$$

where $g[n] = (-1)^n h[n]$.

Part c. Is the system S time-invariant?

Briefly explain.

The transformation from $x[n]$ to $y[n]$ can be expressed as a convolution with $g[n] = (-1)^n h[n]$ (see above). Shifting the input to a convolution by n_0 time steps simply shifts the output of the convolution by n_0 time steps. Therefore the system S is time-invariant.

Part d. Can system S be regarded as a lowpass filter or as a highpass filter or as a bandpass filter? If so, describe which and specify the cutoff frequency or frequencies.

Because the system is LTI, it can be regarded as a filter. The passband includes $\Omega = \pi$, therefore it is highpass. The cutoff frequency of the highpass filter is $2\pi/3$.