Harmonic Aliasing

Consider three periodic signals:

- $x_1(t)$ with period $T_1 = \frac{1}{11}$ seconds
- $x_2(t)$ with period $T_2 = \frac{1}{12}$ seconds
- $x_3(t)$ with period $T_3 = \frac{1}{13}$ seconds

Each of these signals contains a fundamental component (at frequency ω given by 2π divided by its period) as well as harmonics 2, 3, 4, and 5, but not other frequencies.

Each of these continuous-time signals is sampled 40 times per second to generate corresponding discrete-time signals:

- $x_1[n] = x_1(n/40)$
- $x_2[n] = x_2(n/40)$
- $x_3[n] = x_3(n/40)$

Each of these discrete-time signals contains exactly five discrete-time sinusoidal components with frequencies in the range $0 \le \Omega \le \pi$.

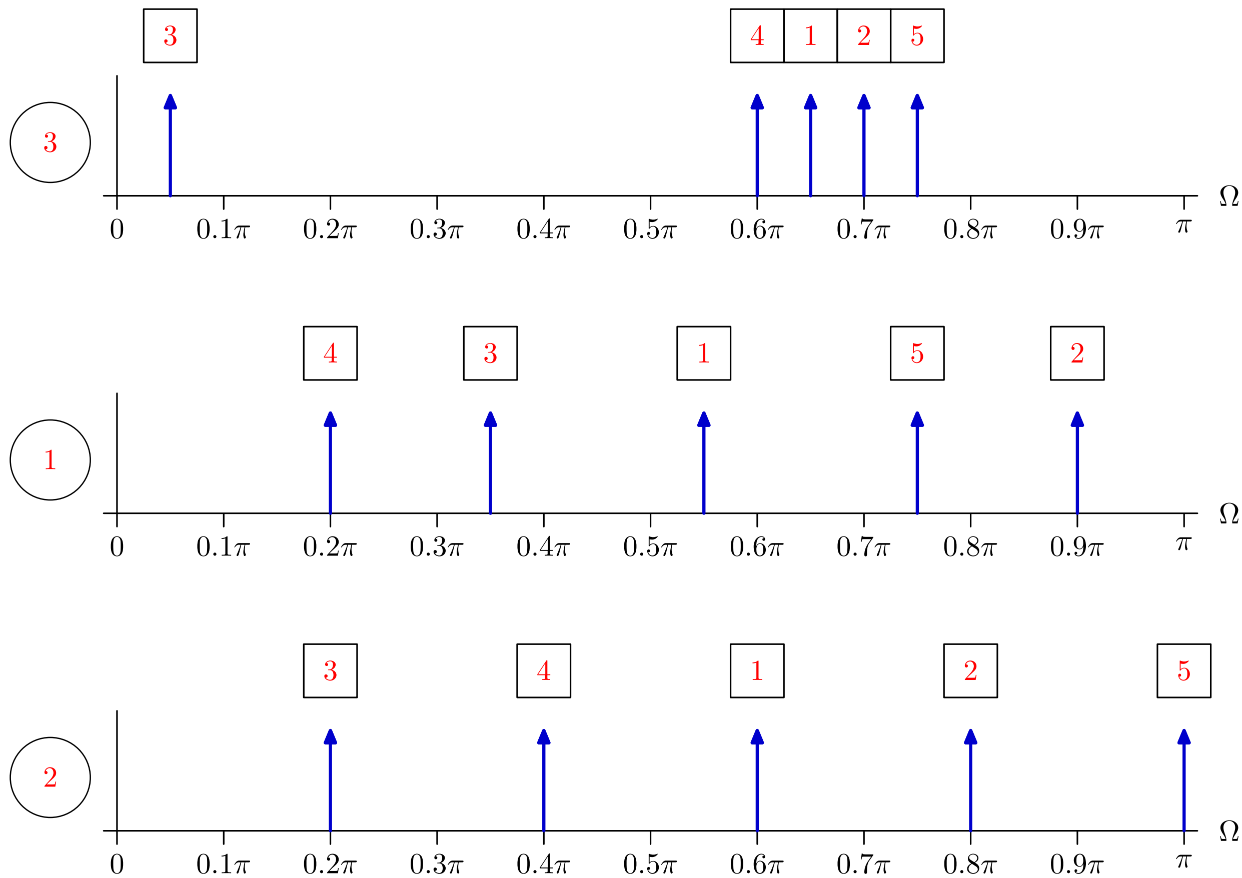
Each plot on the facing page shows the frequencies found in one of these DT signals. In the circle next to each plot, write the name of the corresponding signal (either x_1, x_2, or x_3).

Each of the DT frequency components is associated with one of the harmonics in the original CT signal. For each DT frequency, write the number of the associated CT harmonic (1-5) in the box above that frequency. If none of these harmonics could have produced a given frequency, enter an **X** in its box instead.

The fundamental component of the i^{th} CT signal $x_i(t)$ has the form $e^{j2\pi t/T_i}$. Sampling this signal 40 times per second results in a DT signal of the form $e^{j2\pi n/(40T_i)}$, which has a DT frequency of $2\pi/(40T_i) = \pi/(20T_i)$. Thus the frequency of the fundamental component of $x_1[n]$ is $11\pi/20$.

The second harmonic frequency is $22\pi/20$, which is greater than π. But its alias at $2\pi - 22\pi/20 = 18\pi/20$ is in the range $[0, \pi]$. Thus the second harmonic of $x_1(t)$ generates a DT frequency of $18\pi/20$. The third harmonic frequency is $33\pi/20$, which is greater than π. It aliases to $2\pi - 33\pi/20 = 7\pi/20$ is in the range $[0, \pi]$. The fourth harmonic frequency is $44\pi/20$, which is greater than 2π. It aliases to $44\pi/20 - 2\pi = 4\pi/20$ is in the range $[0, \pi]$. The fifth harmonic frequency is $66\pi/20$, which is greater than 2π. It aliases to $55\pi/20 - 2\pi = 15\pi/20$ is in the range $[0, \pi]$.

These results for $x_1[n]$ and analogous results for $x_2[n]$ and $x_3[n]$ are summarized in the table on the next page.



signal	harmonic number	harmonic frequency	base-band alias $[0, \pi]$
x_1	1	$11\pi/20$	$11\pi/20$
x_1	2	$22\pi/20$	$2\pi - 22\pi/20 = 18\pi/20$
x_1	3	$33\pi/20$	$2\pi - 33\pi/20 = 7\pi/20$
x_1	4	$44\pi/20$	$44\pi/20 - 2\pi = 4\pi/20$
x_1	5	$55\pi/20$	$55\pi/20 - 2\pi = 15\pi/20$
x_2	1	$12\pi/20$	$12\pi/20$
x_2	2	$24\pi/20$	$2\pi - 24\pi/20 = 16\pi/20$
x_2	3	$36\pi/20$	$2\pi - 36\pi/20 = 4\pi/20$
x_2	4	$48\pi/20$	$48\pi/20 - 2\pi = 8\pi/20$
x_2	5	$60\pi/20$	$60\pi/20 - 2\pi = 20\pi/20$
x_3	1	$13\pi/20$	$13\pi/20$
x_3	2	$26\pi/20$	$2\pi - 26\pi/20 = 14\pi/20$
x_3	3	$39\pi/20$	$2\pi - 39\pi/20 = 1\pi/20$
x_3	4	$52\pi/20$	$52\pi/20 - 2\pi = 12\pi/20$
x_3	5	$65\pi/20$	$4\pi - 65\pi/20 = 15\pi/20$