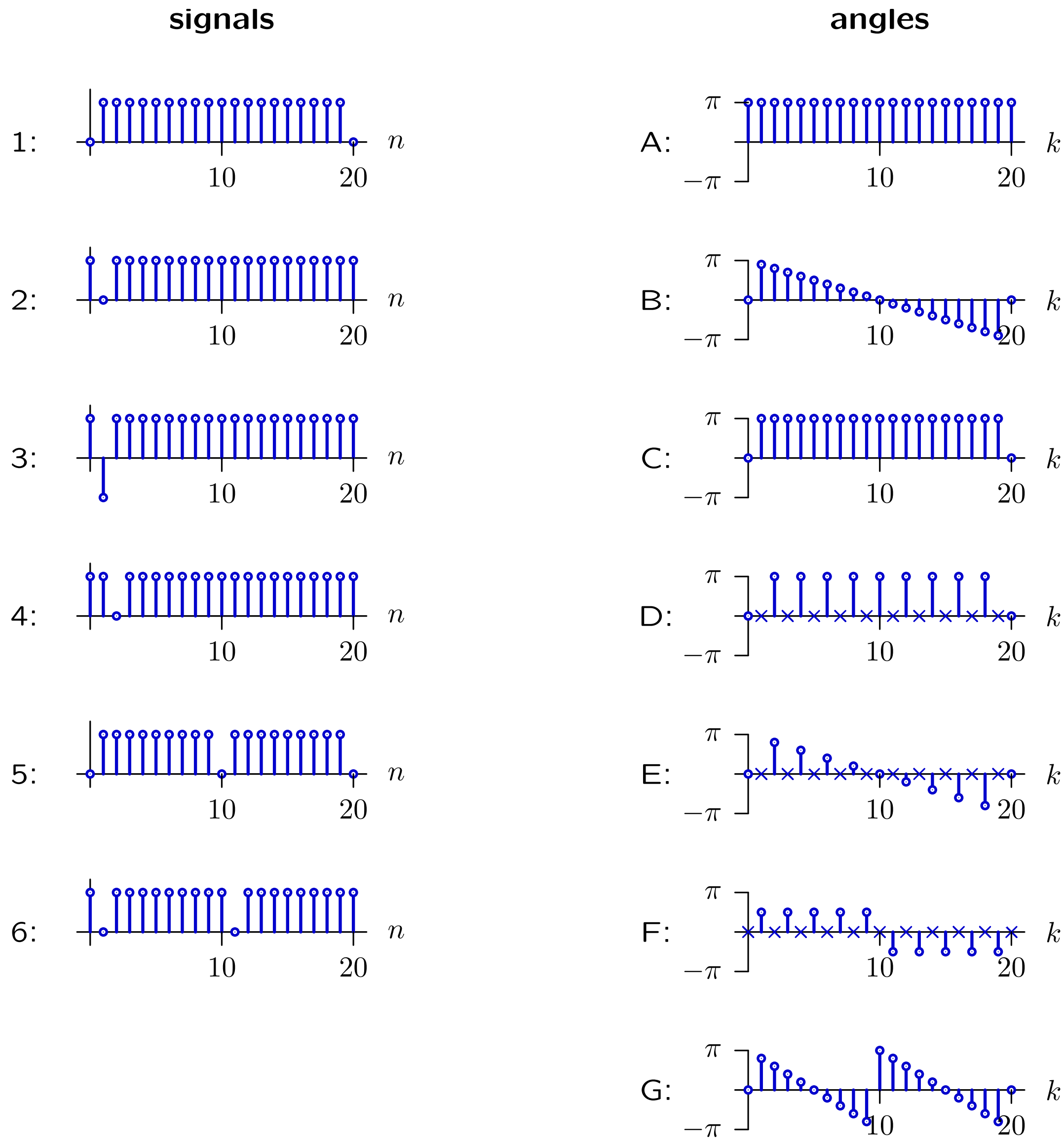
Find the Phase in 1D

The signals shown in the left column are periodic in n with period $N = 20$. The functions shown in the right column show the angles of Fourier series. For each signal, indicate which (if any) of the angle plots represents the angles of that signal's Fourier series coefficients (where an "X" indicates an undefined phase).



Which angle plot matches with each signal?

For all of these questions, the intention is not to directly apply the analysis formula, but rather to think about properties, symmetries, and well-known transform pairs, to find a solution more easily.

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For $x_1[\cdot]$, importantly, notice that $x_1[n] = 1 - \delta[n]$. By linearity, then $X_1[k] = \delta[k] - \frac{1}{20}$. Thus:

$$X_1[k] = \begin{cases} 19/20 & \text{if } k = 0 \\ 1/20 & \text{otherwise} \end{cases}$$

Thus, the phase is:

$$\angle X_1[k] = \begin{cases} 0 & \text{if } k = 0 \\ \pi & \text{otherwise} \end{cases}$$

which corresponds to graph **C**.

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For $x_2[\cdot]$, notice that $x_2[n] = x_1[n-1]$, so by the time shift property, $X_2[k] = e^{-j\frac{2\pi k}{20}} X_1[k]$. Thus, the phase $\angle X_2[k] = \angle X_1[k] - \frac{2\pi k}{20}$, which corresponds to graph **B**.

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Here, we can apply similar reasoning to $x_2[\cdot]$. We can look at $x_3[n]$ as a time-shifted version of a variant of $x_1[\cdot]$:

$x_3[n] = x_1'[n-1]$ where $x_1'[n] = 1 - 2\delta[n]$.

Since $\angle X_1'[k] = \angle X_1[k]$ and $\angle X_3[k] = \angle X_1'[k] - \frac{2\pi k}{20}$, we again end up at graph **B**.

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For $x_4[\cdot]$, we can again leverage the time-shift property:

$x_4[n] = x_1[n-2] \Rightarrow X_4[k] = e^{-j\frac{4\pi k}{20}} X_1[k]$

Thus, $\angle X_4[k] = \angle X_1[k] - \frac{4\pi k}{20}$, which corresponds to graph **G**.

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$x_5[\cdot]$ looks like a version of $x_1[\cdot]$ that has been compressed by a factor of 2 in the time domain, so we can anticipate $X_5[\cdot]$ to be a stretched version of $X_1[\cdot]$ in the frequency domain, stretched by a factor of 2. The matching graph is graph **D**.

Alternatively, we have a couple of ways that we could approach this analytically. For one, we could notice that this signal actually has a period of $N = 10$ (not 20!) and perform our analysis there, and then think about the consequences of analyzing such a signal with $N = 20$.

Or we can take a more direct approach. Note that $x_5[n] = 1 - \delta[n] - \delta[n-10]$.

Thus, by linearity, $X_5[k] = \delta[k] - 1/20 - (1/20)e^{\frac{-j2\pi 10k}{20}} = \delta[k] - \frac{(1+(-1)^k)}{20}$

Note that that second term alternates between being $-1/10$ and 0. So have:

$$X_5[k] = \begin{cases} 19/20 & \text{if } k = 0 \\ -1/10 & \text{if } k \text{ is even and } k \neq 0 \\ 0 & \text{otherwise} \end{cases}$$

Thus,

$$\angle X_5[k] = \begin{cases} 0 & \text{if } k = 0 \\ \pi & \text{if } k \text{ is even and } k \neq 0 \\ \text{undefined} & \text{otherwise} \end{cases}$$

which, again, matches graph **D**.

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$x_6[\cdot]$ looks like a shifted version of $x_5[\cdot]$: $x_6[n] = x_5[n-1]$. Thus, $X_6[k] = e^{-j\frac{2\pi k}{20}} X_5[k]$, so $\angle X_6[k] = \angle X_5[k] - \frac{2\pi k}{20}$, which corresponds to graph **E**.