Two-Dimensional Convolution
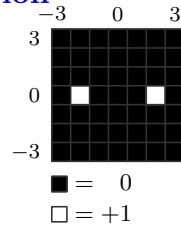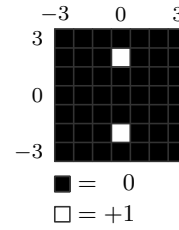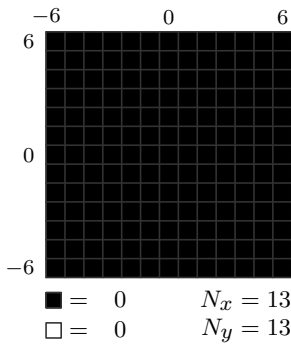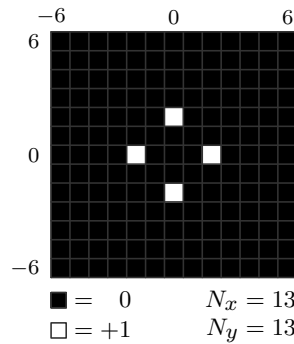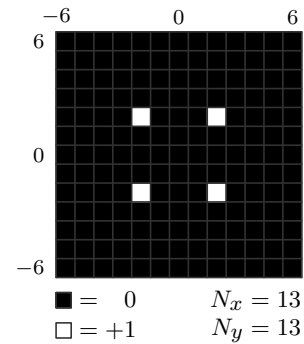
A

B

Part a. Which of the three numbered options below corresponds to the conventional (non-circular) convolution of A and B above? Briefly explain your reasoning.

$N_x = 13$
$N_y = 13$
№ 1

$N_x = 13$
$N_y = 13$
№ 2

$N_x = 13$
$N_y = 13$
№ 3

Convolving the left white pixel of A with the two white pixels in B produces white pixels at $(r, c) = (-2, 2)$ and $(-2, -2)$. Convolving the right white pixel of A with the two white pixels in B produces white pixels at $(r, c) = (2, 2)$ and $(2, -2)$. The answer is the sum of these parts, which is the patterns shown in Number 3.
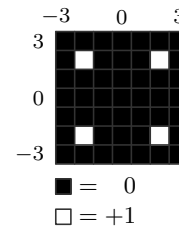
Part b. Which of the three numbered options below corresponds to the circular convolution of A and B above with an analysis window that is 7 pixels wide and 7 pixels high? Briefly explain your reasoning.
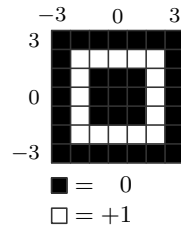
№ 4

№ 5
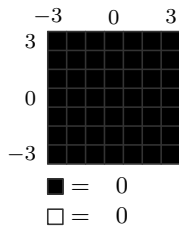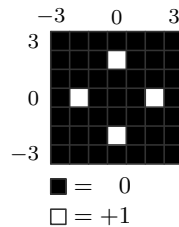
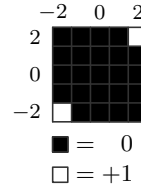**№ 6**

Conventional convolution produces a result that is entire contained in the region $-2 \le r \le 2$ and $-2 \le c \le 2$. Therfore the circular convolution results in the same patterns as conventional convolution, and the answer is Number 6.
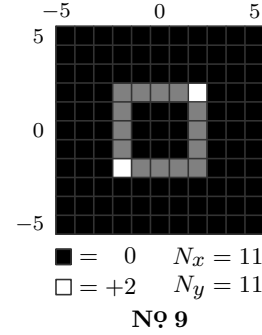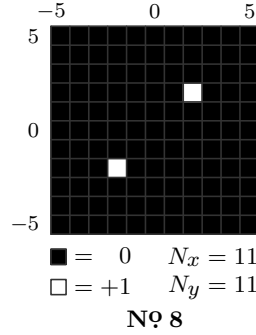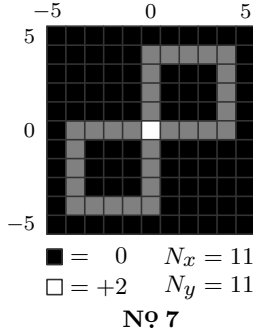
C   **D**

Part c. Which of the three numbered options below corresponds to the conventional (non-circular) convolution of C and D? Briefly explain your reasoning.

№ 7   **№ 8**   **№ 9**

Convolving pattern C with the white pixel at $(r, c) = (2, 2)$ generates a square with the same size as that in C but whose lower left corner is at the origin. Similarly, convolving pattern C with the white pixel at $(r, c) = (-2, -2)$ generates a square with the same size as that in C but whose upper right corner is at the origin. The sum of these results has a bright pixel at the origin and two squares as shown in Number 7, which is the answer.

Part d. Which of the three numbered options below corresponds to the circular convolution of C and D above with an analysis window that is 7 pixels wide and 7 pixels high? Briefly explain your reasoning.

№ 10   **№ 11**   **№ 12**
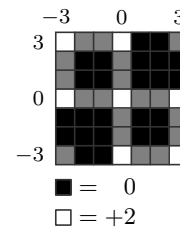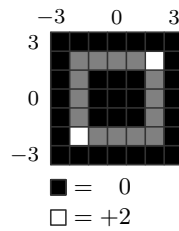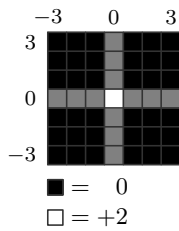
Circular convolution results in aliasing of the result from part c (i.e., Number 7) to the region $-2 \leq r \leq 2$ and $-2 \leq c \leq 2$. This displaces the pixels with $r = 4$ to $r = 4 - 7 = -3$. Similarly, pixels with $r = -4$ are displaced to $r = -4 + 7 = 3$; pixels with $c = 4$ are displaced to $c = 4 - 7 = -2$; and pixels with $c = -4$ are displaced to $c = -4 + 7 = 3$. The resulting pattern is shown in Number 12.