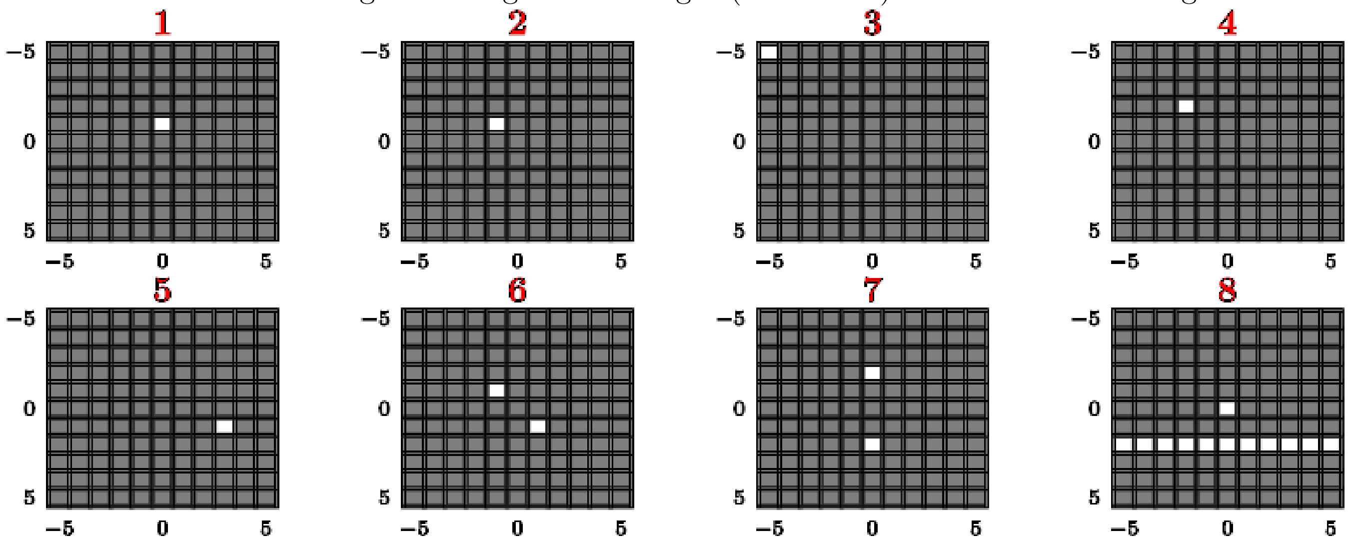
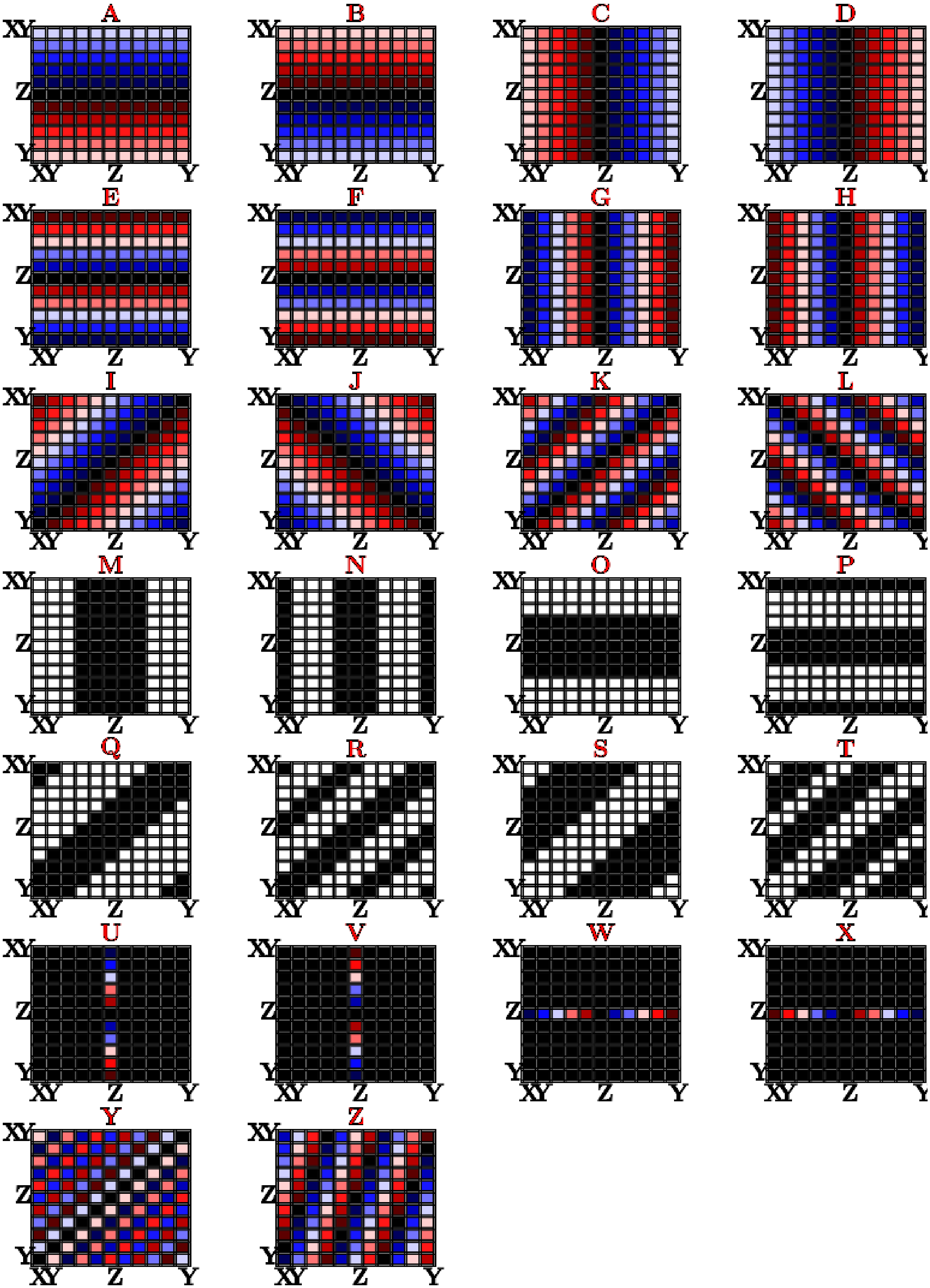
2D Phases

Each of the following panels illustrates an 11×11 pixel image where gray represents a value of 0 and white represents a value of 1. Let r and c denote the row and column, respectively, with r increasing downward and c increasing to the right. The origin ($r = c = 0$) of each of these images is in its center.



For each image above, determine which of the panels on the following page represents the angle of the (11×11) DFT of that signal, where black represents 0, white represents π and $-\pi$, red represents positive angles, and blue represents negative angles (with $\pi/2$ represented by solid red, and $-\pi/2$ represented by solid blue). The origin of each of these panels is in the center of the panel, k_r increases downward, and k_c increases to the right. For reference, image A is a gradient in the vertical direction, starting at a value of $-\pi$ in the first row and increasing linearly as k_r increases. The bottom row of image A has the value π.


A
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T
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V
W
X
Y
Z
XY
Z
Y

1. The single pixel at `(r,c)=(-1,0)` will generate a transform with constant magnitude with an angle that is constant with `k_c` and increases with `k_r`. The total change in phase from top to bottom of the transform will be 2π. Therefore the top half of the angle plot will be negative (blue) and the bottom half will be positive (red). The answer is panel A.

2. The single non-zero pixel again generates a transform with constant magnitude, but now lines of contant angle are diagonally oriented. Pixels to the upper-left part of the diagonal defined by $k_x = -k_y$ will be negative (blue) and those to the lower-right will be positive (red). Notice however that the frequency at $(k_x, k_y) = (-\pi, -\pi)$ has magnitude $\Big|(k_x, k_y)\Big| = \sqrt{2}\pi$. Therefore the angle in the upper left corner will be red and that in the lower right corner will be blue. Thus the answer is I.

3. The single non-zero pixel again geneates a transform with constant magnitude, but the position at $r = c = -5$ corresponds to the maximum frequency: $2\pi 5/11$. The angle pattern is symmetric about $k_x = k_y$. Therefore the answer is Y.

4. The single non-zero pixel again generates a transform with constant magnitude, and lines of contant angle are diagonally oriented. Pixels to the upper-left part of the diagonal defined by $k_x = -k_y$ will be negative (blue) and those to the lower-right will be positive (red). Notice however that there will now be twice as many phase contours as there were for image 2. Therefore the answer is K.

5. The single non-zero pixel again generates a transform with constant magnitude, and lines of contant angle are diagonally oriented. The position of this pixel at $(r, c) = (1, 3)$ will generate a total phase change of 2π in the vertical direction and 6π in the horizontal direction. Therefore the answer is Z.

6. Non-zero pixels at $(r, c) = (1, 1)$ and $(-1, -1)$ will generate a cosine pattern of phase with a total of 2π radians in both the horizontal and vertical directions. Since the transform is real, the phases are either 0 or $\pm\pi$. Therefore the answer is Q.

7. Non-zero pixels at $(r, c) = (2, 0)$ and $(-2, 0)$ will generate a constant pattern of phase in the horizontal direction and a cosine pattern of phase with a total of 4π radians in the vertical direction. Since the transform is real, the phases are either 0 or $\pm\pi$. Therefore the answer is P.

8. The white line at $r = 2$ will generate a vertical line in the frequency domain. Phase along that line will change by 4π from the top to bottom of the transform. The phase will be zero at the origin, and tend toward blue as k_r increases from 0 to 1 The single white pixel at the origin will generate a constant offset. Therefore the answer is U.

answer = AIYKZQPU