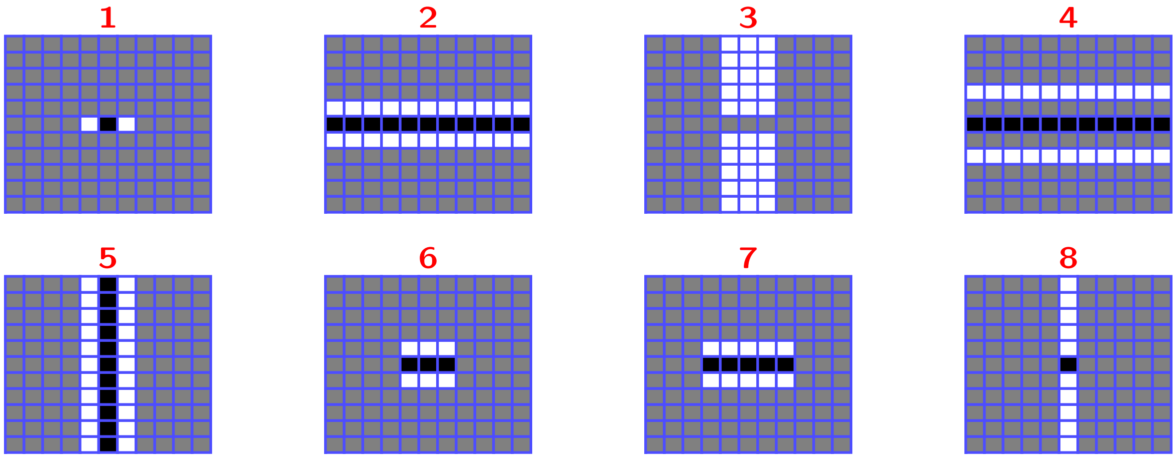
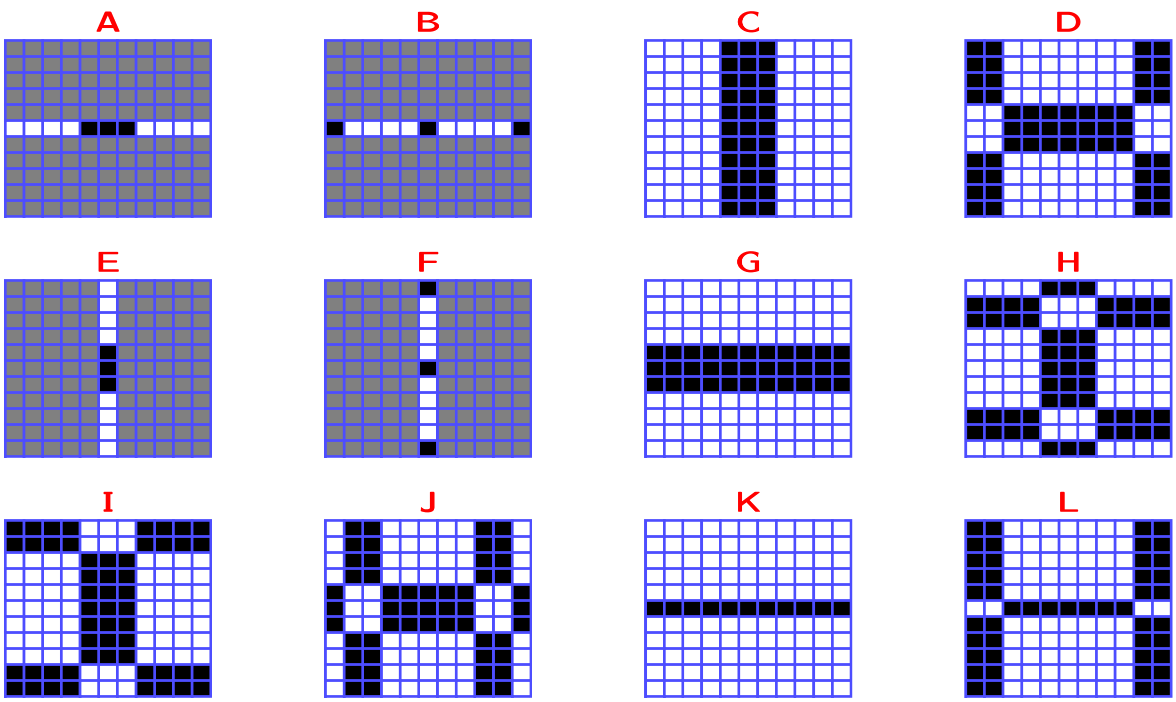
## Find the Phase in 2D

Panels 1-8 illustrate eight 2D discrete-time signals. Each signal has 11 rows and 11 columns. Black represents $-1$, grey represents 0 and white represents 1. The origin of each of these panels is in the center of the panel, $r$ increases downward, and $c$ increases to the right.




For each signal, determine which of the following panels represents the angle of the $(11 \times 11)$ DFT of that signal, where black represents 0, white represents $\pi$, and gray represents samples for which the angle is indeterminate (because the corresponding magnitude is 0). The origin of each of these panels is in the center of the panel, $k_r$ increases downward, and $k_c$ increases to the right.




Which angle plot matches with each signal?

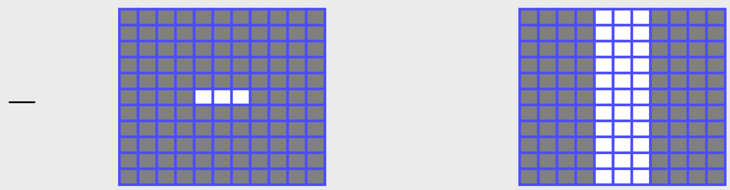
1. The Fourier transform of image 1 is

$$\frac{-1+2\cos(2\pi k_c/11)}{121}$$

There is no dependence on $k_r$ therefore the same pattern results for all rows in the phase plot. The transform will be positive if $2\cos(2\pi k_c/11) > 1$, i.e., if $k_c$ is $-1, 0$, or $1$. Therefore the answer is C.

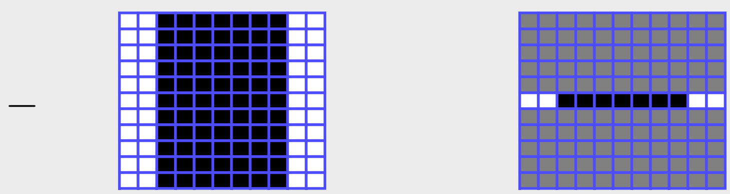
2. The brightness in each row of image 2 is constant. Therefore its Fourier transform has a single non-zero column at $c = 0$. The non-zero column in the Fourier transform of image 2 has the same structure as each row in the Fourier transform (C) of image 1. Therefore the answer is E.

3. Image 3 can be constructed by subtracting a rectangle of height 1 and width 3 from a rectangle of height 11 and width 3, as shown below.



The transform of the smaller rectangle is similar to the transform of image 1 except that the region with zero phase is wider (due to the positive DC term). Instead of being 3 pixels wide (as in C), the transform of a rectangle of width 3 is now 7 pixels wide. See the right panel of the following figure.

Since each column in the large rectangle is constant, the transform of that rectangle has a single non-zero row at $r = 0$. Thus the white rectangle of height 11 and width 3 is positive if $r = 0$ and $|c| \leq 3$, negative if $r = 0$ and $|c| > 3$, and zero if $r \neq 0$. See the left panel of the following figure.



Combining these images leads to 9 interesting regions. The DC value of the large rectangle is 11 times larger than that of the small rectangle. Similarly all of the pixels in the center region ($r = 0, -3 \leq c \leq 3$) are larger in the left image, so the subtraction leads to a positive magnitude with a phase of 0.

The regions above and below the central image have magnitudes of 0 in the left and positive magnitudes in the right. The difference is therefore negative and the angle is $\pi$.

The regions left and right of the central image are both negative, but that of the left image is more negative (by a factor of 11) than that in the right image. Subtraction therefore leads to a negative magnitude and an angle of $\pi$.

The remaining regions have zero magnitude in the left image and negative magnitudes in the right image. Subtraction therefore leads to a positive magnitude and an angle of 0.

The resulting answer is L.

4. The transform of 4 is similar to the transform of 2 except that there are now 2 periods of the cosine along the vertical axis instead of one. The answer is therefore F.

5. The transform of 5 is a $\pi/2$ rotation of that for image 2. The answer is therefore A.

6. Image 6 is the product of image 2 times a white rectangle that is 3 pixels wide and 11 pixels high. Therefore the transform of 6 is the transform of 2 convolved with the transform of the white rectangle. The transform of 2 is E. The transform of the white rectangle is given in the solution for image 3. Convolving those patterns generates panel D.

7. The transform of 7 is similar to that of 6 except that the pattern in E is now convolved with a narrower vertical stripe. The result is the pattern in J.

8. Image 8 is a vertical white stripe minus 2 times a delta function at the origin. The transform oa a vertical white stripe is a horizontal white stripe with height of 1/11. Subtracting a DC value of 1/121 for all the pixels has little effect on the amplitudes The result is positive (i.e., phase is zer0) for the horizontal stripe through $r = 0$ and negative (i.e., phase is $\pi$) elsewhere. The result is K.

answer = CELFADJK