6.300: Signal Processing

Systems: Linearity and Time-Invariance

Additivity: $x_1[n] + x_2[n] \rightarrow \boxed{\mathcal{H}} \rightarrow y_1[n] + y_2[n]$

Homogeneity: $cx[n] \rightarrow \boxed{\mathcal{H}} \rightarrow cy[n]$ for any constant c

Linearity: $c_1x_1[n] + c_2x_2[n] \rightarrow \boxed{\mathcal{H}} \rightarrow c_1y_1[n] + c_2y_2[n]$
Together, additivity and homogeneity imply linearity.

Time-Invariance: $x[n-n_0] \rightarrow \boxed{\mathcal{H}} \rightarrow y[n-n_0]$ for all n_0

March 10, 2026

Agenda for Recitation

- System properties: Linearity and time-invariance

What questions do you have from lecture?

All 6.300 students are invited to join our staff meeting next **Monday, March 16** in **38-466** (Jackson Room) from **5:00 p.m.** to around 6:00 p.m. Please join us! We'd like to hear your thoughts on 6.300 thus far.

Representations of LTI Systems

Linearity (L) and **time-invariance (TI)** are properties that greatly simplify the analysis and design of systems.

In this class, we will consider three equivalent representations of **discrete-time LTI systems**.

- linear, constant-coefficient difference equation
- unit-sample response $h[n]$
- frequency response $H(\Omega)$

At the same time, we will also consider three equivalent representations of **continuous-time LTI systems**.

- linear, constant-coefficient differential equation
- impulse response $h(t)$
- frequency response $H(\omega)$

Today, we will practice determining if a system defined by a difference (DT) or differential (CT) equation is **LTI**.

Discrete-Time Systems

Determine if each system is linear and time-invariant.
You may assume that each system is initially "at rest."

$$x[n] \rightarrow \boxed{\mathcal{H}_1} \rightarrow y[n] = x[n] + 1$$

$$x[n] \rightarrow \boxed{\mathcal{H}_2} \rightarrow y[n] = x[2n]$$

$$x[n] \rightarrow \boxed{\mathcal{H}_3} \rightarrow y[n] = x[-n]$$

$$x[n] \rightarrow \boxed{\mathcal{H}_4} \rightarrow y[n] = y[n-1] + x[n]$$

$$x[n] \rightarrow \boxed{\mathcal{H}_5} \rightarrow y[n] = |x[n]|$$

$$x[n] \rightarrow \boxed{\mathcal{H}_6} \rightarrow y[n] = x[|n|]$$

$$x[n] \rightarrow \boxed{\mathcal{H}_7} \rightarrow y[n] = C \text{ (constant)}$$

Discrete-Time Systems

Determine if each system is linear and time-invariant.
You may assume that each system is initially "at rest."

$x[n] \rightarrow \boxed{\mathcal{H}_1} \rightarrow y[n] = x[n] + 1$	L? No. TI? Yes.
$x[n] \rightarrow \boxed{\mathcal{H}_2} \rightarrow y[n] = x[2n]$	L? Yes. TI? No.
$x[n] \rightarrow \boxed{\mathcal{H}_3} \rightarrow y[n] = x[-n]$	L? Yes. TI? No.
$x[n] \rightarrow \boxed{\mathcal{H}_4} \rightarrow y[n] = y[n-1] + x[n]$	L? Yes. TI? Yes.
$x[n] \rightarrow \boxed{\mathcal{H}_5} \rightarrow y[n] = \lvert x[n] \rvert$	L? No. TI? Yes.
$x[n] \rightarrow \boxed{\mathcal{H}_6} \rightarrow y[n] = x[\lvert n \rvert]$	L? Yes. TI? No.
$x[n] \rightarrow \boxed{\mathcal{H}_7} \rightarrow y[n] = C$ (constant)	L? Depends. TI? Yes.

Continuous-Time Systems

Determine if each system is linear and time-invariant.
You may assume that each system is initially "at rest."

$$x(t) \rightarrow \boxed{\mathcal{H}_1} \rightarrow y(t) = \frac{d}{dt}x(t)$$
$$x(t) \rightarrow \boxed{\mathcal{H}_2} \rightarrow y(t) = \int_0^t x(\tau)\,d\tau$$
$$x(t) \rightarrow \boxed{\mathcal{H}_3} \rightarrow y(t) = \int_{-\infty}^t x(\tau)\,d\tau$$
$$x(t) \rightarrow \boxed{\mathcal{H}_4} \rightarrow y(t) = \int_t^{\infty} x(\tau)\,d\tau$$
$$x(t) \rightarrow \boxed{\mathcal{H}_5} \rightarrow y(t) = \int_{-\infty}^{\infty} x(\tau)\,d\tau$$

Continuous-Time Systems

Determine if each system is linear and time-invariant.
You may assume that each system is initially "at rest."

$x(t) \rightarrow \boxed{\mathcal{H}_1} \rightarrow y(t) = \frac{d}{dt}x(t)$ L? Yes. TI? Yes.

$x(t) \rightarrow \boxed{\mathcal{H}_2} \rightarrow y(t) = \int_0^t x(\tau)d\tau$ L? Yes. TI? No.

$x(t) \rightarrow \boxed{\mathcal{H}_3} \rightarrow y(t) = \int_{-\infty}^t x(\tau)d\tau$ L? Yes. TI? Yes.

$x(t) \rightarrow \boxed{\mathcal{H}_4} \rightarrow y(t) = \int_t^\infty x(\tau)d\tau$ L? Yes. TI? Yes.

$x(t) \rightarrow \boxed{\mathcal{H}_5} \rightarrow y(t) = \int_{-\infty}^\infty x(\tau)d\tau$ L? Yes. TI? Yes.

Common Confusion

A causal continuous-time (CT) system described by a linear, constant-coefficient differential equation (LCCDE) **with initial rest conditions** is linear and time-invariant.

$$\sum_k a_k \frac{d^k y(t)}{dt^k} = \sum_k b_k \frac{d^k x(t)}{dt^k}$$

A causal discrete-time (DT) system described by a linear, constant-coefficient difference equation (LCCDE) **with initial rest conditions** is linear and time-invariant.

$$\sum_k a_k y[n-k] = \sum_k b_k x[n-k]$$

A system specified by an LCCDE is **not necessarily LTI**. We need initial rest conditions to force the homogeneous solution to be identically zero. A system specified by an LCCDE **with initial rest conditions** is LTI.

Lessons Learned

Linearity (L) and **time-invariance (TI)** are properties that greatly simplify the analysis and design of systems.

Additivity: $x_1[n] + x_2[n] \rightarrow \boxed{\mathcal{H}} \rightarrow y_1[n] + y_2[n]$

Homogeneity: $cx[n] \rightarrow \boxed{\mathcal{H}} \rightarrow cy[n]$ for any constant c

Linearity: $c_1 x_1[n] + c_2 x_2[n] \rightarrow \boxed{\mathcal{H}} \rightarrow c_1 y_1[n] + c_2 y_2[n]$

Together, additivity and homogeneity imply linearity.

Time-Invariance: $x[n - n_0] \rightarrow \boxed{\mathcal{H}} \rightarrow y[n - n_0]$ for all n_0

Question of the Day

For the system below to be linear and time-invariant, what must be true about the constants M and B?

$$x[n] \rightarrow \boxed{\mathcal{H}} \rightarrow y[n] = Mx[n] + B$$