6.300: Signal Processing

Systems: Linearity and Time-Invariance

Additivity: $x_1[n] + x_2[n] \rightarrow \boxed{\mathcal{H}} \rightarrow y_1[n] + y_2[n]$

Homogeneity: $cx[n] \rightarrow \boxed{\mathcal{H}} \rightarrow cy[n]$ for any constant c

Linearity: $c_1x_1[n] + c_2x_2[n] \rightarrow \boxed{\mathcal{H}} \rightarrow c_1y_1[n] + c_2y_2[n]$

Together, additivity and homogeneity imply linearity.

Time-Invariance: $x[n-n_0] \rightarrow \boxed{\mathcal{H}} \rightarrow y[n-n_0]$ for all n_0

March 10, 2026

Agenda for Recitation

- System properties: Linearity and time-invariance

What questions do you have from lecture?

All 6.300 students are invited to join our staff meeting next **Monday, March 16** in **38-466** (Jackson Room) from **5:00 p.m.** to around 6:00 p.m. Please join us! We'd like to hear your thoughts on 6.300 thus far.

Discrete-Time Systems

Determine if each system is linear and time-invariant. You may assume that each system is initially "at rest."

$$x[n] \rightarrow \boxed{\mathcal{H}_1} \rightarrow y[n] = x[n] + 1$$
$$x[n] \rightarrow \boxed{\mathcal{H}_2} \rightarrow y[n] = x[2n]$$
$$x[n] \rightarrow \boxed{\mathcal{H}_3} \rightarrow y[n] = x[-n]$$
$$x[n] \rightarrow \boxed{\mathcal{H}_4} \rightarrow y[n] = y[n-1] + x[n]$$
$$x[n] \rightarrow \boxed{\mathcal{H}_5} \rightarrow y[n] = |x[n]|$$
$$x[n] \rightarrow \boxed{\mathcal{H}_6} \rightarrow y[n] = x[|n|]$$
$$x[n] \rightarrow \boxed{\mathcal{H}_7} \rightarrow y[n] = C \text{ (constant)}$$

Continuous-Time Systems

Determine if each system is linear and time-invariant.
You may assume that each system is initially "at rest."

$$x(t) \rightarrow \boxed{\mathcal{H}_1} \rightarrow y(t) = \frac{d}{dt}x(t)$$

$$x(t) \rightarrow \boxed{\mathcal{H}_2} \rightarrow y(t) = \int_0^t x(\tau)d\tau$$

$$x(t) \rightarrow \boxed{\mathcal{H}_3} \rightarrow y(t) = \int_{-\infty}^t x(\tau)d\tau$$

$$x(t) \rightarrow \boxed{\mathcal{H}_4} \rightarrow y(t) = \int_t^{\infty} x(\tau)d\tau$$

$$x(t) \rightarrow \boxed{\mathcal{H}_5} \rightarrow y(t) = \int_{-\infty}^{\infty} x(\tau)d\tau$$

Lessons Learned

Linearity (L) and **time-invariance (TI)** are properties that greatly simplify the analysis and design of systems.

Additivity: $x_1[n] + x_2[n] \rightarrow \boxed{\mathcal{H}} \rightarrow y_1[n] + y_2[n]$

Homogeneity: $cx[n] \rightarrow \boxed{\mathcal{H}} \rightarrow cy[n]$ for any constant c

Linearity: $c_1 x_1[n] + c_2 x_2[n] \rightarrow \boxed{\mathcal{H}} \rightarrow c_1 y_1[n] + c_2 y_2[n]$

Together, additivity and homogeneity imply linearity.

Time-Invariance: $x[n - n_0] \rightarrow \boxed{\mathcal{H}} \rightarrow y[n - n_0]$ for all n_0