6.300: Signal Processing

Series and Symmetry

The **synthesis equation** tells us how to represent a periodic signal as a Fourier series.

$$f(t) = f(t+T) = c_0 + \sum_{k=1}^{\infty} c_k \cos\left(k\frac{2\pi}{T}t\right) + \sum_{k=1}^{\infty} d_k \sin\left(k\frac{2\pi}{T}t\right)$$

The **analysis equations** tell us how to calculate these Fourier series coefficients.

$$c_0 = \frac{1}{T}\int_T f(t)\,dt \qquad \text{average value over a period}$$

$$c_k = \frac{2}{T}\int_T f(t)\cos\left(k\frac{2\pi}{T}t\right)dt \qquad \text{for } k \ge 1$$

$$d_k = \frac{2}{T}\int_T f(t)\sin\left(k\frac{2\pi}{T}t\right)dt \qquad \text{for } k \ge 1$$

February 05, 2026

Agenda for Recitation

- Definitions of **symmetry**, **anti-symmetry**, **asymmetry**
- Using **symmetry** to simplify Fourier series calculations

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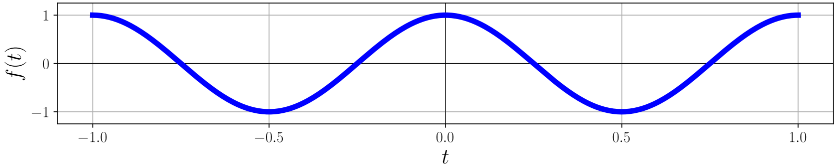
What questions do you have from lecture?

Symmetry, Anti-Symmetry, Asymmetry

A **symmetric** signal satisfies $f(t) = f(-t)$.



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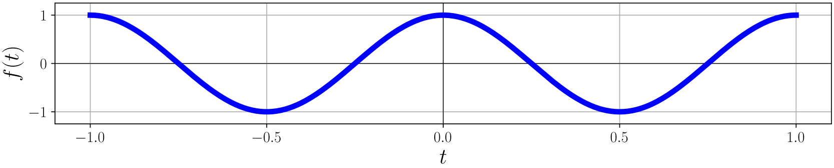
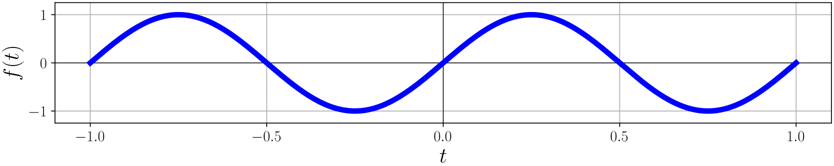


An **anti-symmetric** signal satisfies $f(t) = -f(-t)$.



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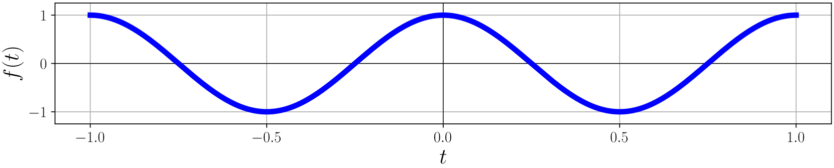

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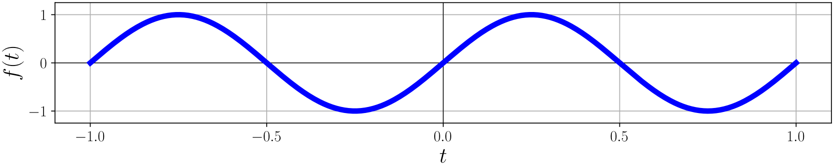
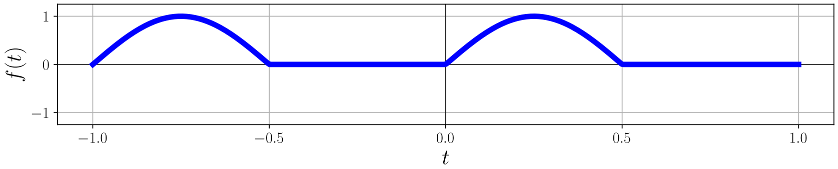


What we've called **symmetry/anti-symmetry** is also called **even/odd symmetry**. Might as well know both.

Symmetry, Anti-Symmetry, Asymmetry

An **asymmetric** signal is neither symmetric nor anti-symmetric — it lacks any semblance of symmetry.



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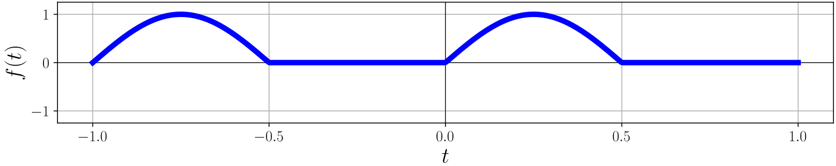

We can express an **asymmetric** signal as the **sum** of a **symmetric part** and an **anti-symmetric part**.

$$f(t) = f_{\text{symmetric}}(t) + f_{\text{anti-symmetric}}(t)$$

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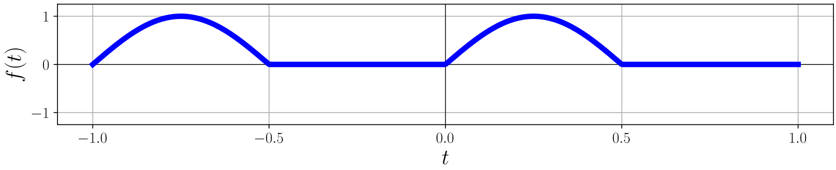

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$$f(t) = f_{\text{symmetric}}(t) + f_{\text{anti-symmetric}}(t)$$

How do we get $f_{\text{symmetric}}(t)$ from $f(t)$?
How do we get $f_{\text{anti-symmetric}}(t)$ from $f(t)$?

Symmetry, Anti-Symmetry, Asymmetry

How do we get $f_{\text{symmetric}}(t)$ from $f(t)$?
How do we get $f_{\text{anti-symmetric}}(t)$ from $f(t)$?

Add $f(t)$ and $f(-t)$ to cancel out the anti-symmetric part. This leaves the symmetric part.

$$f_{\text{symmetric}}(t) = \frac{f(t) + f(-t)}{2}$$

Add $f(t)$ and $-f(-t)$ to cancel out the symmetric part. This leaves the anti-symmetric part.

$$f_{\text{anti-symmetric}}(t) = \frac{f(t) - f(-t)}{2}$$

Symmetry, Anti-Symmetry, Asymmetry

Before we make any calculations, we can use **symmetry** (and **anti-symmetry**) to make some conclusions about the Fourier series coefficients.

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What can we say about the Fourier series coefficients (c_k and d_k) for a **real, symmetric** signal?

What can we say about the Fourier series coefficients (c_k and d_k) for a **real, anti-symmetric** signal?

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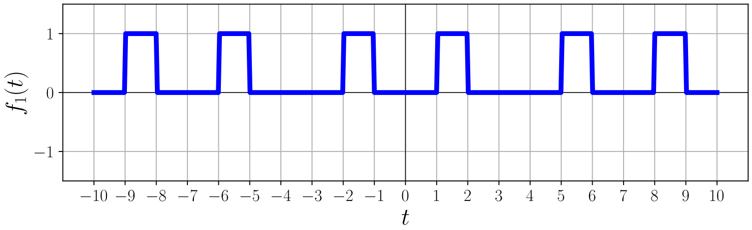
Instead of keeping the discussion abstract, let's work through examples and draw some conclusions from there.

Fourier Series and Symmetry

Determine the Fourier series coefficients (c_k and d_k) for the periodic function $f_1(t)$, shown below.



What is the fundamental period (T) of $f_1(t)$?

Does it matter which interval you integrate over?
(0 to T? $-\frac{1}{2}T$ to $\frac{1}{2}T$? $\frac{5}{16}T$ to $\frac{21}{16}T$?)

Do you notice anything about the c_k and/or d_k terms?

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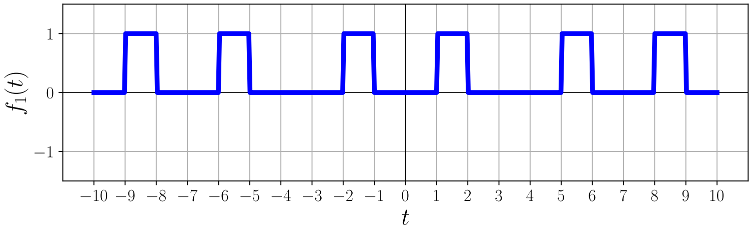


The fundamental period is $T = 7$.

Integrating over any length-T interval yields the same result. (The calculations are on the next slide.)

Because $f_1(t)$ is symmetric, $d_k = 0$ for all k.

Fourier Series and Symmetry

c_0 is the average value of $f_1(t)$ over one period.

$$c_0 = \frac{1}{7}\int_{-\frac{7}{2}}^{\frac{7}{2}} f_1(t)\,dt = \frac{2}{7}$$

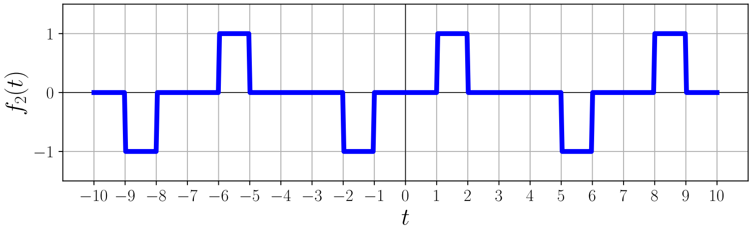
Compute the remaining coefficients via integration.

$$c_k = \frac{1}{7}\int_{-\frac{7}{2}}^{\frac{7}{2}} f_1(t)\cos\!\left(k\tfrac{2\pi}{7}t\right)dt = \frac{2}{\pi k}\left(\sin\left(\frac{4\pi}{7}k\right) - \sin\left(\frac{2\pi}{7}k\right)\right)$$

$$d_k = \frac{1}{7}\int_{-\frac{7}{2}}^{\frac{7}{2}} f_1(t)\sin\!\left(k\tfrac{2\pi}{7}t\right)dt = 0$$

Fourier Series and Symmetry

Determine the Fourier series coefficients (c_k and d_k) for the periodic function $f_2(t)$, shown below.



Do you notice anything about the c_k and/or d_k terms?

Fourier Series and Symmetry

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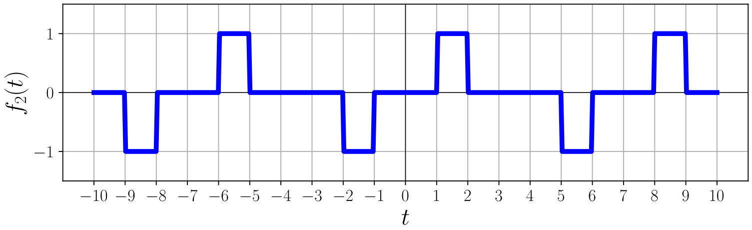


Do you notice anything about the c_k and/or d_k terms?

Because $f_2(t)$ is anti-symmetric, $c_k = 0$ for all k.

Fourier Series and Symmetry

c_0 is the average value of $f_2(t)$ over one period.

$$c_0 = \frac{1}{7} \int_{-\frac{7}{2}}^{\frac{7}{2}} f_2(t)\, dt = 0$$

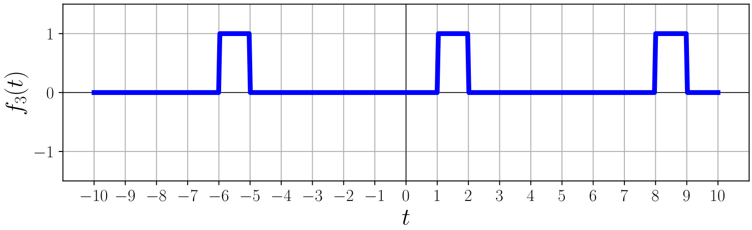
Compute the remaining coefficients via integration.

$$c_k = \frac{1}{7} \int_{-\frac{7}{2}}^{\frac{7}{2}} f_2(t) \cos\!\left(k \tfrac{2\pi}{7} t\right) dt = 0$$

$$d_k = \frac{1}{7} \int_{-\frac{7}{2}}^{\frac{7}{2}} f_2(t) \sin\!\left(k \tfrac{2\pi}{7} t\right) dt = \frac{2}{\pi k} \left(\cos\left(\frac{2\pi}{7} k \right) - \cos\left(\frac{4\pi}{7} k \right) \right)$$

Fourier Series and Symmetry

Determine the Fourier series coefficients (c_k and d_k) for the periodic function $f_3(t)$, shown below.



Do you notice anything about the c_k and/or d_k terms?

How does $f_3(t)$ relate to $f_1(t)$ and $f_2(t)$?

How do the Fourier series coefficients for $f_3(t)$ relate to the coefficients for $f_1(t)$ and $f_2(t)$?

Fourier Series and Symmetry

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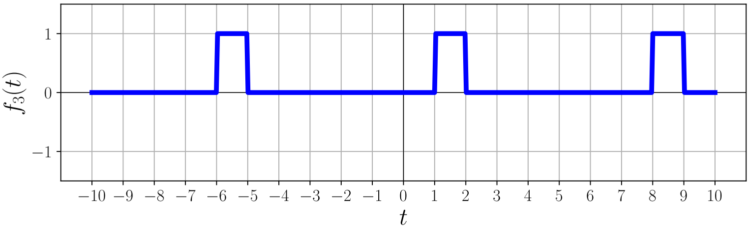

$f_3(t)$ is asymmetric. Both c_k and d_k are non-zero.

Notice that $f_3(t) = \frac{1}{2} f_1(t) + \frac{1}{2} f_2(t)$. So, the coefficients for $f_3(t)$ are half those for $f_1(t)$ plus half those for $f_2(t)$.

Fourier Series and Symmetry

c_0 is the average value of $f_3(t)$ over one period.

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Compute the remaining coefficients via integration.

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Fourier Series and Symmetry

We have looked at how to use **symmetry** to simplify calculations with Fourier series.

Real, symmetric signals: Only c_k are non-zero.
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We will soon learn many more **Fourier properties**.

e.g., If we know c_k/d_k for $f(t)$, what about c_k/d_k for ...

- $f(t-1)$?
- $f(2t)$?
- $f(t)\cos(t)$?
- $f^2(t)$?

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Next time: Simplifying the math even further using **complex numbers — replacing trig with algebra!**

Lessons Learned

The **synthesis equation** tells us how to represent a periodic signal as a Fourier series.

$$f(t) = f(t+T) = c_0 + \sum_{k=1}^{\infty} c_k \cos\left(k \frac{2\pi}{T} t\right) + \sum_{k=1}^{\infty} d_k \sin\left(k \frac{2\pi}{T} t\right)$$

The **analysis equations** tell us how to calculate these Fourier series coefficients.

$c_0 = \frac{1}{T} \int_T f(t)\, dt$ average value over a period

$c_k = \frac{2}{T} \int_T f(t) \cos\left(k \frac{2\pi}{T} t\right) dt$ for $k \geq 1$

$d_k = \frac{2}{T} \int_T f(t) \sin\left(k \frac{2\pi}{T} t\right) dt$ for $k \geq 1$

Real, symmetric signals: Only c_k are non-zero.

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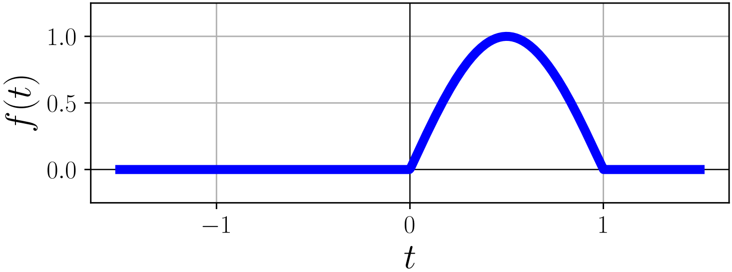
Question of the Day

An **asymmetric** function $f(t)$ is shown below.
The function is zero outside the indicated bounds.



Sketch $f_{\text{symmetric}}(t)$, the **symmetric part**.

Sketch $f_{\text{anti-symmetric}}(t)$, the **anti-symmetric part**.