6.300: Signal Processing

Quiz Review

Quiz #2 takes place in **Walker Memorial (50-340)** on **Thursday, April 16** from **2:00 to 4:00 p.m.**

View the **Quiz #2 Information** page on the course website: `https://sigproc.mit.edu/spring26/q2_info`.

April 14, 2026

Agenda for Recitation

- Review: Fourier transforms and LTI systems
- Solving problems: Quiz #2 from spring 2025

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Calculus and Signal Processing

You wouldn't want to walk into a calculus quiz without knowing

$$\frac{d\sin(\theta)}{d\theta} = \cos(\theta)$$

by heart, right? Going back to the derivation wastes precious time!

$$\lim_{\phi \to 0} \frac{\sin(\theta + \phi) - \sin(\theta)}{\phi}$$

$$\lim_{\phi \to 0} \frac{\sin(\theta)\cos(\phi) + \sin(\phi)\cos(\theta) - \sin(\theta)}{\phi}$$

$$\lim_{\phi \to 0} \underbrace{\left[\frac{\sin(\phi)}{\phi}\right]}_{1 \text{ as } \phi \to 0} \cos(\theta) + \lim_{\phi \to 0} \underbrace{\left[\frac{\cos(\phi) - 1}{\phi}\right]}_{0 \text{ as } \phi \to 0} \sin(\theta)$$

Calculus and Signal Processing

To do well on a **calculus** quiz,
you need to know at least a few things by heart.

common functions and their **derivatives**

$$\frac{d(t^n)}{dt} = nt^{n-1} \qquad \frac{d\left(e^{\lambda t}\right)}{dt} = \lambda e^{\lambda t} \qquad \frac{d\sin(t)}{dt} = \cos(t)$$

differentiation rules

$$\frac{d\left[c_1 f(t) + c_2 g(t)\right]}{dt} = c_1\frac{df}{dt} + c_2\frac{dg}{dt}$$

$$\frac{dg\big(f(t)\big)}{dt} = \frac{dg}{df}\cdot\frac{df}{dt}$$

Calculus and Signal Processing

To do well on a **signal processing** quiz,
you need to know at least a few things by heart.

common signals and their **Fourier transforms**

$$\delta(t-t_0) \Longleftrightarrow e^{-j\omega t_0} \qquad e^{j\omega_0 t} \Longleftrightarrow 2\pi\delta(\omega-\omega_0)$$

Fourier properties

$$c_1x_1(t)+c_2x_2(t) \Longleftrightarrow c_1X_1(\omega)+c_2X_2(\omega)$$

$$x(t-t_0) \Longleftrightarrow e^{-j\omega t_0}X(\omega)$$

$$e^{j\omega_0 t}x(t) \Longleftrightarrow X(\omega-\omega_0)$$

If you're getting caught up on "level one" problems, it's difficult to do "level two" problems.

Fourier Representations

	type of time-domain signal			domain	period
CTFS	CT	period T	infinite length	integer k	none
DTFS	DT	period N	infinite length	integer k	N
CTFT	CT	aperiodic	infinite length	real ω	none
DTFT	DT	aperiodic	infinite length	real Ω	2π
DFT	DT	aperiodic	length N	integer k	N

Periodic signals have both **Fourier series** and **Fourier transform** representations.

- Fourier series coefficients: $X[k]$
- Fourier transform: $X(\omega) = \sum_k 2\pi X[k]\delta(\omega - k\omega_0)$

Duality of the continuous-time Fourier transform:

If $x(t) \iff X(\omega)$, then $X(t) \iff 2\pi x(-\omega)$, too.

Fourier Transform Pairs

	time	frequency
CTFT	$\delta(t-t_0)$	$e^{-j\omega t_0}$
CTFT	$e^{j\omega_0 t}$	$2\pi\delta(\omega-\omega_0)$
CTFT	$e^{-\sigma t}u(t)$ where $\sigma > 0$	$1/(\sigma+j\omega)$
DTFT	$\delta[n-n_0]$	$e^{-j\Omega n_0}$
DTFT	$e^{j\Omega_0 n}$	$2\pi\delta((\Omega-\Omega_0) \bmod 2\pi)$
DTFT	$r^n u[n]$ where $\|r\| < 1$	$1/(1-re^{-j\Omega})$
DFT	$\delta[n-n_0]$	$\frac{1}{N}e^{-jk\frac{2\pi}{N}n_0}$
DFT	$e^{jk_0\frac{2\pi}{N}n}$	$\delta[(k-k_0) \bmod N]$

All DTFTs are periodic in 2π. All DFTs are periodic in N.

Fourier Transform Properties

There are a few properties worth knowing by heart, too.

- A few CTFT properties are given below.
- Analogous properties apply for other representations.

	time	**frequency**
time shift	$x(t-t_0)$	$e^{-j\omega t_0}X(\omega)$
frequency shift	$e^{j\omega_0 t}x(t)$	$X(\omega-\omega_0)$
scaling time	$x(at)$	$\|a\|^{-1}X(a^{-1}\omega)$
scaling time	$x(a^{-1}t)$	$\|a\|X(a\omega)$
time derivative	$\frac{d}{dt}x(t)$	$j\omega X(\omega)$
frequency derivative	$t\times x(t)$	$j\frac{d}{d\omega}X(\omega)$

LTI Systems

Three equivalent representations of LTI systems:

- difference (DT) or differential (CT) equations
- unit-sample (DT) or impulse (CT) response
- frequency response

Example: System with feedback

$$y[n] = \tfrac{1}{2}y[n-1] + x[n]$$

$$h[n] = (\tfrac{1}{2})^n u[n] \iff H(\Omega) = \frac{1}{1 - \frac{1}{2}e^{-j\Omega}}$$

Multiple equivalent ways to perform convolution:

- directly compute the sum or integral
- superposition
- flip and shift
- transform, multiply transforms, inverse transform

Discrete Fourier Transform (DFT)

The **discrete Fourier transform (DFT)** is a discrete-time, discrete-frequency Fourier transform.

- for aperiodic discrete-time (n) signals $x[n]$
- yields discrete-frequency (k) representation $X[k]$
- finite length (N) in both time and frequency

Two equivalent perspectives of the DFT:

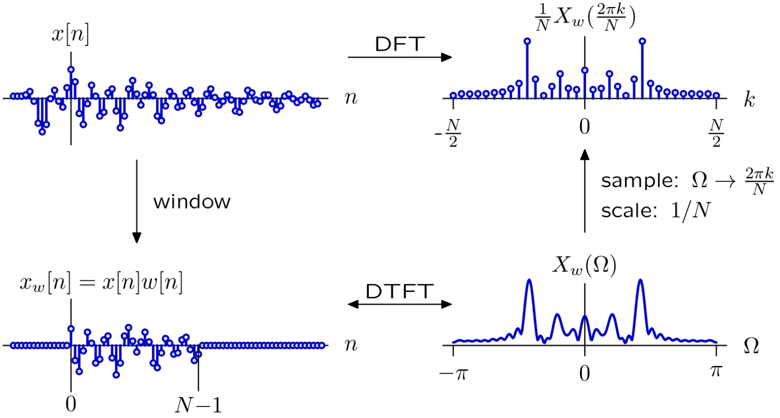
- DTFS of periodically-extended $x_w[n] = x[n]w[n]$
- Sampled ($\Omega \to 2\pi k/N$), scaled ($1/N$) DTFT of $x_w[n]$

Circular convolution: $\frac{1}{N}(x \circledast h)[n] \iff X[k]H[k]$

- Start by computing the usual convolution $(x * h)[n]$.
- Wrap into base period: $(x \circledast h)[n] = (x * h)[n \bmod N]$.
- Finally, scale by $1/N$.

Relation Between DFT and DTFT

Graphical depiction of relation between DFT and DTFT.



Graphic: Professor Denny Freeman (freeman@mit.edu)

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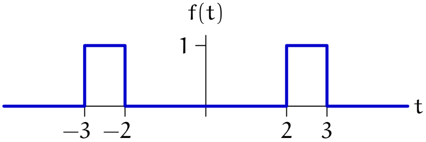
#1. Short-Answer Questions

The left column of the following table shows the input-output relations for four different systems, where x_i represents the input and y_i represents the output. For each system, determine if that system is **additive** and/or **homogeneous** and/or **time-invariant**. (Enter `YES` or `NO`.)

	Add.	**Hom.**	**Time-Inv.**
$y_1[n] = (-\frac{1}{2})^n (x_1[n] + 1)$			
$y_2[n] = \sin(x_2[n])$			
$y_3(t) = \int_{-\infty}^{t} x_3(\tau)\, d\tau$			
$y_4(t) = t\, x_4(t)$			

#1. Short-Answer Questions



Determine a closed-form expression (e.g., no integrals) for the frequency response $F(\omega)$ of a linear, time-invariant (LTI) system with impulse response

$$f(t) = \begin{cases} 1 & 2 \leq |t| \leq 3 \\ 0 & \text{otherwise.} \end{cases}$$

$F(\omega) =$

#1. Short-Answer Questions

Let $G(\Omega)$ denote the frequency response of a linear, time-invariant (LTI) system with unit-sample response

$$g[n] = \delta[n] - \delta[n-2].$$

Sketch $|G(\Omega)|$ and $\angle G(\Omega)$. Label key features of your plots.

#2. Cascaded Systems

$$x[n] \rightarrow \boxed{\mathcal{S}_1} \rightarrow y_1[n]$$

Let $\mathcal{S}_1$ denote the LTI system with unit-sample response

$$h_1[n] = u[n] = \begin{cases} 1 & n \geq 0 \\ 0 & n < 0. \end{cases}$$

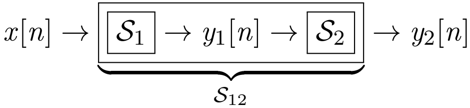
Suppose the input to this system is

$$x[n] = (0.9)^n u[n] = \begin{cases} (0.9)^n & n \geq 0 \\ 0 & n < 0. \end{cases}$$

Determine a closed-form expression for the response $y_1[n]$.

$y_1[n] =$

#2. Cascaded Systems



Let $\mathcal{S}_{12}$ represent the LTI system that results when two LTI systems ($\mathcal{S}_1$ and $\mathcal{S}_2$) are connected in cascade (so that the output of $\mathcal{S}_1$ is the input to $\mathcal{S}_2$) as shown in the figure above.

Let $h_i[n]$ denote the unit-sample response of system $\mathcal{S}_i$.

$$h_1[n] = h_2[n] = u[n] = \begin{cases} 1 & n \geq 0 \\ 0 & n < 0 \end{cases}$$

Determine a closed-form expression for $h_{12}[n]$.

$h_{12}[n] =$

#2. Cascaded Systems

$$x[n] \rightarrow \underbrace{\boxed{\boxed{\mathcal{S}_1} \rightarrow y_1[n] \rightarrow \boxed{\mathcal{S}_2} \rightarrow y_2[n] \rightarrow \boxed{\mathcal{S}_3}}}_{\mathcal{S}_{123}} \rightarrow y_3[n]$$

Let $\mathcal{S}_{123}$ represent the LTI system that results when three LTI systems ($\mathcal{S}_1$, $\mathcal{S}_2$, and $\mathcal{S}_3$) are connected in cascade, as shown in the figure above.

Let $h_i[n]$ denote the unit sample response of system $\mathcal{S}_i$.

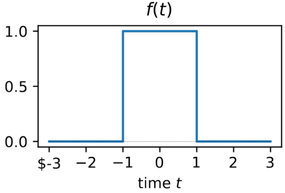
$$h_1[n] = h_2[n] = h_3[n] = u[n] = \begin{cases} 1 & n \geq 0 \\ 0 & n < 0 \end{cases}$$

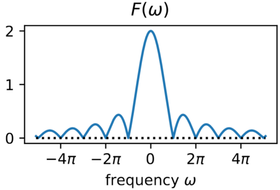
Determine a closed-form expression for $h_{123}[n]$.

$h_{123}[n] =$

#3. Pulsed Relations

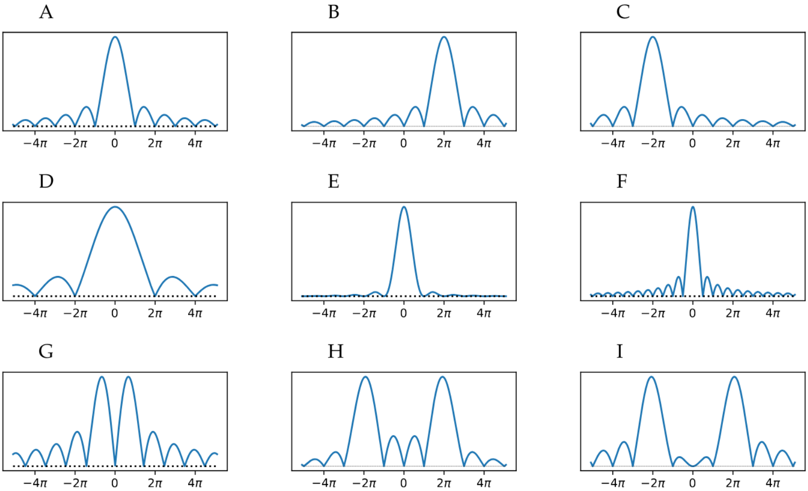
$$f(t) = \begin{cases} 1 & |t| \leq 1 \\ 0 & \text{otherwise} \end{cases} \quad \Longleftrightarrow \quad F(\omega) = \int_{-\infty}^{\infty} f(t) e^{-j\omega t} dt = 2\frac{\sin(\omega)}{\omega}$$





Match each transformation of $f(t)$ below to the plot on the next page that shows the magnitude of its CTFT.

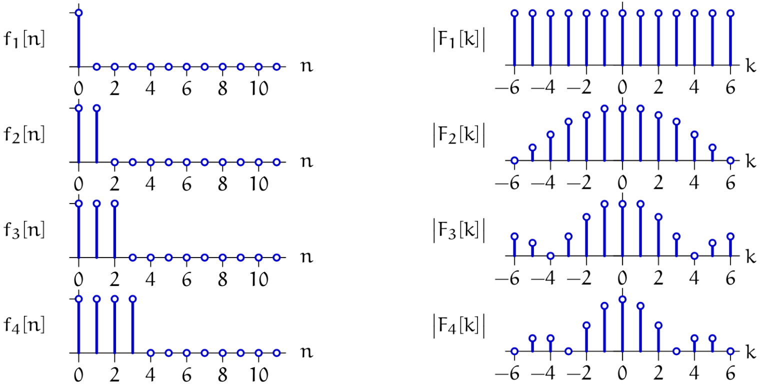
$f(\frac{1}{2}t)$	$f(2t)$	$f(t-1)$	$f(t+1)$	$t\,f(t)$
$(f*f)(t)$	$f(t)e^{j2\pi t}$	$f(t)e^{-j2\pi t}$	$f(t)\sin(2\pi t)$	$f(t)\cos(2\pi t)$



$f(\frac{1}{2}t)$	$f(2t)$	$f(t-1)$	$f(t+1)$	$t\,f(t)$
$(f * f)(t)$	$f(t)\,e^{j2\pi t}$	$f(t)\,e^{-j2\pi t}$	$f(t)\sin(2\pi t)$	$f(t)\cos(2\pi t)$

#4. Pulses In, Pulses Out

The first 12 samples of four periodic signals, each periodic in $N = 12$, are shown in the left column below. The magnitudes of their Fourier series coefficients are shown in the right column.



#4. Pulses In, Pulses Out

Consider a system $\mathcal{S}_{24}$ that produces $f_4[n]$ as output when $f_2[n]$ is the input.

$$f_2[n] \rightarrow \boxed{\mathcal{S}_{24}} \rightarrow f_4[n]$$

Determine a closed-form expression for $h_{24}[n]$.

$h_{24}[n] =$

Determine a closed-form expression for $H_{24}[k]$.

$H_{24}[k] =$

Sketch $|H_{24}[k]|$, the magnitude of $H_{24}[k]$, over $-6 \leq k \leq 6$.

#4. Pulses In, Pulses Out

Consider 16 systems, each defined by input $f_i[n]$ and output $f_o[n]$.

$$\text{input } f_i[n] \rightarrow \boxed{\mathcal{S}_{io}} \rightarrow \text{output } f_o[n]$$

Some of these systems could be LTI; others cannot. Determine which of the 16 systems cannot possibly be LTI, and enter an **X** in the corresponding box below.

$\mathcal{S}_{io}$	$o = 1$	$o = 2$	$o = 3$	$o = 4$
$i = 1$				
$i = 2$				
$i = 3$				
$i = 4$				

Question of the Day

Quiz #2 is this Thursday. What are your strengths? What are weaknesses you'll address before then?