

# 6.300: Signal Processing

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## Quiz Review

**Quiz #2** takes place in **Walker Memorial (50-340)** on **Thursday, April 16** from **2:00 to 4:00 p.m.**

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View the **Quiz #2 Information** page on the course website: [https://sigproc.mit.edu/spring26/q2\\_info](https://sigproc.mit.edu/spring26/q2_info).

*April 14, 2026*

# Agenda for Recitation

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- Review: Fourier transforms and LTI systems
- Solving problems: Quiz #2 from spring 2025

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# Calculus and Signal Processing

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You wouldn't want to walk into a calculus quiz without knowing

$$\frac{d \sin(\theta)}{d\theta} = \cos(\theta)$$

by heart, right? Going back to the derivation wastes precious time!

$$\begin{aligned} & \lim_{\phi \rightarrow 0} \frac{\sin(\theta + \phi) - \sin(\theta)}{\phi} \\ & \lim_{\phi \rightarrow 0} \frac{\sin(\theta) \cos(\phi) + \sin(\phi) \cos(\theta) - \sin(\theta)}{\phi} \\ & \lim_{\phi \rightarrow 0} \underbrace{\left[ \frac{\sin(\phi)}{\phi} \right]}_{1 \text{ as } \phi \rightarrow 0} \cos(\theta) + \lim_{\phi \rightarrow 0} \underbrace{\left[ \frac{\cos(\phi) - 1}{\phi} \right]}_{0 \text{ as } \phi \rightarrow 0} \sin(\theta) \end{aligned}$$

# Calculus and Signal Processing

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To do well on a **calculus** quiz, you need to know at least a few things by heart.

## common functions and their derivatives

$$\frac{d(t^n)}{dt} = nt^{n-1} \quad \frac{d(e^{\lambda t})}{dt} = \lambda e^{\lambda t} \quad \frac{d \sin(t)}{dt} = \cos(t)$$

## differentiation rules

$$\frac{d[c_1 f(t) + c_2 g(t)]}{dt} = c_1 \frac{df}{dt} + c_2 \frac{dg}{dt}$$

$$\frac{dg(f(t))}{dt} = \frac{dg}{df} \cdot \frac{df}{dt}$$

# Calculus and Signal Processing

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To do well on a **signal processing** quiz, you need to know at least a few things by heart.

**common signals** and their **Fourier transforms**

$$\delta(t - t_0) \iff e^{-j\omega t_0} \quad e^{j\omega_0 t} \iff 2\pi\delta(\omega - \omega_0)$$

**Fourier properties**

$$c_1x_1(t) + c_2x_2(t) \iff c_1X_1(\omega) + c_2X_2(\omega)$$

$$x(t - t_0) \iff e^{-j\omega t_0} X(\omega)$$

$$e^{j\omega_0 t} x(t) \iff X(\omega - \omega_0)$$

If you're getting caught up on "level one" problems, it's difficult to do "level two" problems.

# Fourier Representations

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	type of time-domain signal			domain	period
CTFS	CT	period $T$	infinite length	integer $k$	none
DTFS	DT	period $N$	infinite length	integer $k$	$N$
CTFT	CT	aperiodic	infinite length	real $\omega$	none
DTFT	DT	aperiodic	infinite length	real $\Omega$	$2\pi$
DFT	DT	aperiodic	length $N$	integer $k$	$N$

**Periodic signals** have both **Fourier series** and **Fourier transform** representations.

- Fourier series coefficients:  $X[k]$
- Fourier transform:  $X(\omega) = \sum_k 2\pi X[k]\delta(\omega - k\omega_0)$

**Duality** of the continuous-time Fourier transform:

If  $x(t) \iff X(\omega)$ , then  $X(t) \iff 2\pi x(-\omega)$ , too.

# Fourier Transform Pairs

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	<b>time</b>	<b>frequency</b>
<b>CTFT</b>	$\delta(t - t_0)$	$e^{-j\omega t_0}$
<b>CTFT</b>	$e^{j\omega_0 t}$	$2\pi\delta(\omega - \omega_0)$
<b>CTFT</b>	$e^{-\sigma t}u(t)$ where $\sigma > 0$	$1/(\sigma + j\omega)$
<b>DTFT</b>	$\delta[n - n_0]$	$e^{-j\Omega n_0}$
<b>DTFT</b>	$e^{j\Omega_0 n}$	$2\pi\delta((\Omega - \Omega_0) \bmod 2\pi)$
<b>DTFT</b>	$r^n u[n]$ where $ r  < 1$	$1/(1 - re^{-j\Omega})$
<b>DFT</b>	$\delta[n - n_0]$	$\frac{1}{N} e^{-jk \frac{2\pi}{N} n_0}$
<b>DFT</b>	$e^{jk_0 \frac{2\pi}{N} n}$	$\delta[(k - k_0) \bmod N]$

All DTFTs are periodic in  $2\pi$ . All DFTs are periodic in  $N$ .

# Fourier Transform Properties

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There are a few properties worth knowing by heart, too.

- A few CTFT properties are given below.
- Analogous properties apply for other representations.

	<b>time</b>	<b>frequency</b>
time shift	$x(t - t_0)$	$e^{-j\omega t_0} X(\omega)$
frequency shift	$e^{j\omega_0 t} x(t)$	$X(\omega - \omega_0)$
scaling time	$x(at)$	$ a ^{-1} X(a^{-1}\omega)$
scaling time	$x(a^{-1}t)$	$ a  X(a\omega)$
time derivative	$\frac{d}{dt} x(t)$	$j\omega X(\omega)$
frequency derivative	$t \times x(t)$	$j \frac{d}{d\omega} X(\omega)$

# LTI Systems

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Three equivalent representations of LTI systems:

- difference (DT) or differential (CT) equations
- unit-sample (DT) or impulse (CT) response
- frequency response

Example: System with feedback

$$y[n] = \frac{1}{2}y[n-1] + x[n]$$
$$h[n] = \left(\frac{1}{2}\right)^n u[n] \iff H(\Omega) = \frac{1}{1 - \frac{1}{2}e^{-j\Omega}}$$

Multiple equivalent ways to perform convolution:

- directly compute the sum or integral
- superposition
- flip and shift
- transform, multiply transforms, inverse transform

# Discrete Fourier Transform (DFT)

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The **discrete Fourier transform (DFT)** is a discrete-time, discrete-frequency Fourier transform.

- for aperiodic discrete-time ( $n$ ) signals  $x[n]$
- yields discrete-frequency ( $k$ ) representation  $X[k]$
- finite length ( $N$ ) in both time and frequency

Two equivalent perspectives of the DFT:

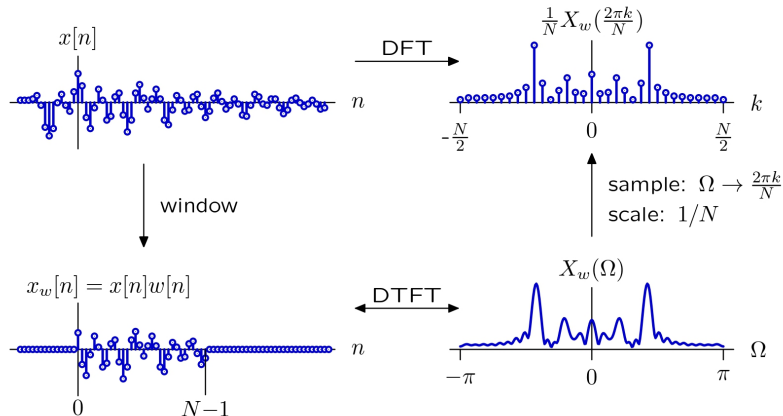
- DTFS of periodically-extended  $x_w[n] = x[n]w[n]$
- Sampled ( $\Omega \rightarrow 2\pi k/N$ ), scaled ( $1/N$ ) DTFT of  $x_w[n]$

**Circular convolution:**  $\frac{1}{N}(x \circledast h)[n] \iff X[k]H[k]$

- Start by computing the usual convolution  $(x * h)[n]$ .
- Wrap into base period:  $(x \circledast h)[n] = (x * h)[n \bmod N]$ .
- Finally, scale by  $1/N$ .

## Relation Between DFT and DTFT

Graphical depiction of relation between DFT and DTFT.



Graphic: Professor Denny Freeman (freeman@mit.edu)

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# #1. Short-Answer Questions

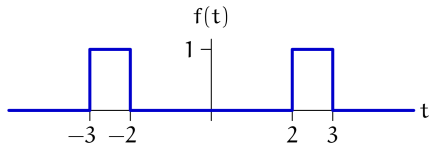
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The left column of the following table shows the input-output relations for four different systems, where  $x_i$  represents the input and  $y_i$  represents the output. For each system, determine if that system is **additive** and/or **homogeneous** and/or **time-invariant**. (Enter YES or NO.)

	<b>Add.</b>	<b>Hom.</b>	<b>Time-Inv.</b>
$y_1[n] = \left(-\frac{1}{2}\right)^n(x_1[n] + 1)$			
$y_2[n] = \sin(x_2[n])$			
$y_3(t) = \int_{-\infty}^t x_3(\tau) d\tau$			
$y_4(t) = t x_4(t)$			

# #1. Short-Answer Questions

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Determine a closed-form expression (e.g., no integrals) for the frequency response  $F(\omega)$  of a linear, time-invariant (LTI) system with impulse response

$$f(t) = \begin{cases} 1 & 2 \leq |t| \leq 3 \\ 0 & \text{otherwise.} \end{cases}$$

$F(\omega) =$

## #1. Short-Answer Questions

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Let  $G(\Omega)$  denote the frequency response of a linear, time-invariant (LTI) system with unit-sample response

$$g[n] = \delta[n] - \delta[n - 2].$$

Sketch  $|G(\Omega)|$  and  $\angle G(\Omega)$ . Label key features of your plots.

## #2. Cascaded Systems

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$$x[n] \rightarrow \boxed{\mathcal{S}_1} \rightarrow y_1[n]$$

Let  $\mathcal{S}_1$  denote the LTI system with unit-sample response

$$h_1[n] = u[n] = \begin{cases} 1 & n \geq 0 \\ 0 & n < 0. \end{cases}$$

Suppose the input to this system is

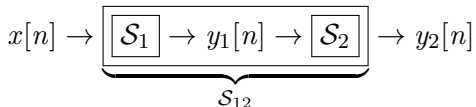
$$x[n] = (0.9)^n u[n] = \begin{cases} (0.9)^n & n \geq 0 \\ 0 & n < 0. \end{cases}$$

Determine a closed-form expression for the response  $y_1[n]$ .

$y_1[n] =$

## #2. Cascaded Systems

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Let  $\mathcal{S}_{12}$  represent the LTI system that results when two LTI systems ( $\mathcal{S}_1$  and  $\mathcal{S}_2$ ) are connected in cascade (so that the output of  $\mathcal{S}_1$  is the input to  $\mathcal{S}_2$ ) as shown in the figure above.

Let  $h_i[n]$  denote the unit-sample response of system  $\mathcal{S}_i$ .

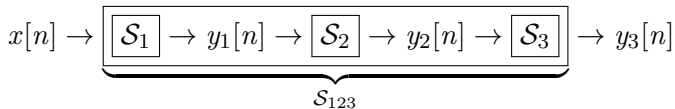
$$h_1[n] = h_2[n] = u[n] = \begin{cases} 1 & n \geq 0 \\ 0 & n < 0 \end{cases}$$

Determine a closed-form expression for  $h_{12}[n]$ .

$h_{12}[n] =$

## #2. Cascaded Systems

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Let  $\mathcal{S}_{123}$  represent the LTI system that results when three LTI systems ( $\mathcal{S}_1$ ,  $\mathcal{S}_2$ , and  $\mathcal{S}_3$ ) are connected in cascade, as shown in the figure above.

Let  $h_i[n]$  denote the unit sample response of system  $\mathcal{S}_i$ .

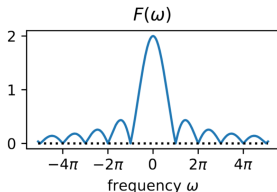
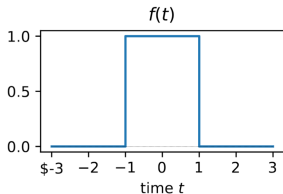
$$h_1[n] = h_2[n] = h_3[n] = u[n] = \begin{cases} 1 & n \geq 0 \\ 0 & n < 0 \end{cases}$$

Determine a closed-form expression for  $h_{123}[n]$ .

$h_{123}[n] =$

## #3. Pulsed Relations

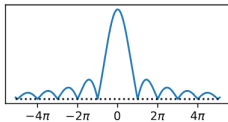
$$f(t) = \begin{cases} 1 & |t| \leq 1 \\ 0 & \text{otherwise} \end{cases} \iff F(\omega) = \int_{-\infty}^{\infty} f(t)e^{-j\omega t} dt = 2 \frac{\sin(\omega)}{\omega}$$



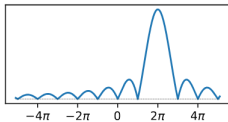
Match each transformation of  $f(t)$  below to the plot on the next page that shows the magnitude of its CTFT.

$f(\frac{1}{2}t)$	$f(2t)$	$f(t-1)$	$f(t+1)$	$tf(t)$
$(f * f)(t)$	$f(t)e^{j2\pi t}$	$f(t)e^{-j2\pi t}$	$f(t)\sin(2\pi t)$	$f(t)\cos(2\pi t)$

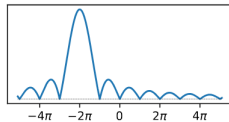
A



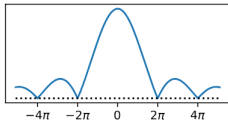
B



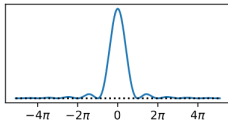
C



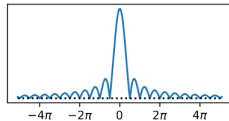
D



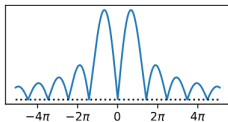
E



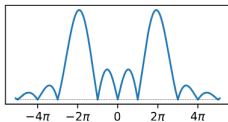
F



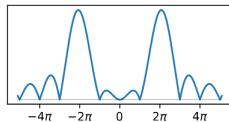
G



H



I

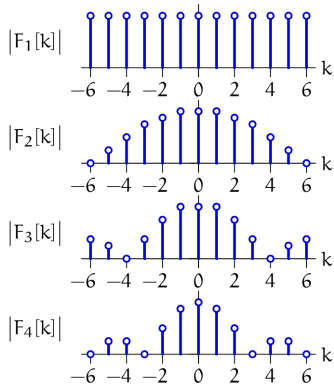
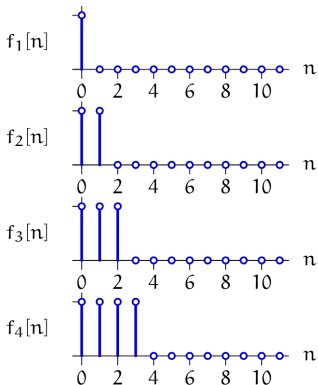


$f(\frac{1}{2}t)$	$f(2t)$	$f(t-1)$	$f(t+1)$	$tf(t)$
$(f * f)(t)$	$f(t)e^{j2\pi t}$	$f(t)e^{-j2\pi t}$	$f(t)\sin(2\pi t)$	$f(t)\cos(2\pi t)$

## #4. Pulses In, Pulses Out

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The first 12 samples of four periodic signals, each periodic in  $N = 12$ , are shown in the left column below. The magnitudes of their Fourier series coefficients are shown in the right column.



## #4. Pulses In, Pulses Out

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Consider a system  $\mathcal{S}_{24}$  that produces  $f_4[n]$  as output when  $f_2[n]$  is the input.

$$f_2[n] \rightarrow \boxed{\mathcal{S}_{24}} \rightarrow f_4[n]$$

Determine a closed-form expression for  $h_{24}[n]$ .

$$h_{24}[n] =$$

Determine a closed-form expression for  $H_{24}[k]$ .

$$H_{24}[k] =$$

Sketch  $|H_{24}[k]|$ , the magnitude of  $H_{24}[k]$ , over  $-6 \leq k \leq 6$ .

## #4. Pulses In, Pulses Out

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Consider 16 systems, each defined by input  $f_i[n]$  and output  $f_o[n]$ .

$$\text{input } f_i[n] \rightarrow \boxed{\mathcal{S}_{io}} \rightarrow \text{output } f_o[n]$$

Some of these systems could be LTI; others cannot. Determine which of the 16 systems cannot possibly be LTI, and enter an **X** in the corresponding box below.

$\mathcal{S}_{io}$	$o = 1$	$o = 2$	$o = 3$	$o = 4$
$i = 1$				
$i = 2$				
$i = 3$				
$i = 4$				