6.300: Signal Processing

Image Processing with the DFT

Multiplication of discrete Fourier transforms corresponds to space-domain **circular convolution**.

$$f[r,c] \rightarrow \boxed{h[r,c]} \rightarrow g[r,c] = \frac{1}{RC}(f \circledast h)[r,c]$$

$$F[k_r,k_c] \rightarrow \boxed{H[k_r,k_c]} \rightarrow G[k_r,k_c] = F[k_r,k_c]H[k_r,k_c]$$

Fast Fourier transform (FFT) algorithms for computing the **discrete Fourier transform (DFT)** make multidimensional signal processing practical.

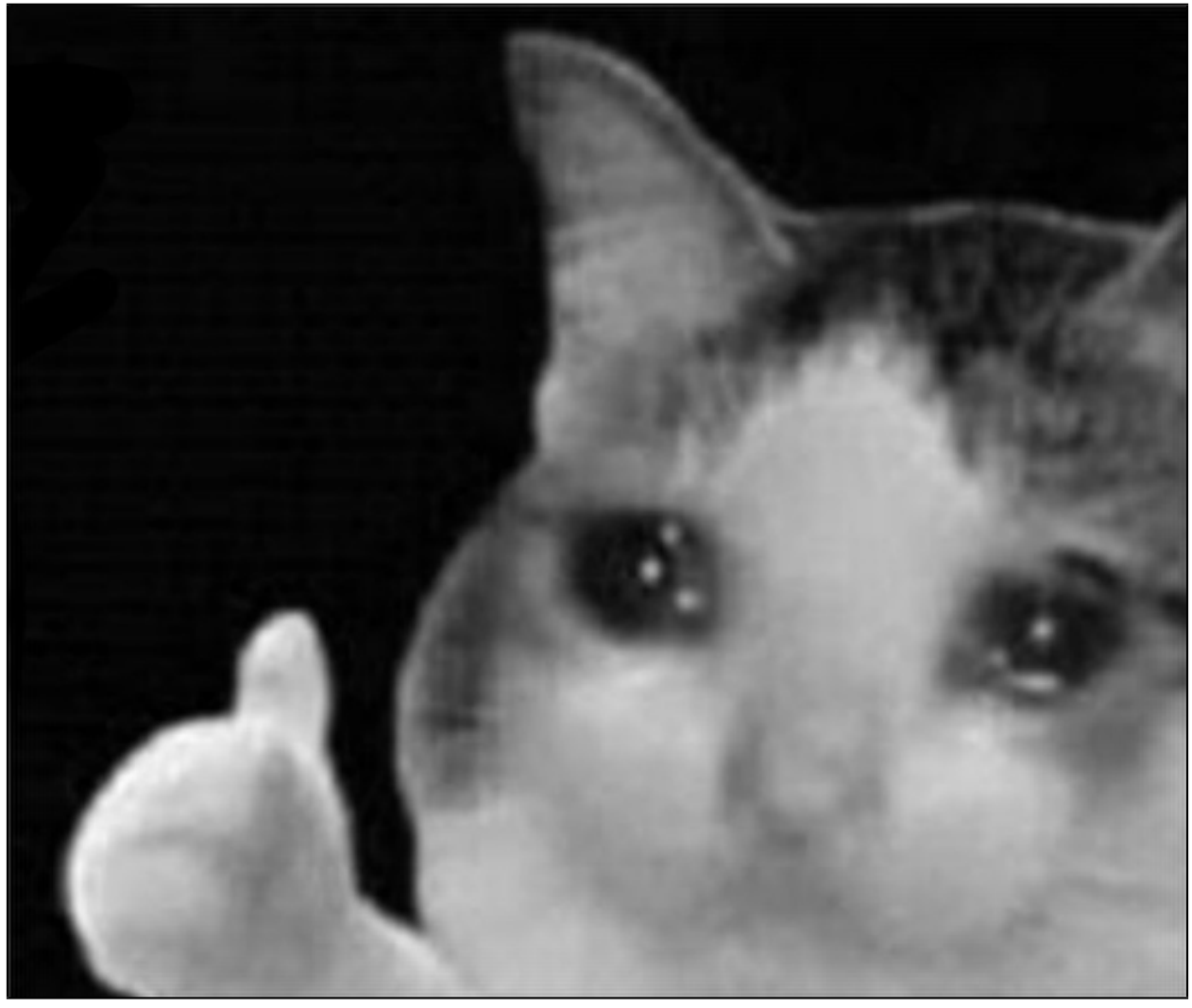
April 30, 2026

Agenda for Recitation

- Image processing with the discrete Fourier transform

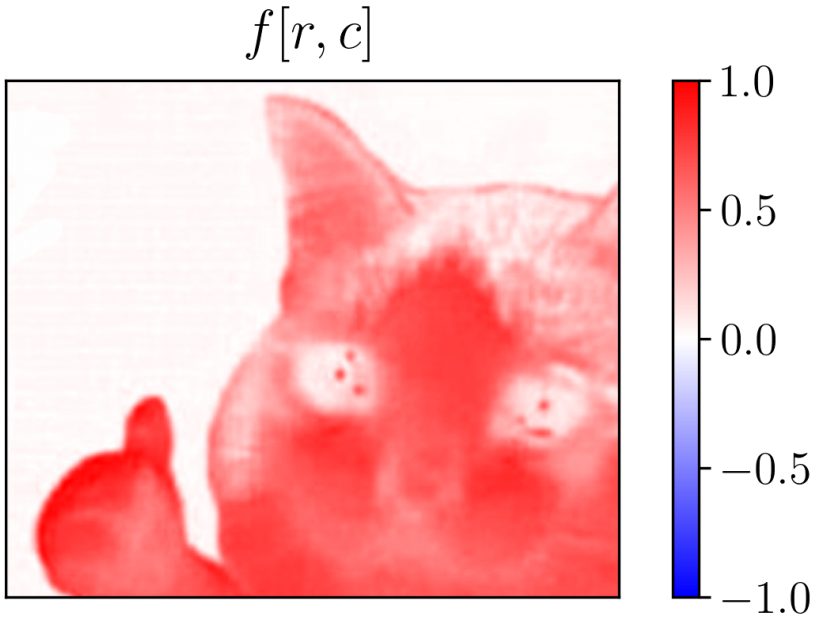
What questions do you have from lecture?

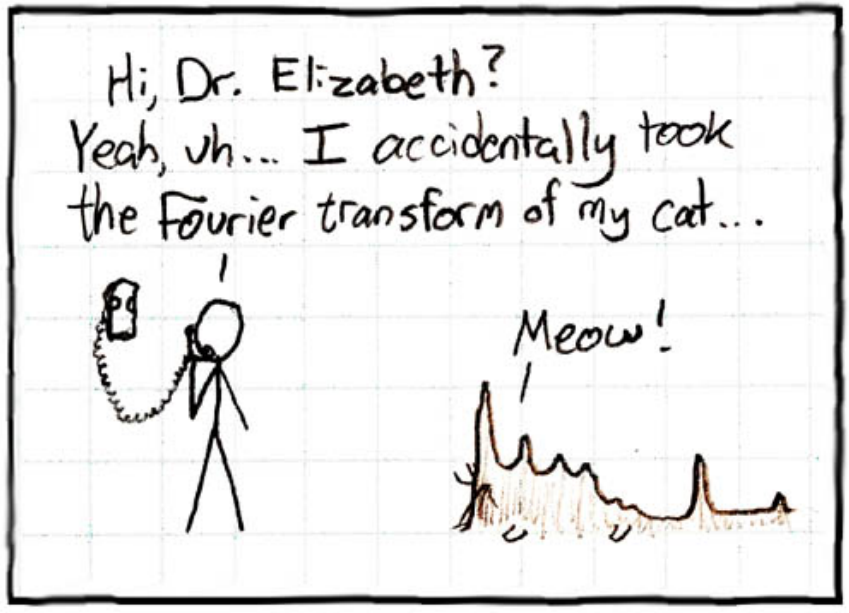
DSP: Digi-tail Signal Paw-cessing



This image shows a crying cat giving a thumbs-up.

DSP: Digi-tail Signal Paw-cessing



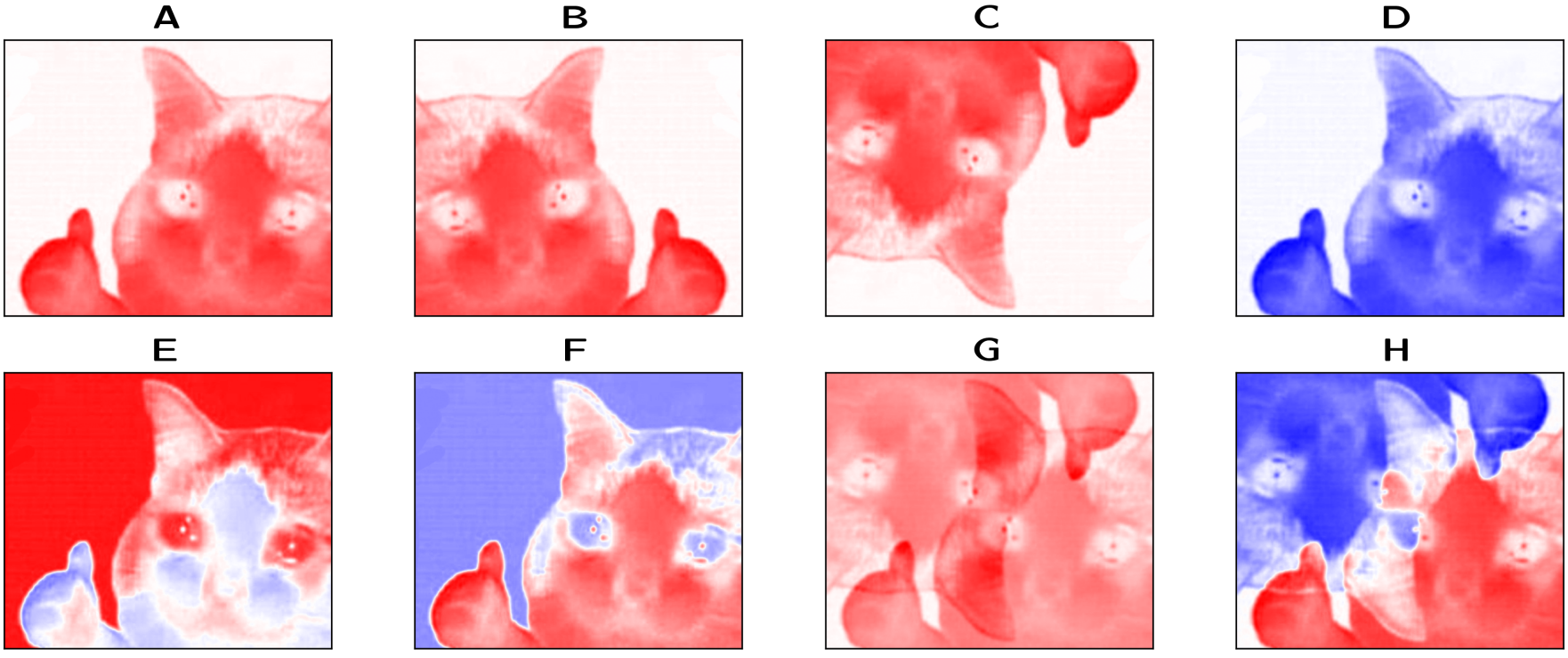


Let $f[r, c]$ represent the space-domain image at left, and let $F[k_r, k_c] = \mathrm{DFT}\{f[r, c]\}$ denote the 2D DFT.

Bonus: Relevant comic (`https://xkcd.com/26/`) at right.

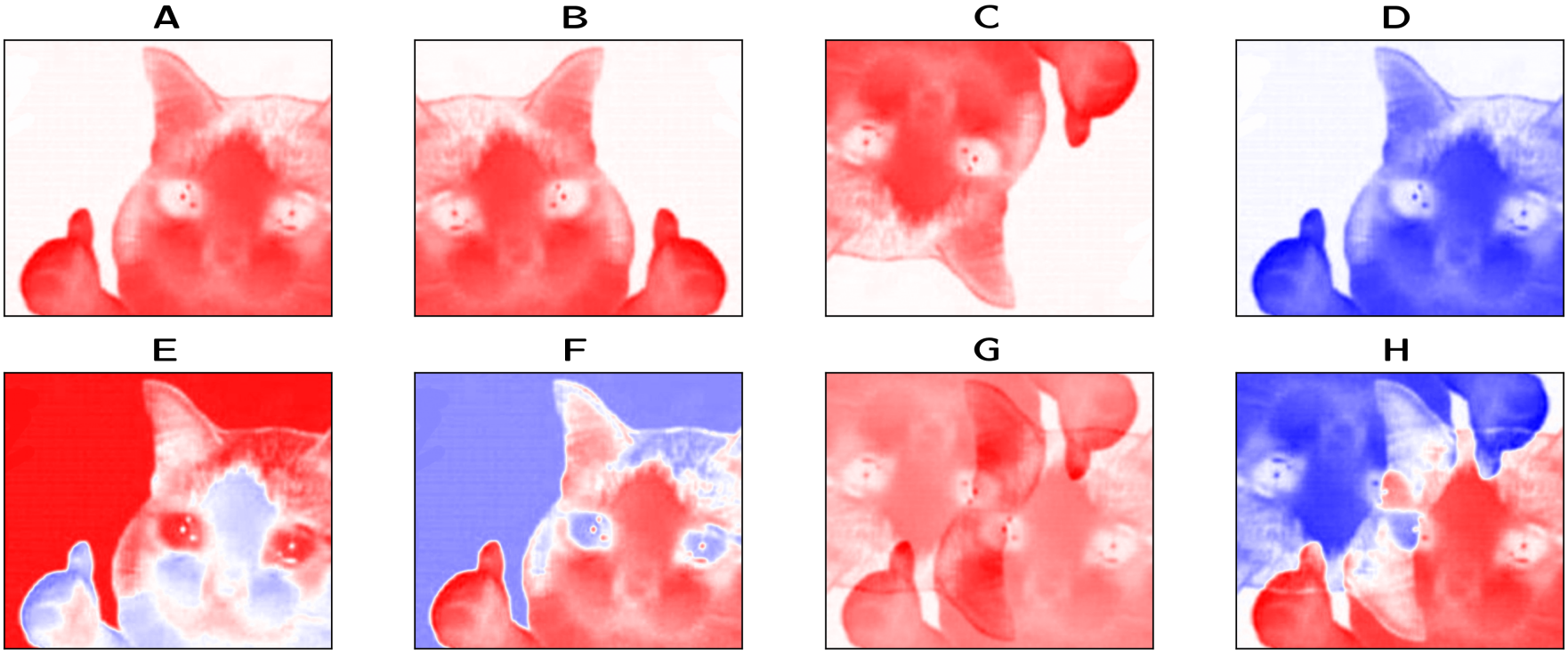
Let's determine what happens to our image when we perform a few operations in the frequency domain.

DSP: Digi-tail Signal Paw-cessing



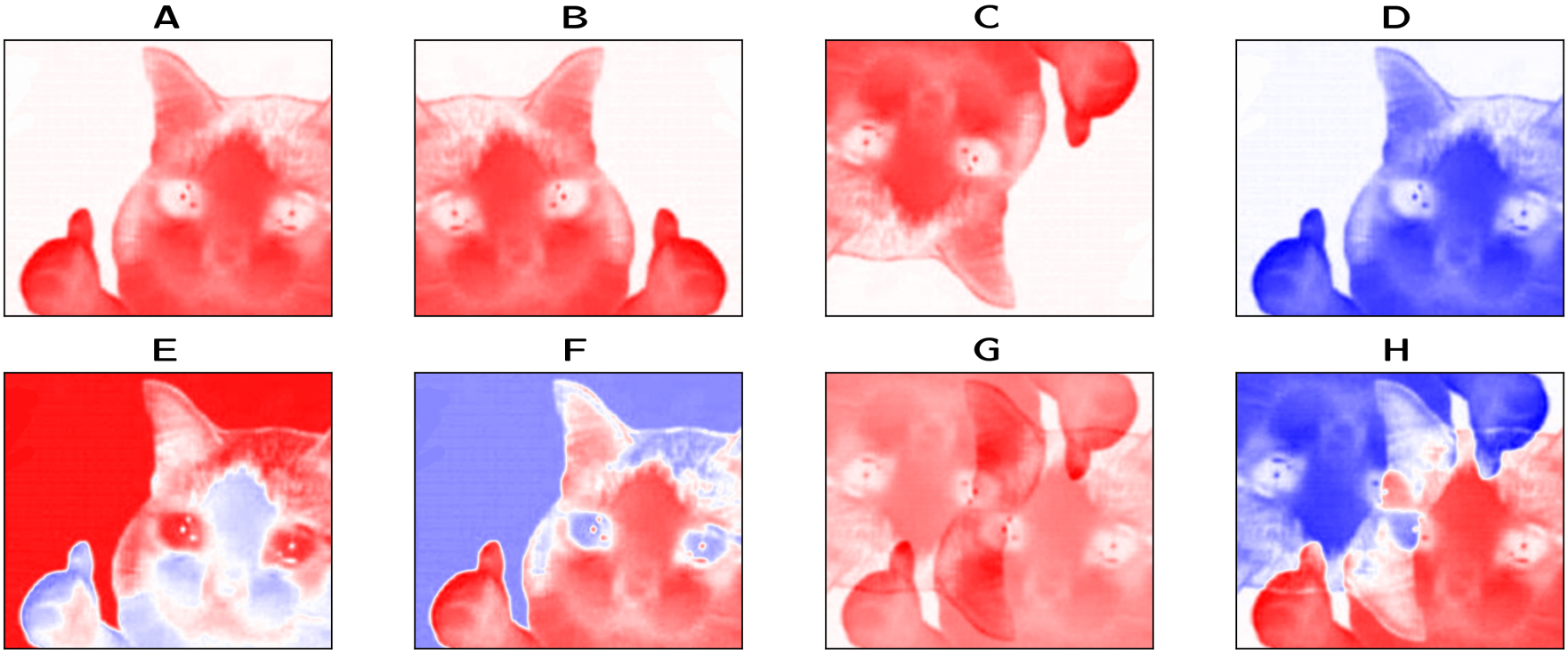
Suppose we compute the inverse DFT (IDFT) of the real part of $F[k_r, k_c]$. Which panel (if any) shows the resulting image?

DSP: Digi-tail Signal Paw-cessing



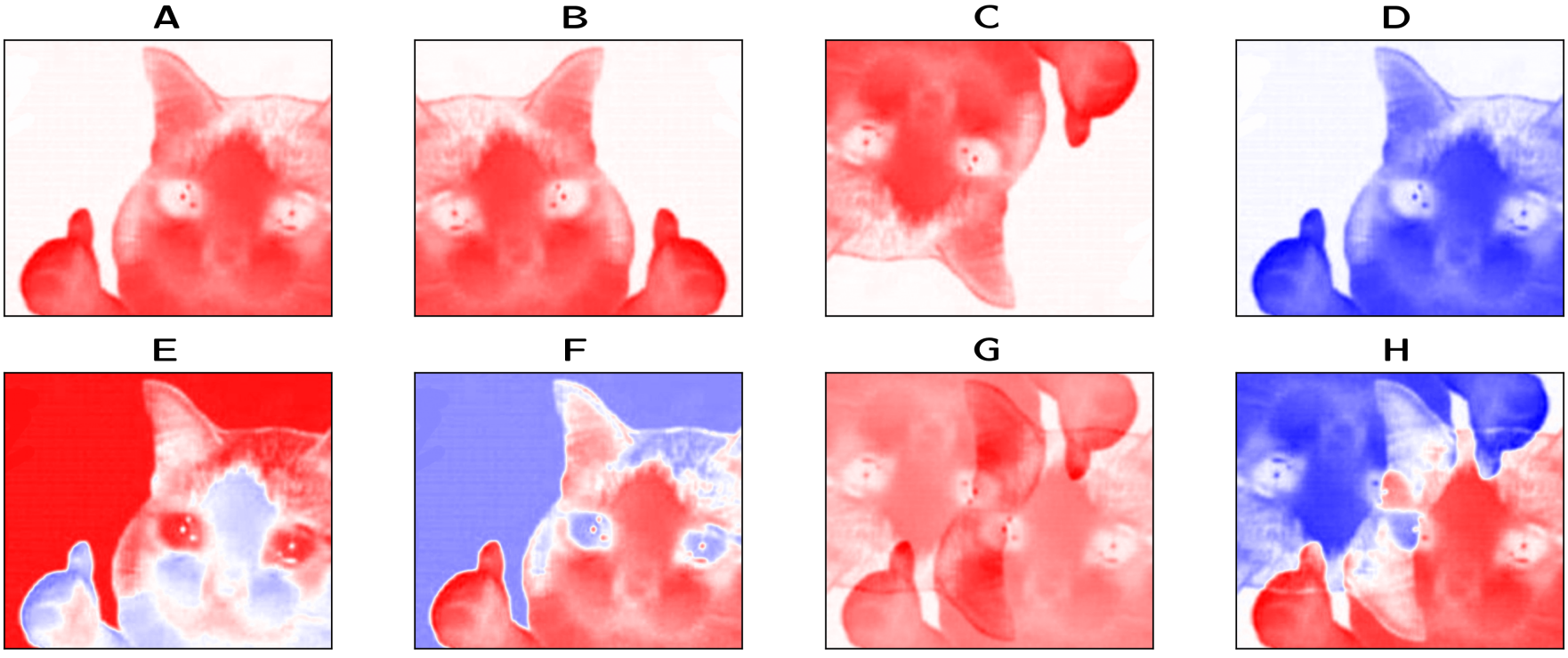
Suppose we compute the inverse DFT (IDFT) of the real part of $F[k_r, k_c]$. Which panel (if any) shows the resulting image? **G**

DSP: Digi-tail Signal Paw-cessing



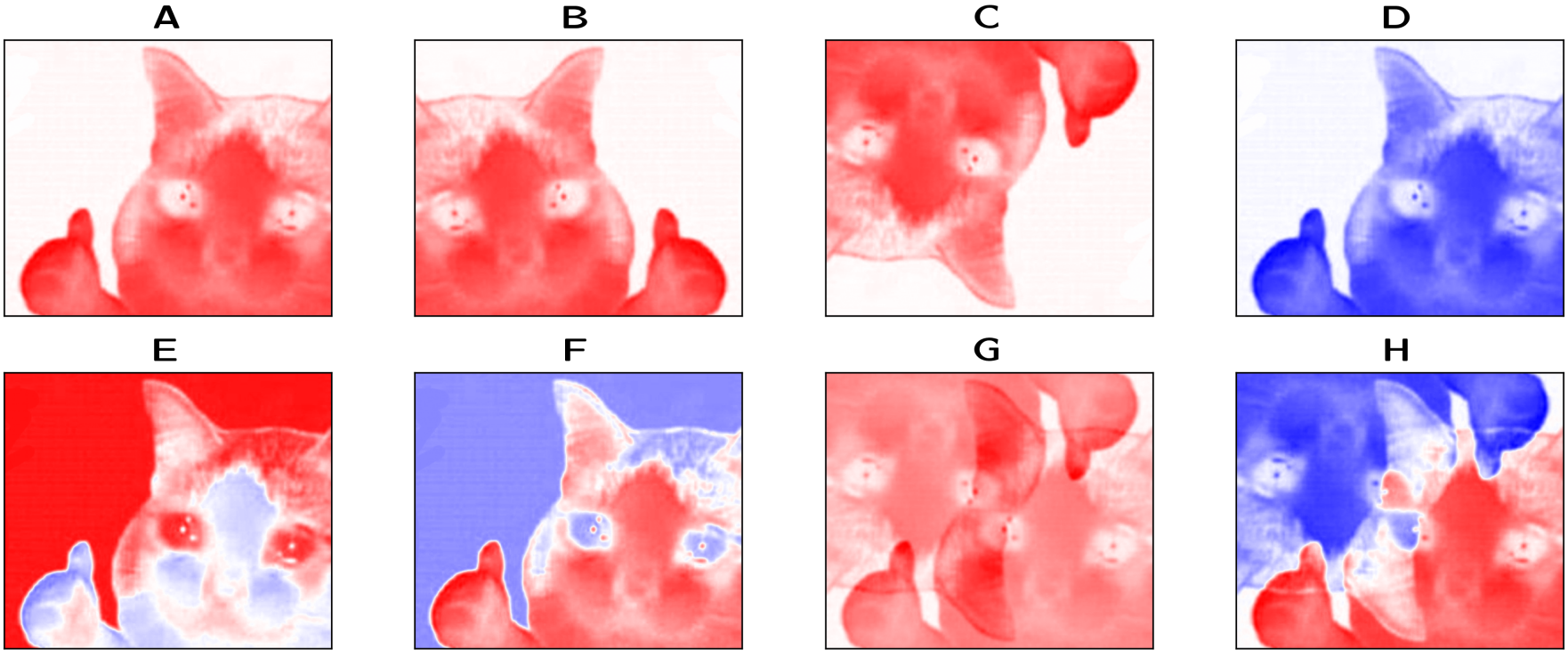
Suppose we compute the IDFT of the imaginary part of $F[k_r, k_c]$. Which panel (if any) shows the resulting image?

DSP: Digi-tail Signal Paw-cessing



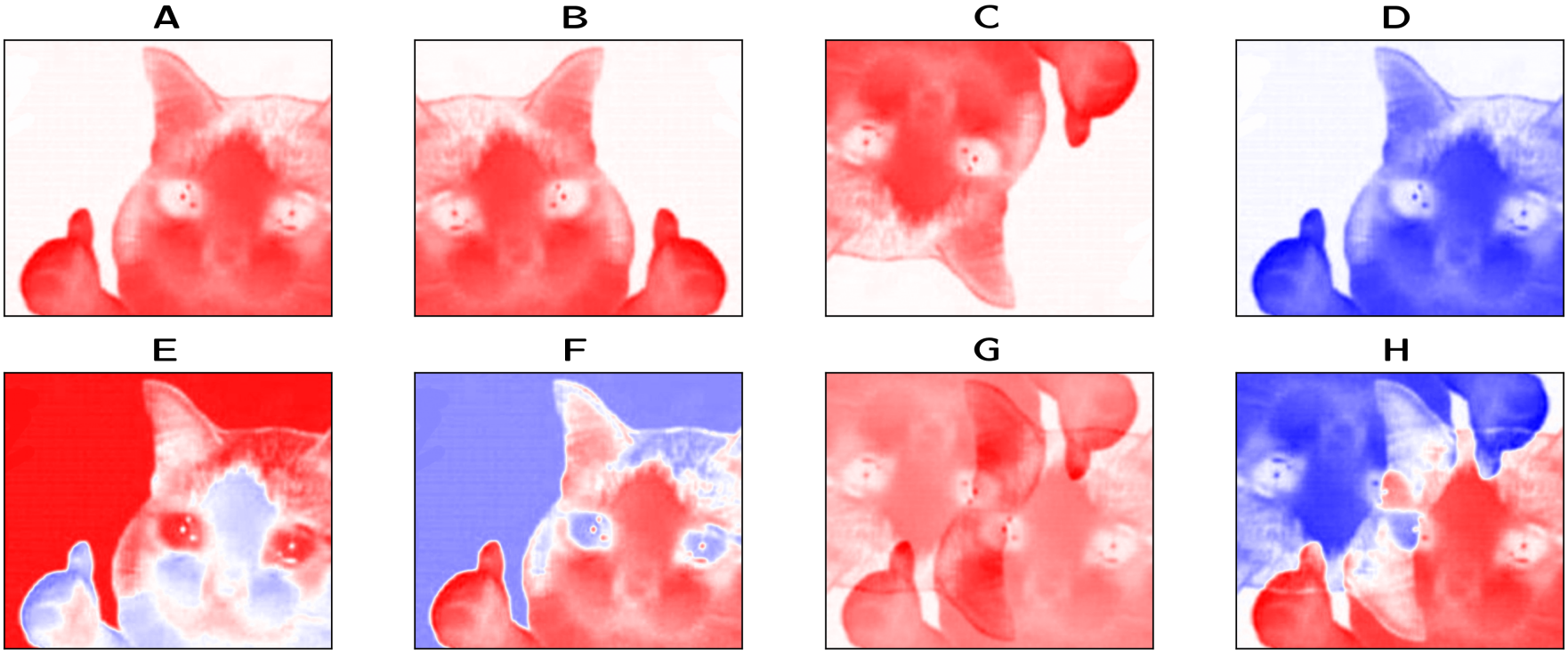
Suppose we compute the IDFT of the imaginary part of $F[k_r, k_c]$. Which panel (if any) shows the resulting image? none

DSP: Digi-tail Signal Paw-cessing



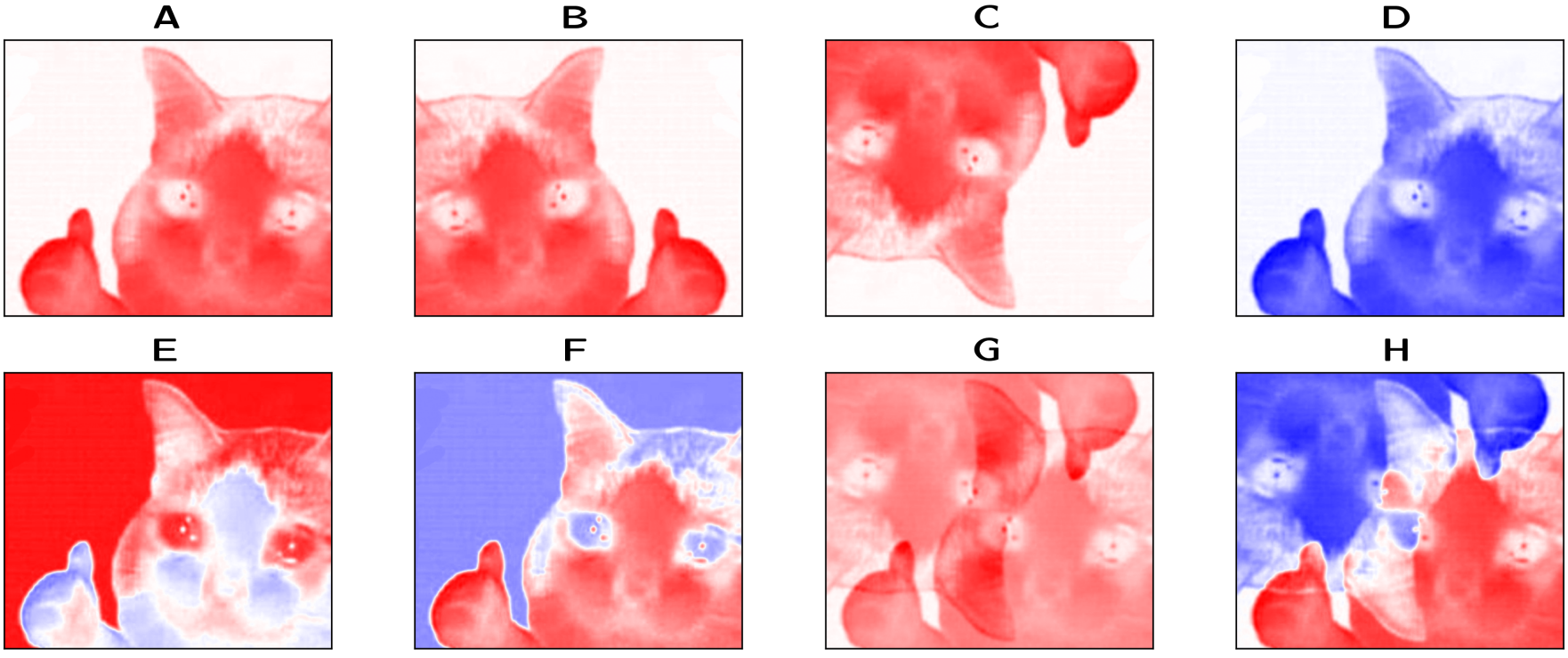
Suppose we compute the IDFT of j times the imaginary part of $F[k_r, k_c]$. Which panel (if any) shows the resulting image?

DSP: Digi-tail Signal Paw-cessing



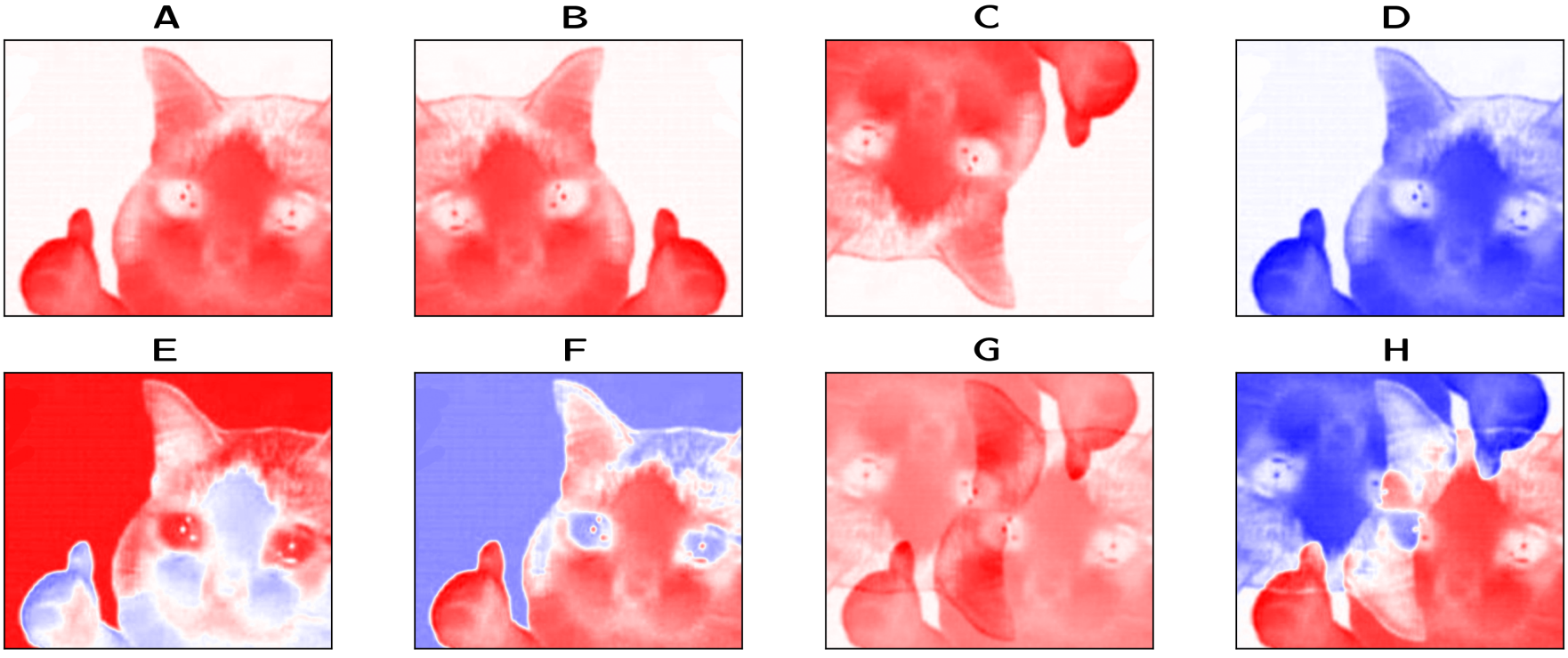
Suppose we compute the IDFT of j times the imaginary part of $F[k_r, k_c]$. Which panel (if any) shows the resulting image? **H**

DSP: Digi-tail Signal Paw-cessing



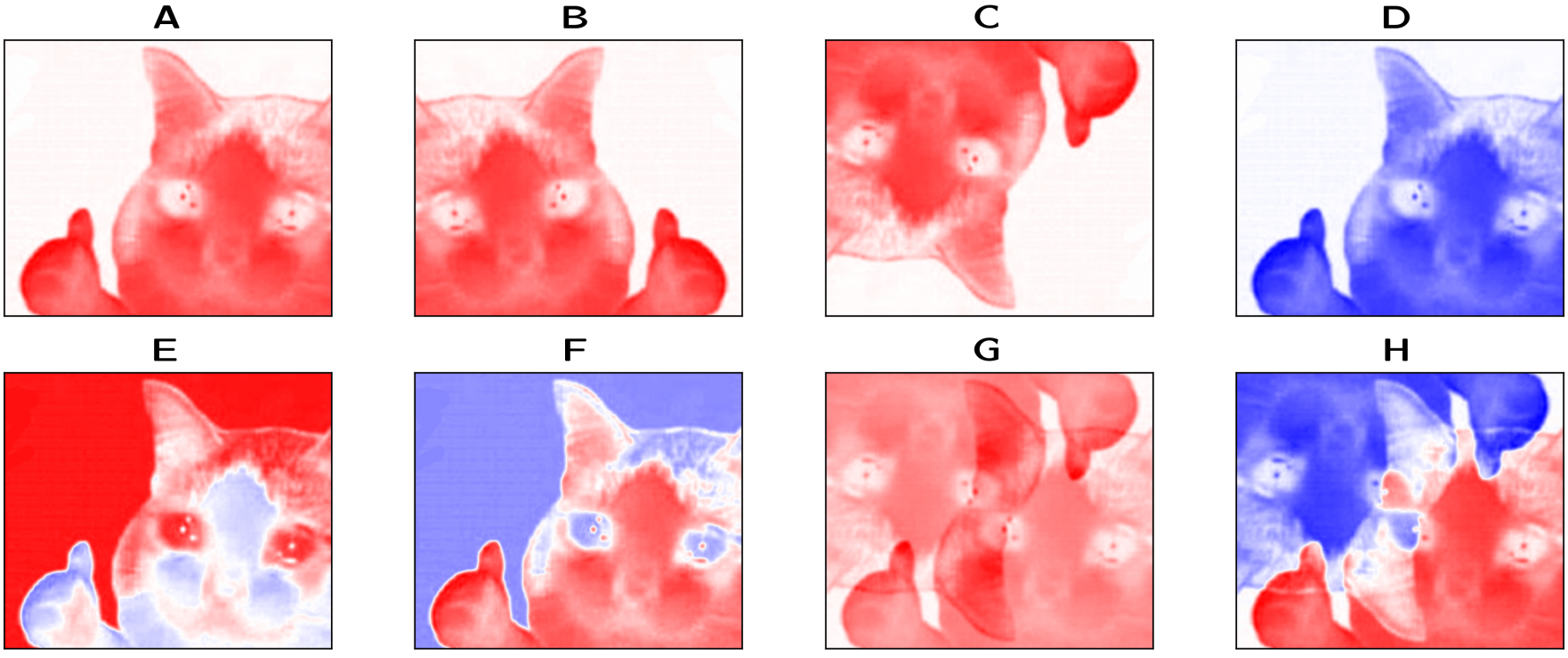
Suppose we set $F[0,0] = 0$ and compute the IDFT. Which panel (if any) shows the resulting image?

DSP: Digi-tail Signal Paw-cessing



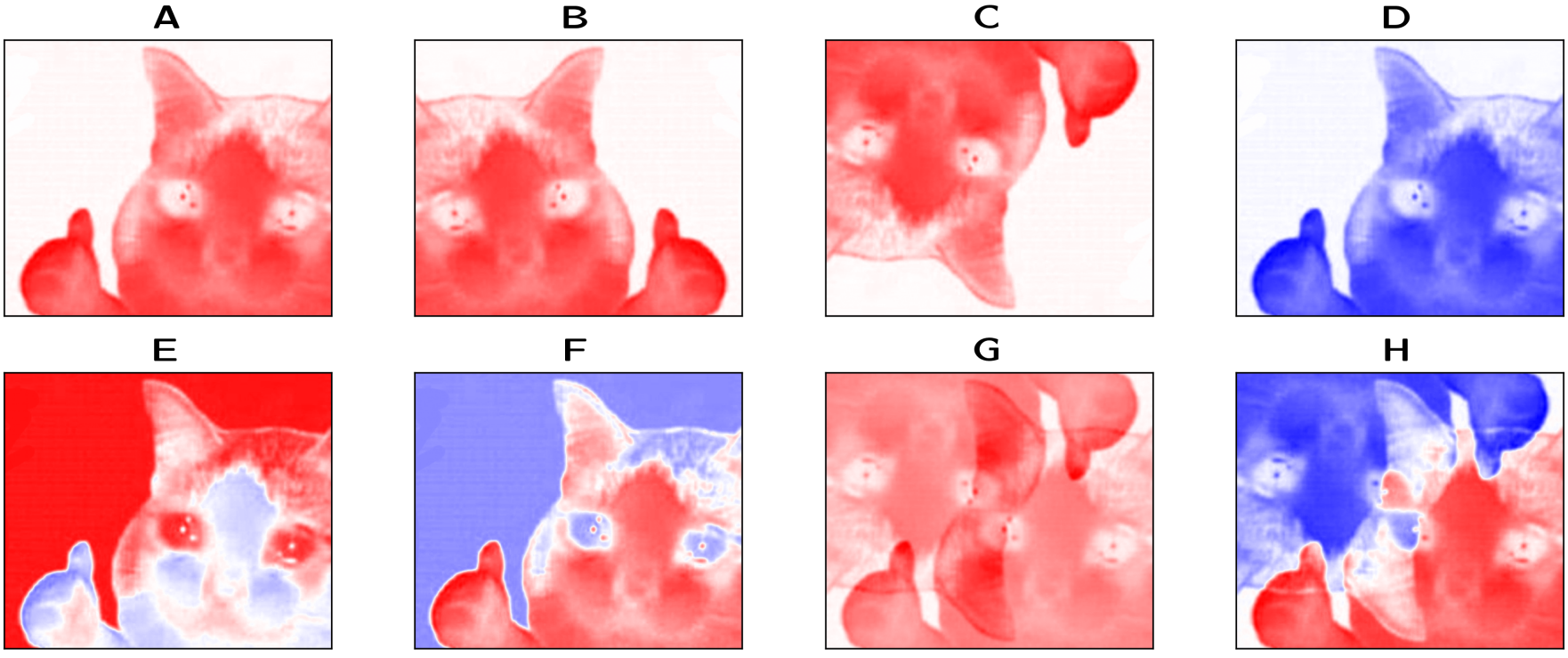
Suppose we set $F[0,0] = 0$ and compute the IDFT. Which panel (if any) shows the resulting image? **F**

DSP: Digi-tail Signal Paw-cessing



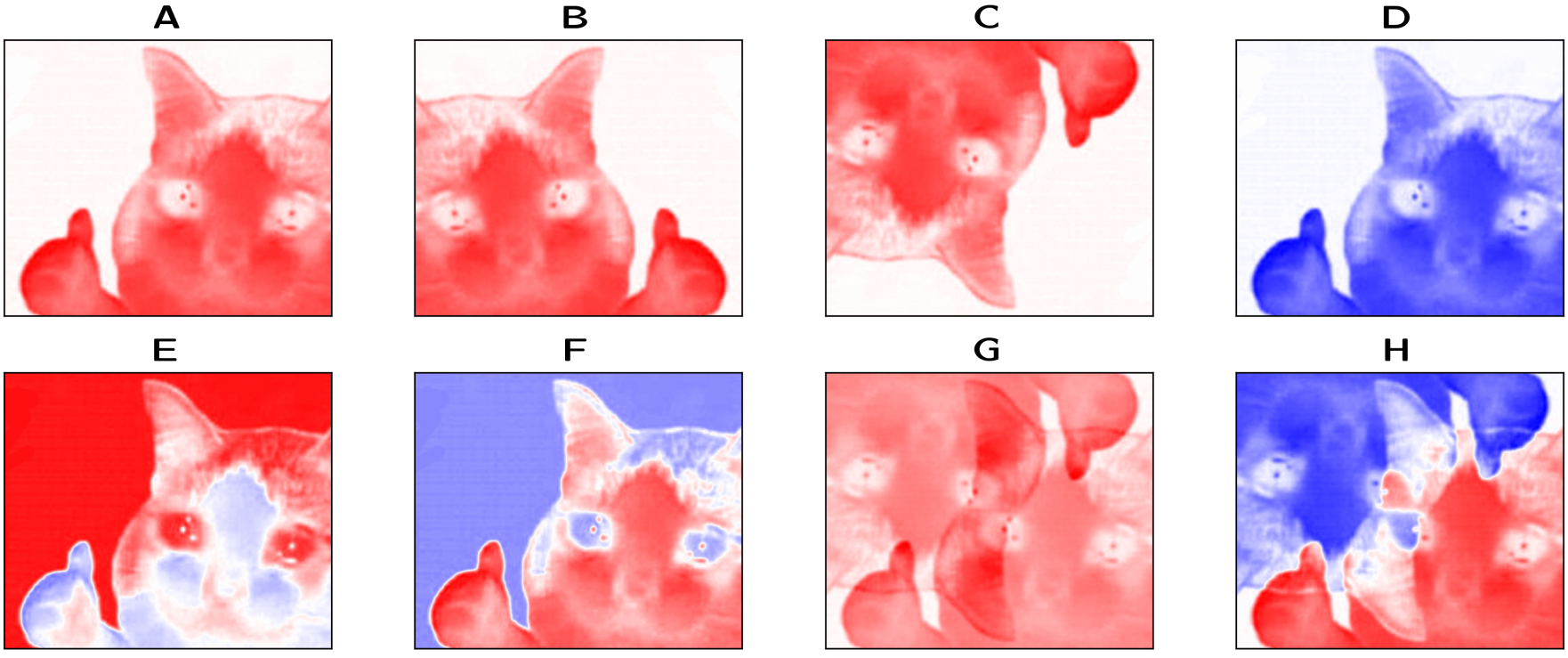
Suppose we multiply each value of $F[k_r, k_c]$ by $e^{-j\pi}$ and compute the IDFT. Which panel (if any) shows the resulting image?

DSP: Digi-tail Signal Paw-cessing



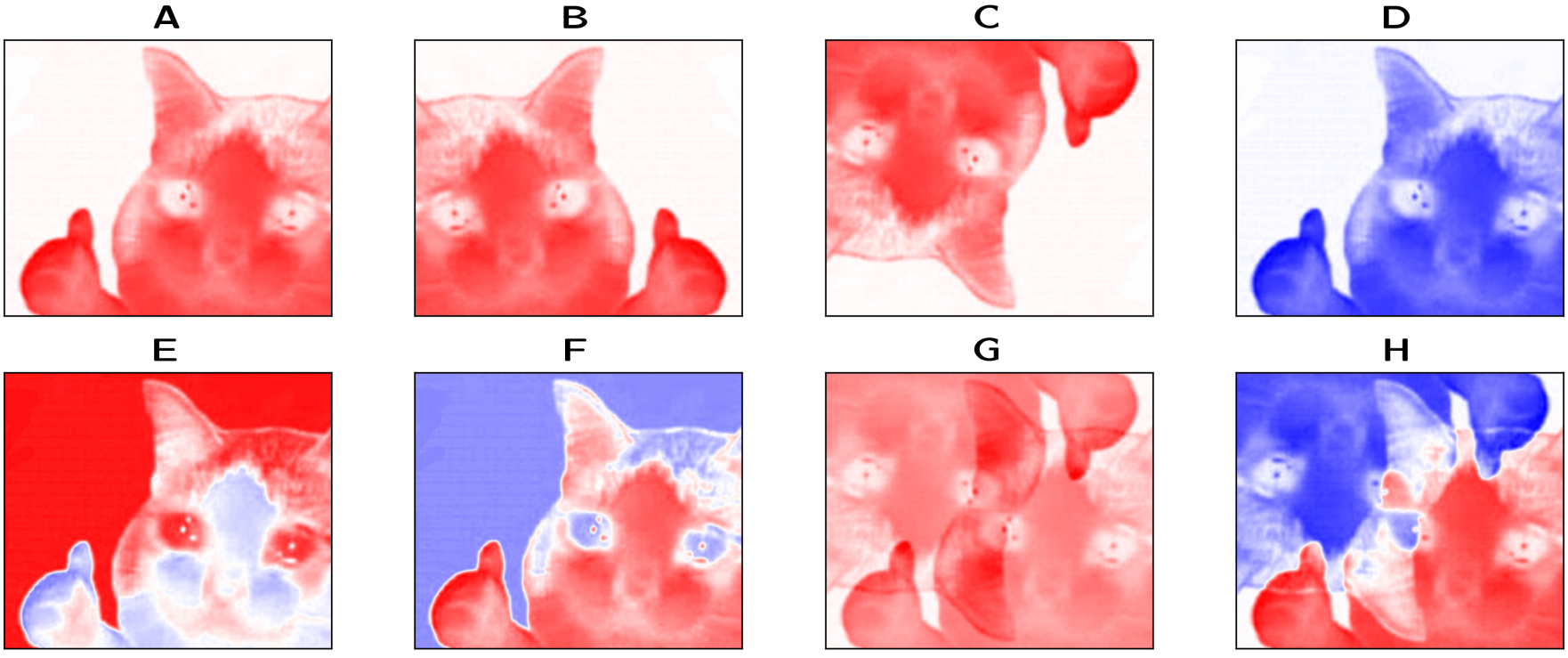
Suppose we multiply each value of $F[k_r, k_c]$ by $e^{-j\pi}$ and compute the IDFT. Which panel (if any) shows the resulting image? **D**

DSP: Digi-tail Signal Paw-cessing



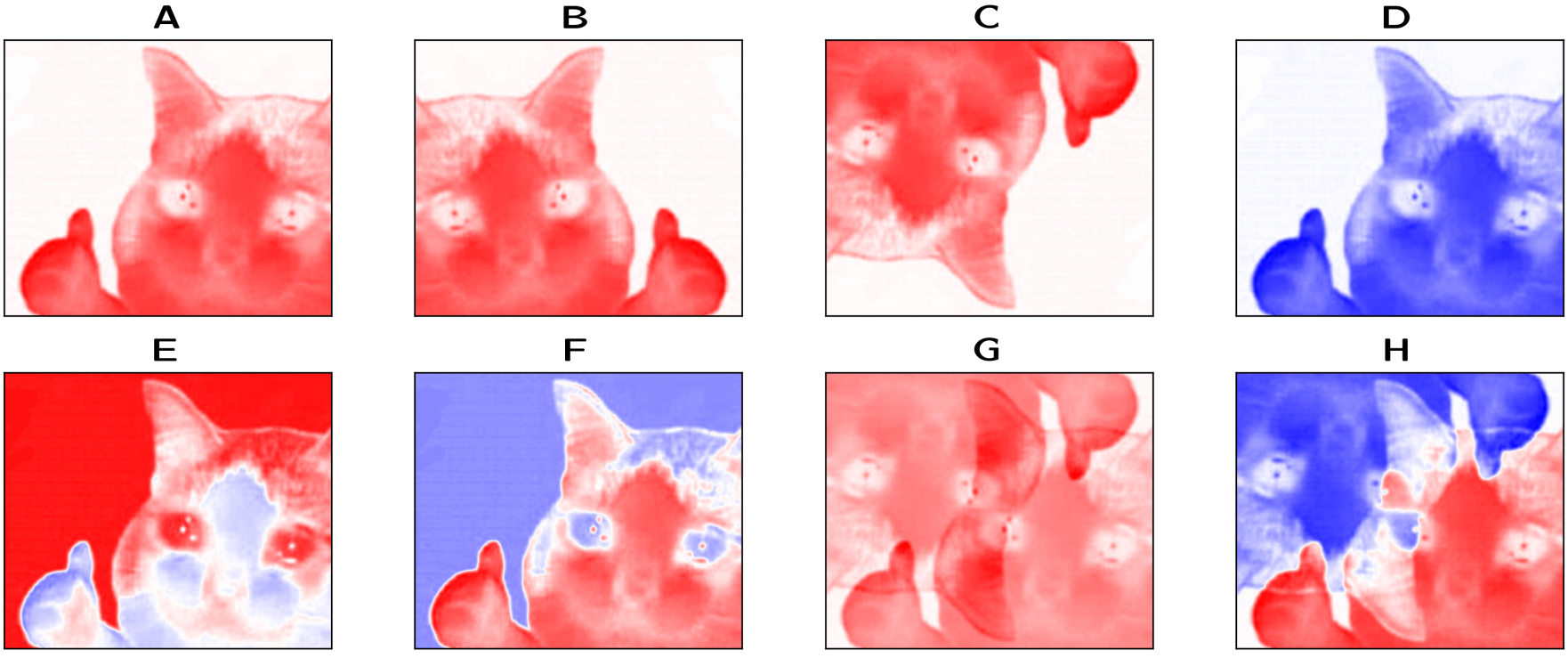
Suppose we multiply each value of $F[k_r, k_c]$ except $F[0, 0]$ by -1 and compute the IDFT. Which panel (if any) shows the resulting image?

DSP: Digi-tail Signal Paw-cessing



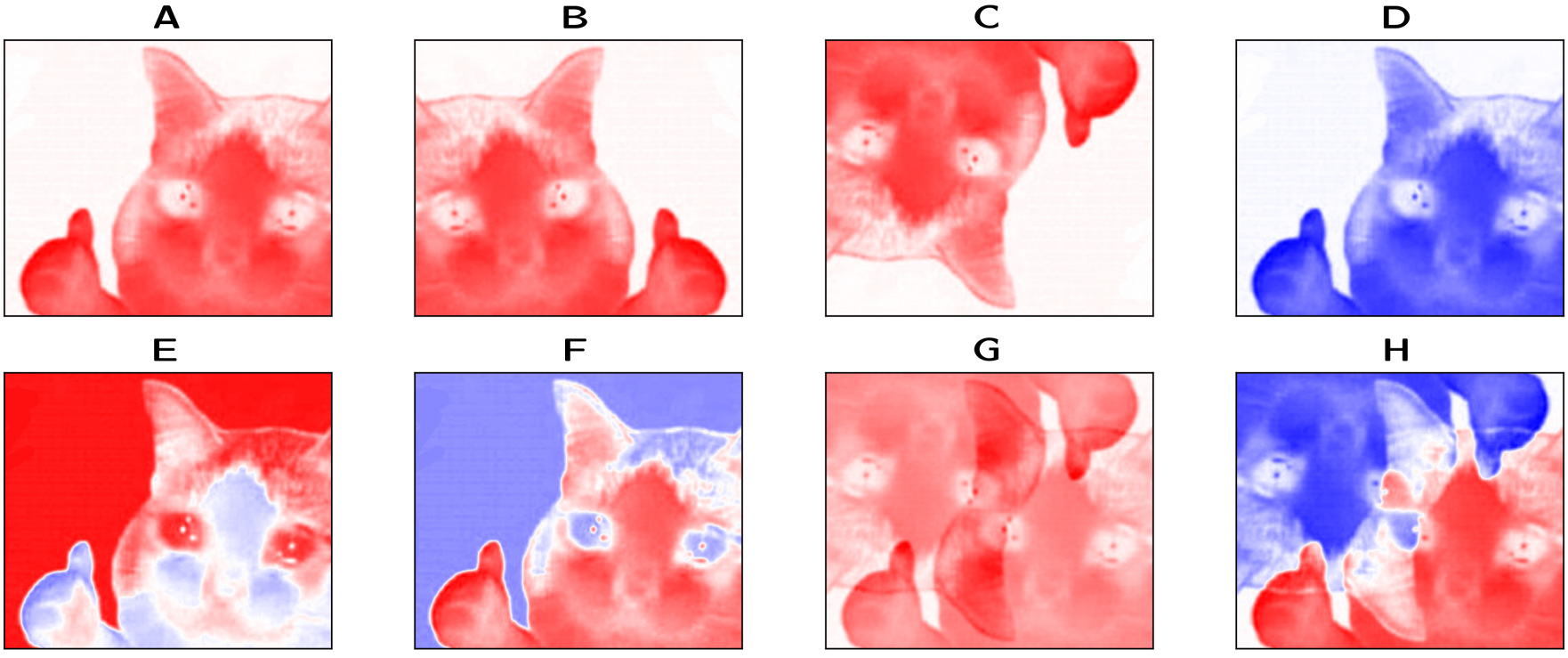
Suppose we multiply each value of $F[k_r, k_c]$ except $F[0, 0]$ by -1 and compute the IDFT. Which panel (if any) shows the resulting image? **E**

DSP: Digi-tail Signal Paw-cessing



Suppose we negate the phase of $F[k_r, k_c]$ and compute the IDFT. Which panel (if any) shows the resulting image?

DSP: Digi-tail Signal Paw-cessing

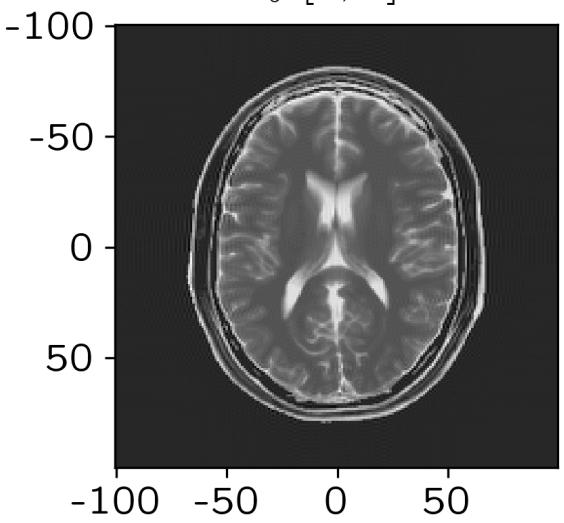


Suppose we negate the phase of $F[k_r, k_c]$ and compute the IDFT. Which panel (if any) shows the resulting image? **C**

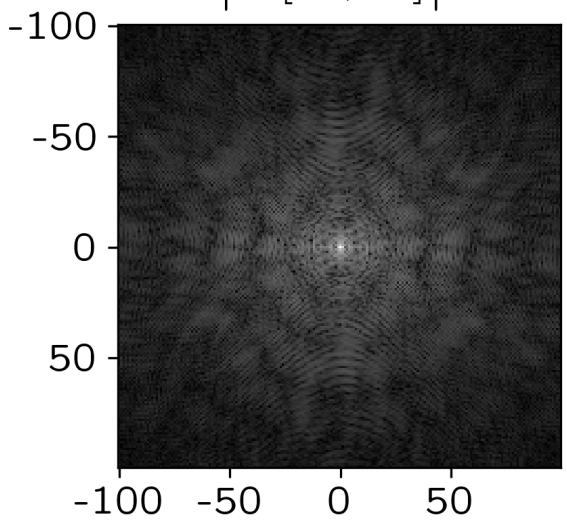
This Is Your Brain on DSP

Let $f[r, c]$ represent the monochrome 200×200 image shown below at left. Let $F[k_r, k_c]$ denote the 2D DFT computed with $R = C = 200$. In each image, black denotes the minimum value and white denotes the maximum value.[1] Note that the minima and maxima may differ between the images here and on the following slides.

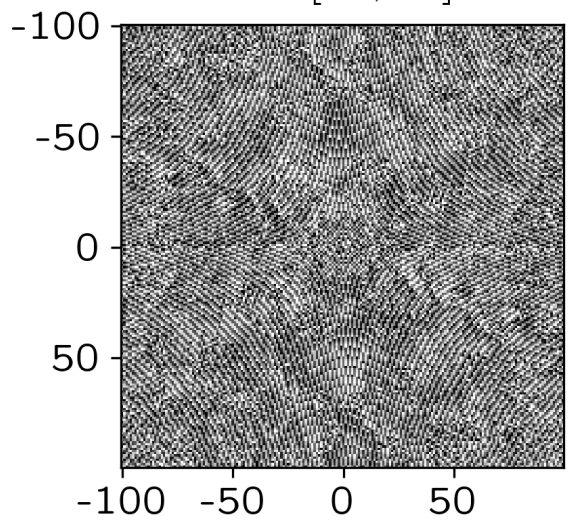
$f[r, c]$



$\left|F[k_r, k_c]\right|$

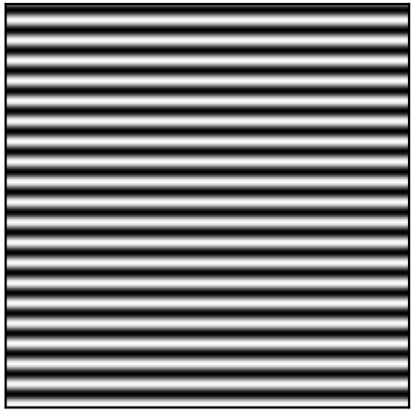


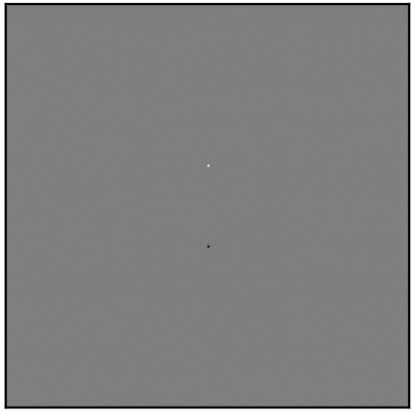
$\measuredangle F[k_r, k_c]$

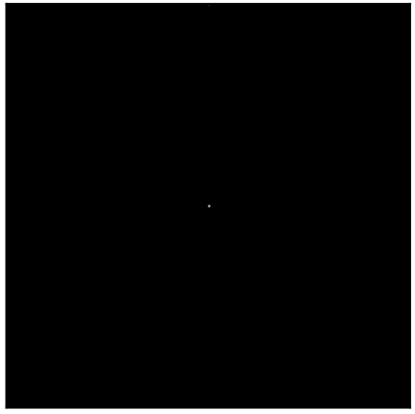


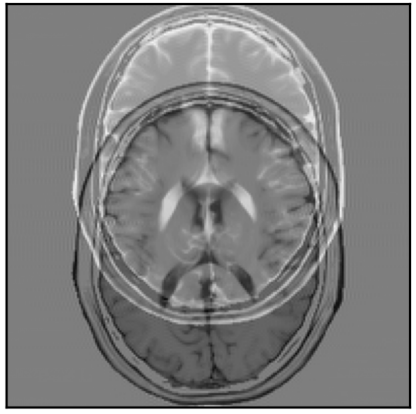
[1] For $f[r, c]$, you may assume that white denotes an arbitrary positive value and black denotes a much smaller positive value.

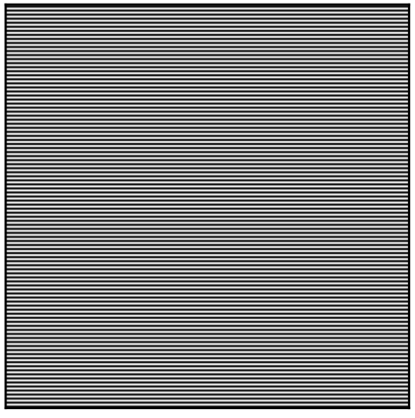
This Is Your Brain on DSP

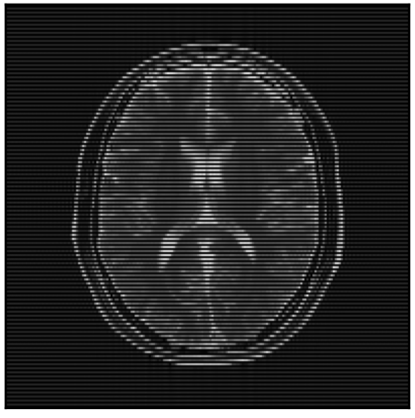
$g_1[r, c]$ 

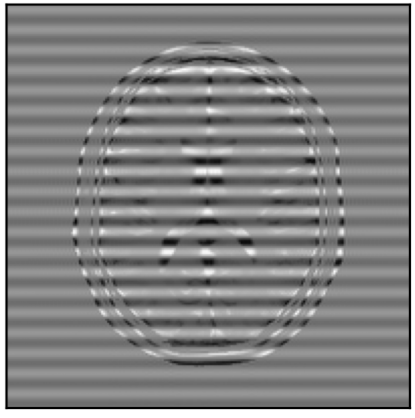
$g_2[r, c]$ 

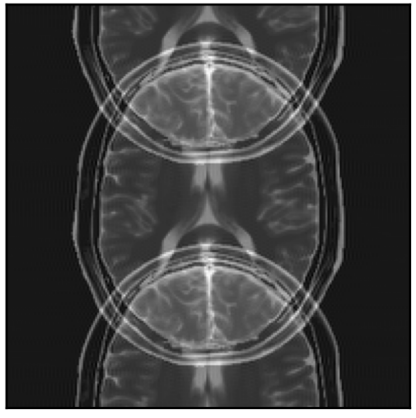
$g_3[r, c]$ 

$g_4[r, c]$ 

$g_5[r, c]$ 

$g_6[r, c]$ 

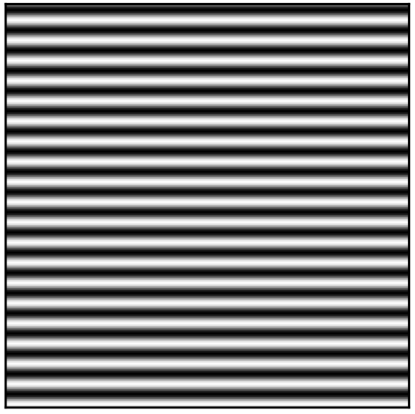
$g_7[r, c]$ 

$g_8[r, c]$ 

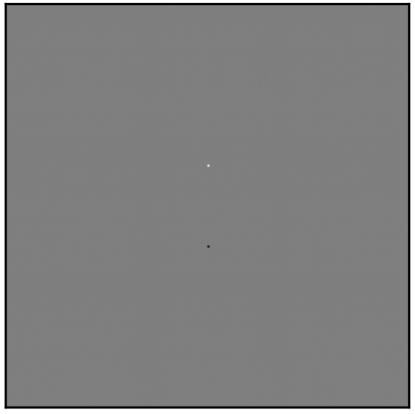
Let $h_1[r, c] = \sin\left(\frac{40\pi}{200} r\right)$. Which is $(f \times h_1)[r, c]$?

This Is Your Brain on DSP

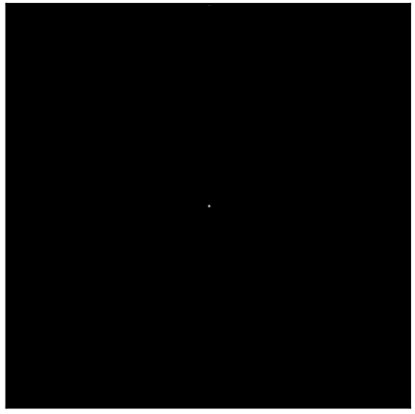
$g_1[r, c]$



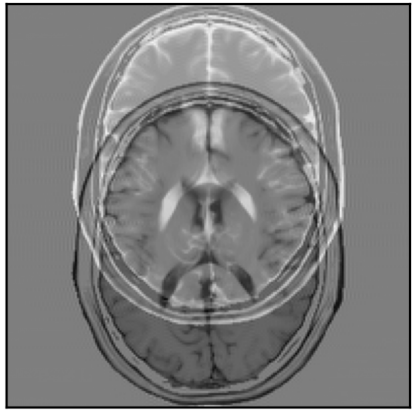
$g_2[r, c]$



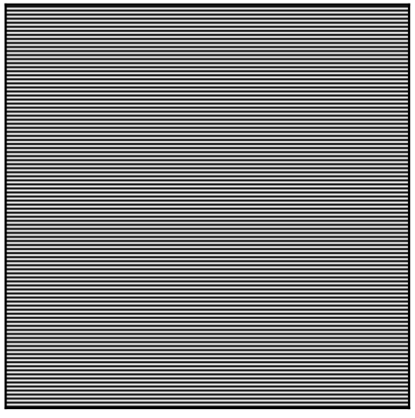
$g_3[r, c]$



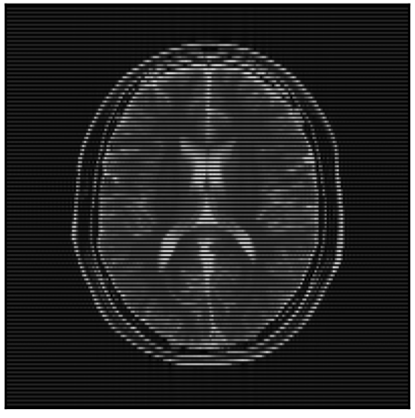
$g_4[r, c]$



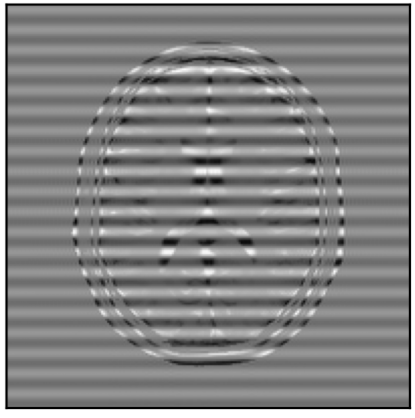
$g_5[r, c]$



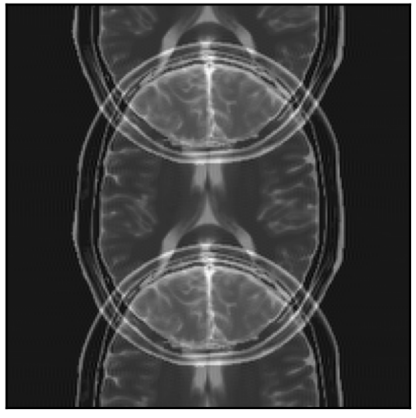
$g_6[r, c]$



$g_7[r, c]$



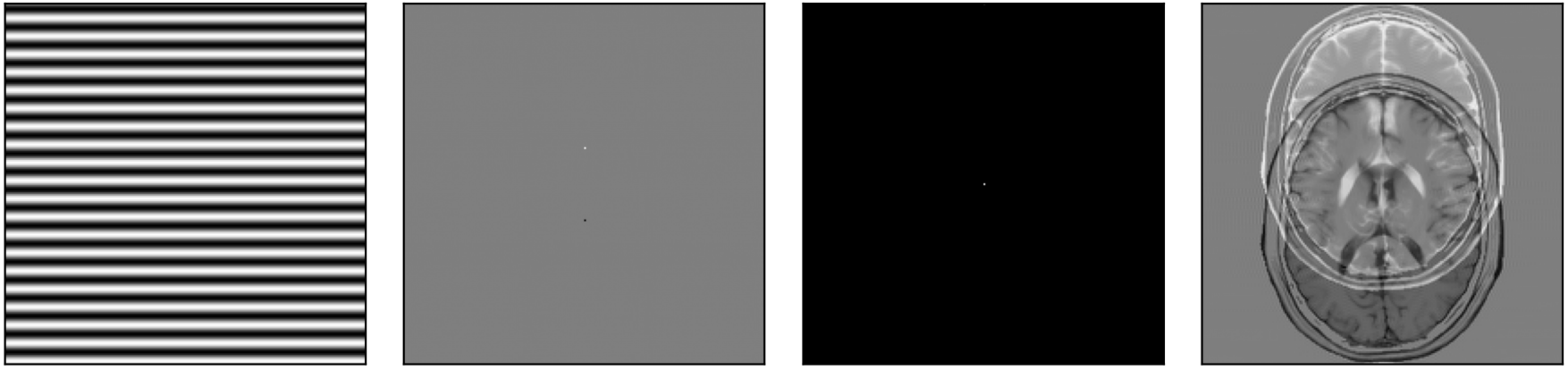
$g_8[r, c]$



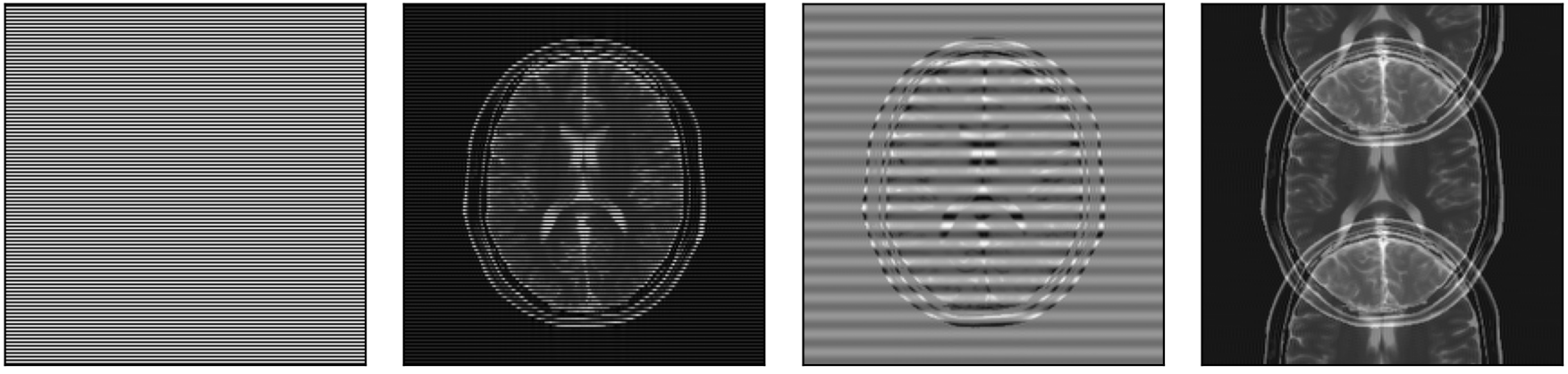
Let $h_1[r, c] = \sin\left(\frac{40\pi}{200} r\right)$. Which is $(f \times h_1)[r, c]$? $g_7[r, c]$

This Is Your Brain on DSP

$g_1[r, c]$ $g_2[r, c]$ $g_3[r, c]$ $g_4[r, c]$

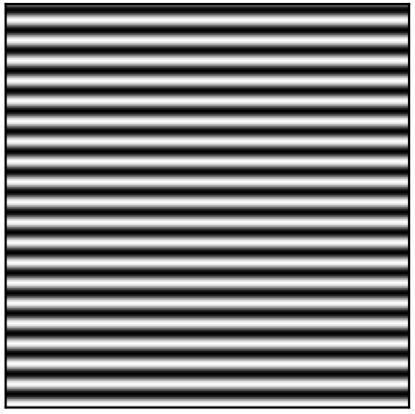


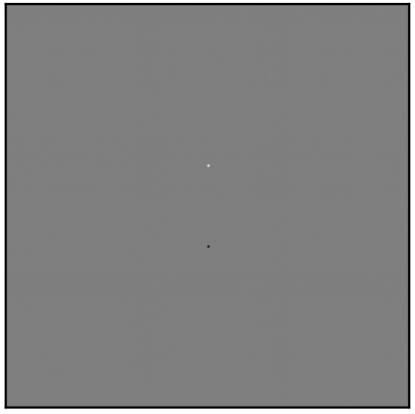
$g_5[r, c]$ $g_6[r, c]$ $g_7[r, c]$ $g_8[r, c]$

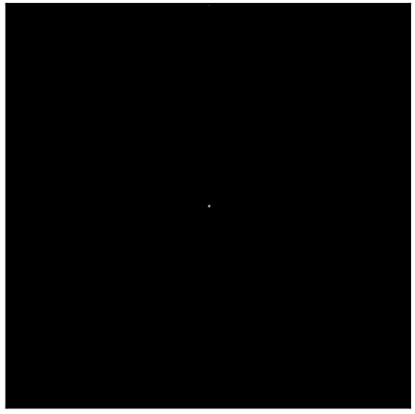


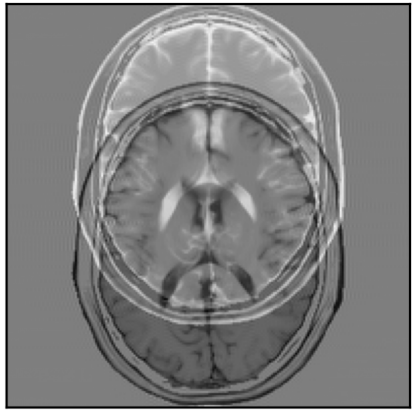
Let $h_1[r, c] = \sin\left(\frac{40\pi}{200} r\right)$. Which is $(f \circledast h_1)[r, c]$?

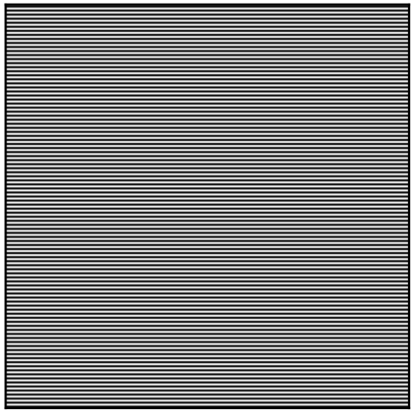
This Is Your Brain on DSP

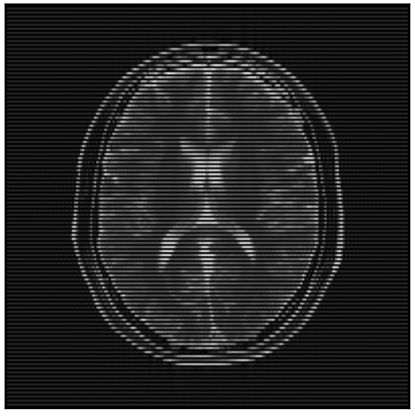
$g_1[r, c]$ 

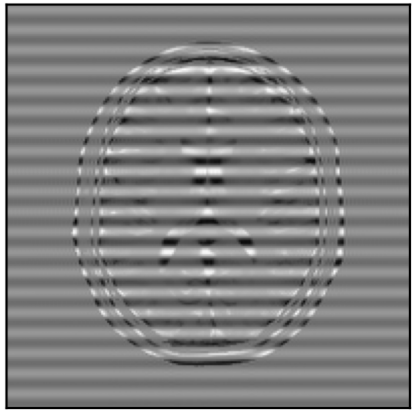
$g_2[r, c]$ 

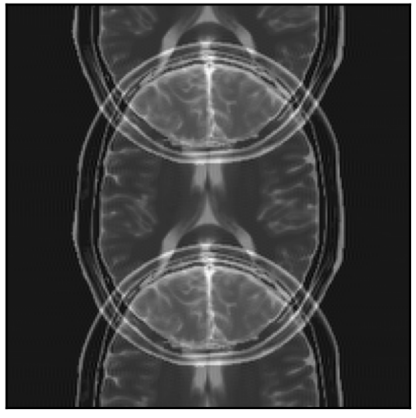
$g_3[r, c]$ 

$g_4[r, c]$ 

$g_5[r, c]$ 

$g_6[r, c]$ 

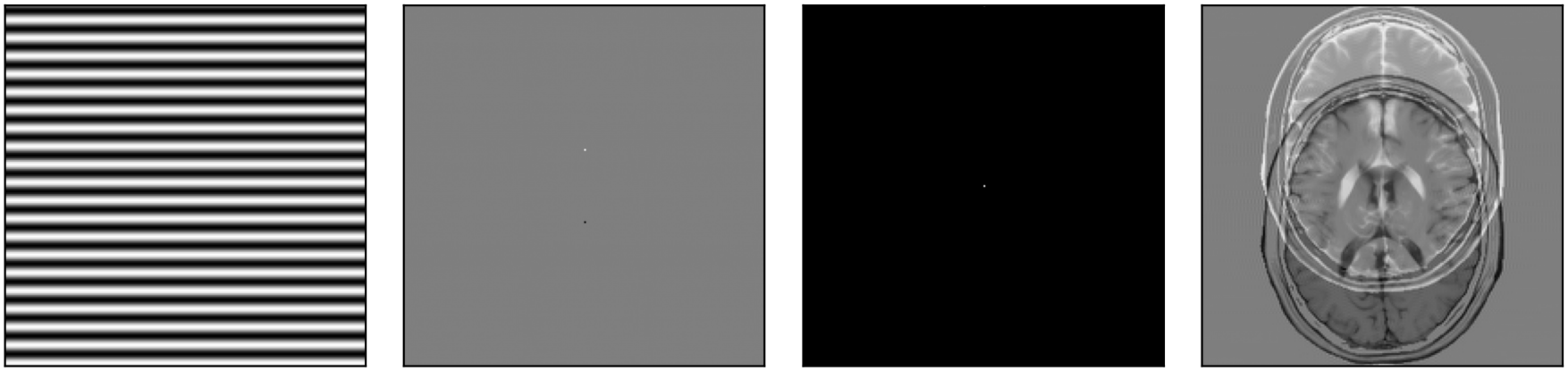
$g_7[r, c]$ 

$g_8[r, c]$ 

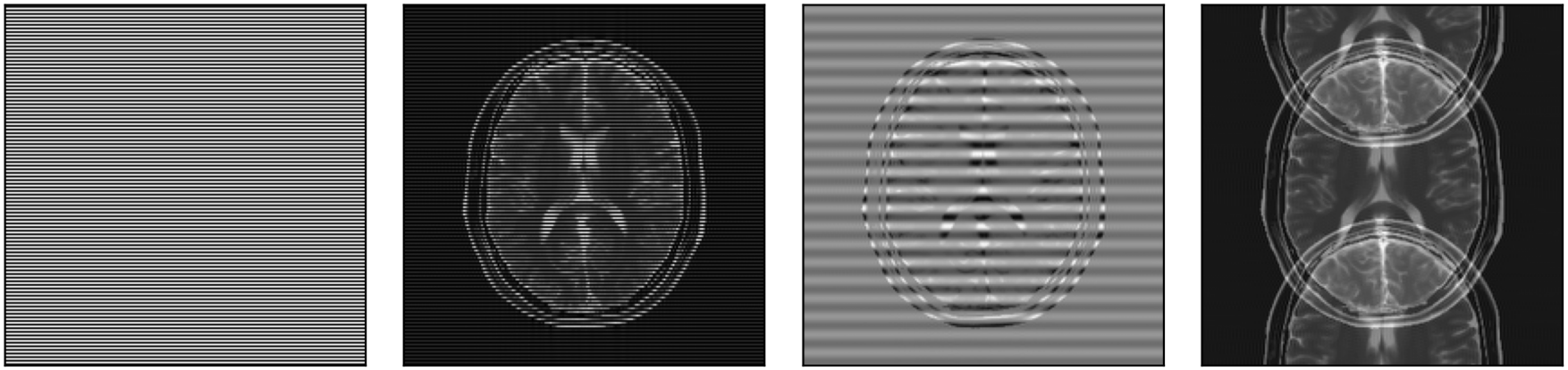
Let $h_1[r, c] = \sin\left(\frac{40\pi}{200} r\right)$. Which is $(f \circledast h_1)[r, c]$? $g_1[r, c]$

This Is Your Brain on DSP

$g_1[r, c]$ $g_2[r, c]$ $g_3[r, c]$ $g_4[r, c]$



$g_5[r, c]$ $g_6[r, c]$ $g_7[r, c]$ $g_8[r, c]$



Let $h_2[r, c] = (-1)^r + 1$. Which is $(f \times h_2)[r, c]$?

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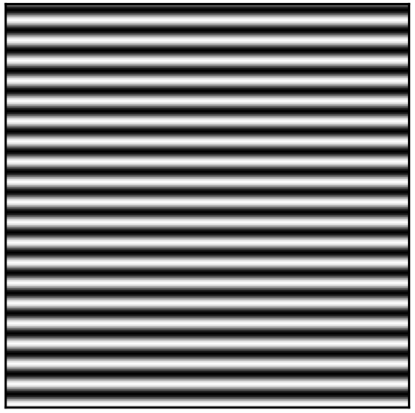
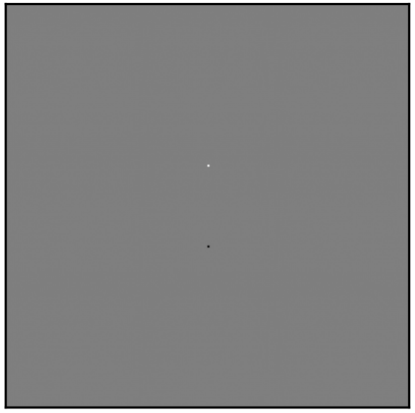
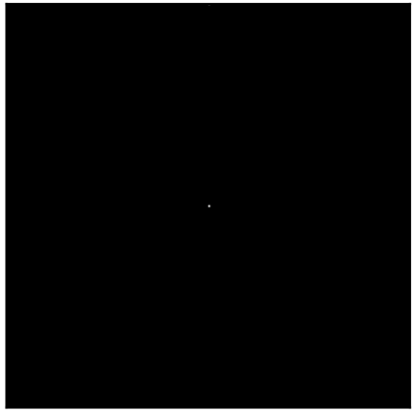
$g_1[r, c]$



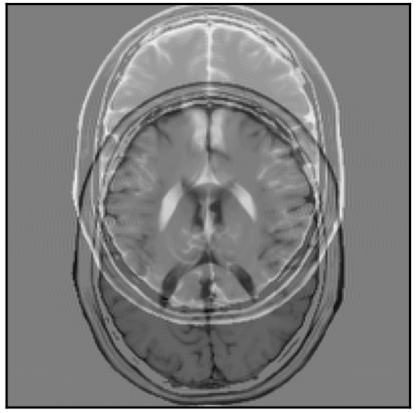
$g_2[r, c]$



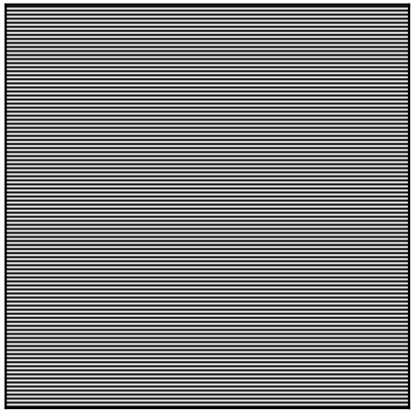
$g_3[r, c]$



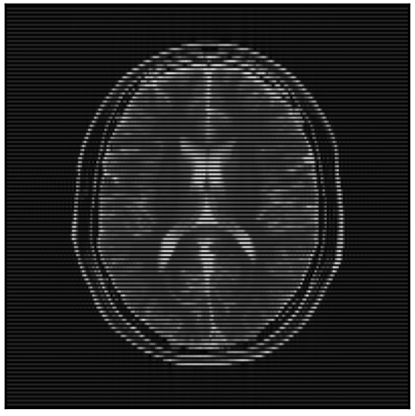
$g_4[r, c]$



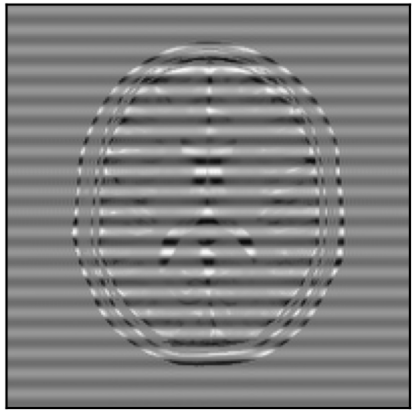
$g_5[r, c]$



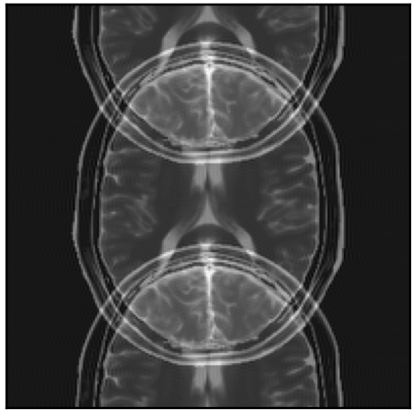
$g_6[r, c]$



$g_7[r, c]$



$g_8[r, c]$



Let $h_2[r, c] = (-1)^r + 1$. Which is $(f \times h_2)[r, c]$? $g_6[r, c]$

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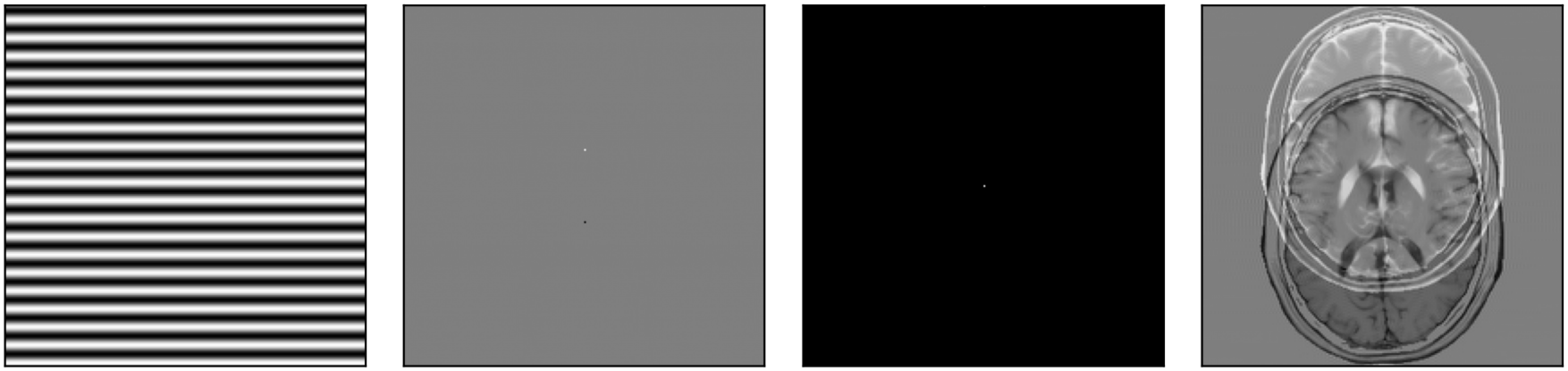
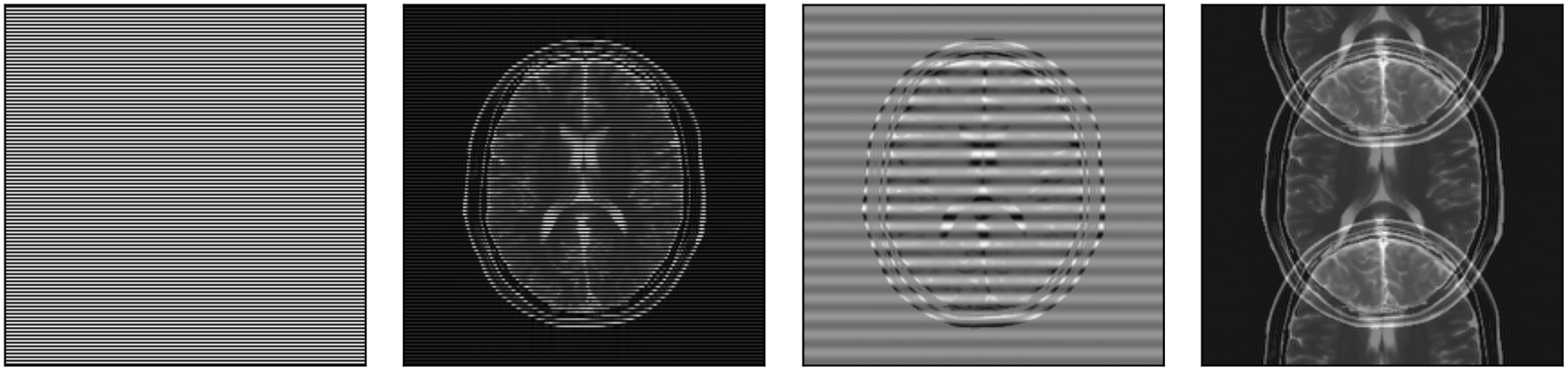
$g_1[r, c]$ $g_2[r, c]$ $g_3[r, c]$ $g_4[r, c]$



$g_5[r, c]$ $g_6[r, c]$ $g_7[r, c]$ $g_8[r, c]$



Let $h_2[r, c] = (-1)^r + 1$. Which is $(f \circledast h_2)[r, c]$?

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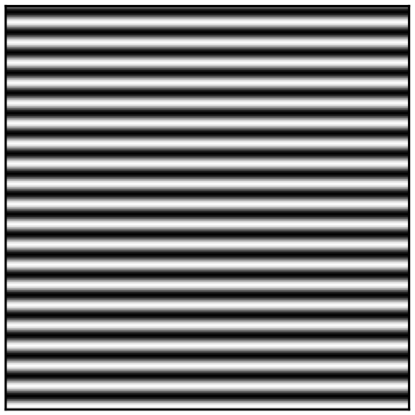
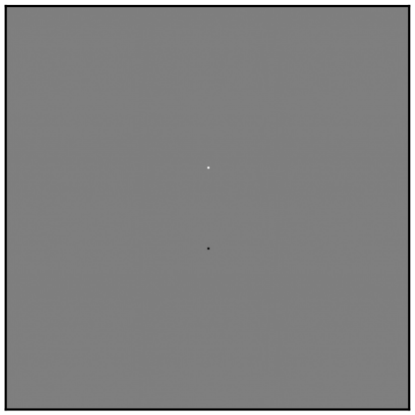
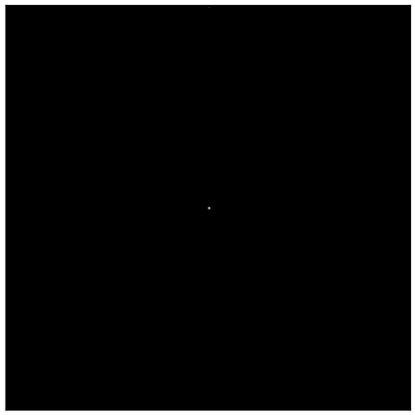
$g_1[r, c]$



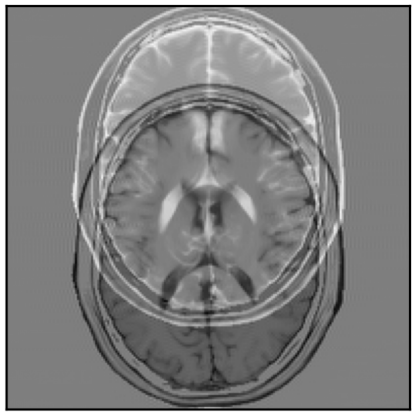
$g_2[r, c]$



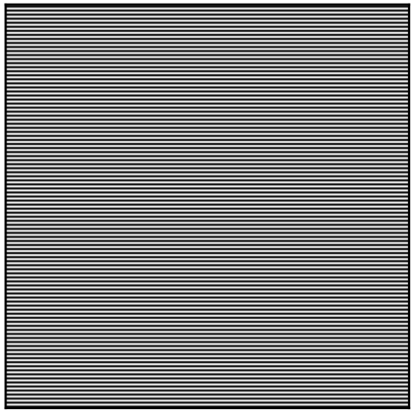
$g_3[r, c]$



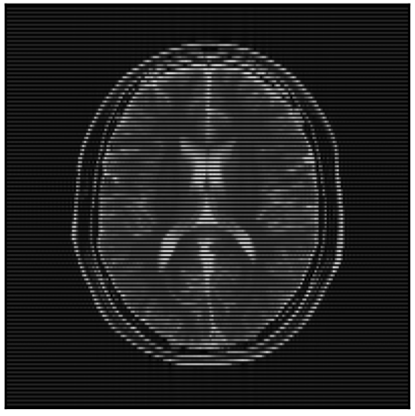
$g_4[r, c]$



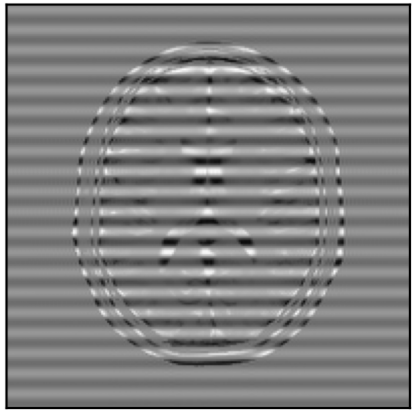
$g_5[r, c]$



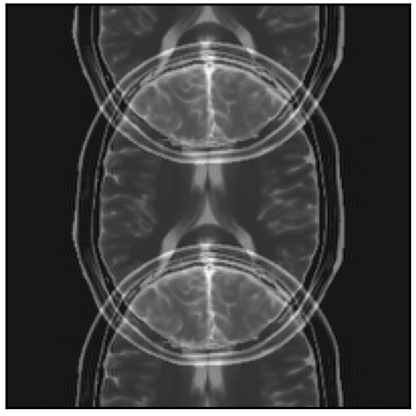
$g_6[r, c]$



$g_7[r, c]$



$g_8[r, c]$



Let $h_2[r, c] = (-1)^r + 1$. Which is $(f \circledast h_2)[r, c]$? $g_5[r, c]$

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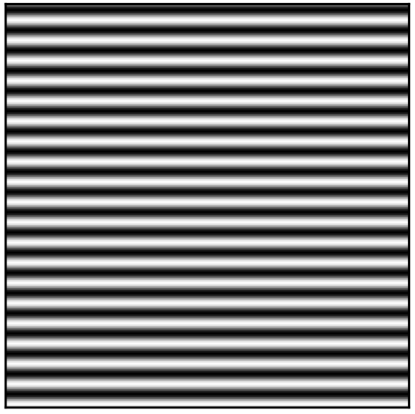
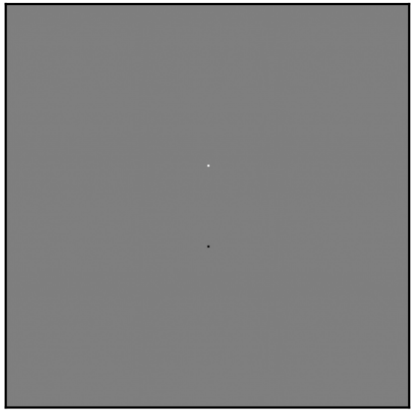
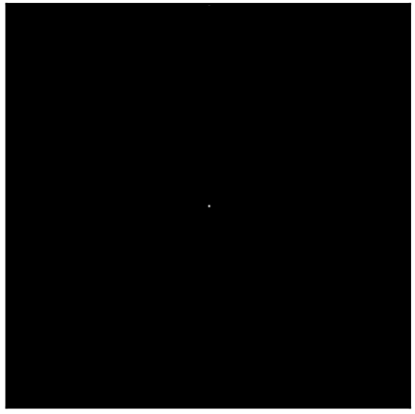
$g_1[r, c]$



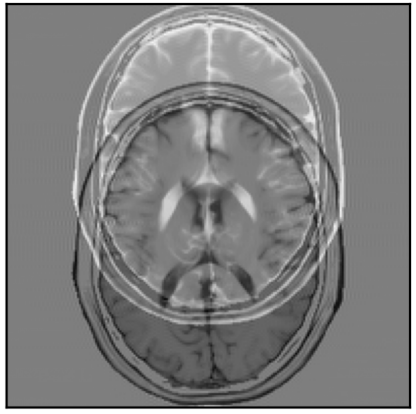
$g_2[r, c]$



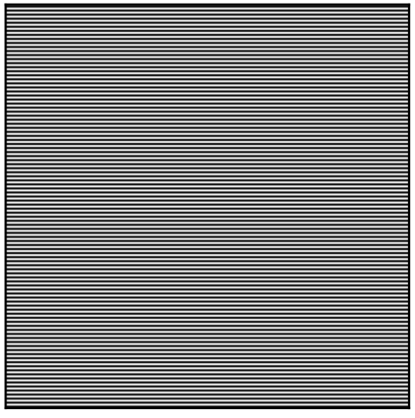
$g_3[r, c]$



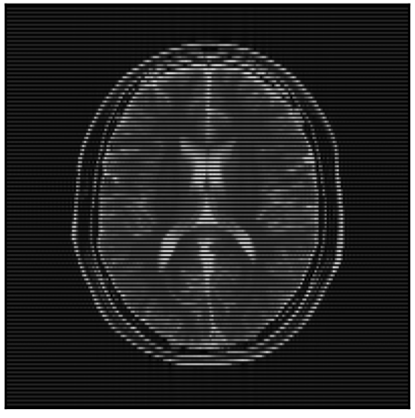
$g_4[r, c]$



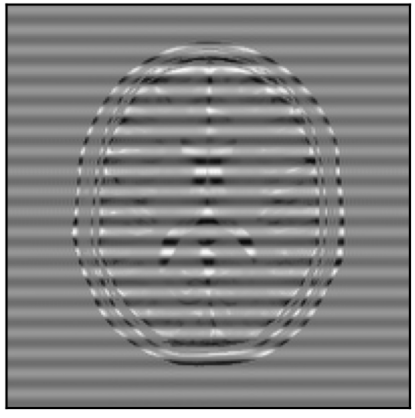
$g_5[r, c]$



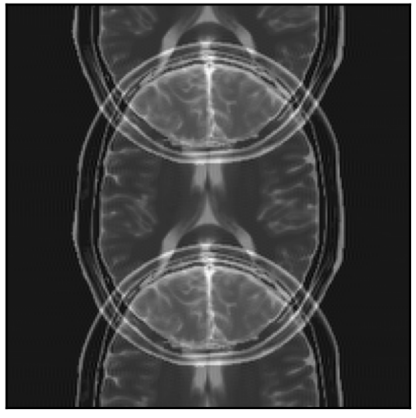
$g_6[r, c]$



$g_7[r, c]$



$g_8[r, c]$



Let $H_3[k_r, k_c] = j \sin\left(\frac{40\pi}{200} k_r\right)$. Which is $(f \times h_3)[r, c]$?

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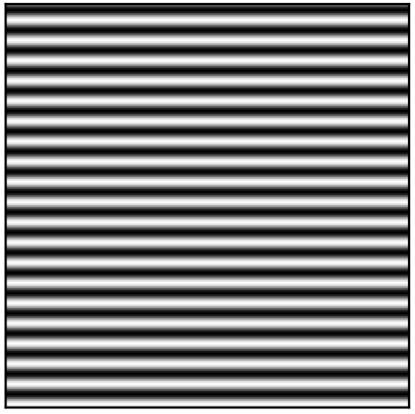
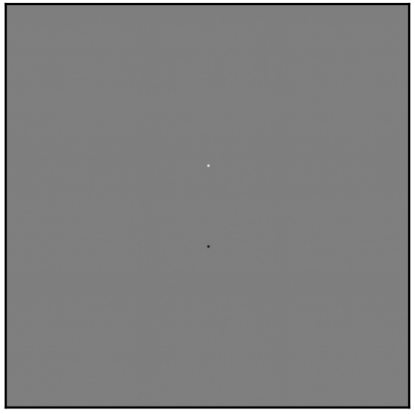
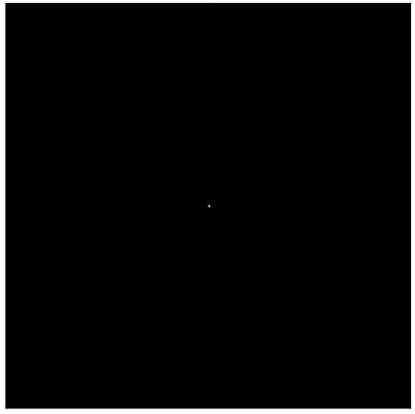
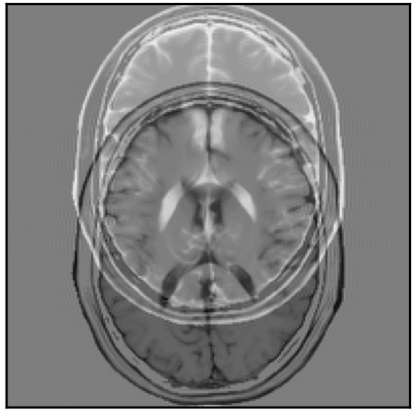
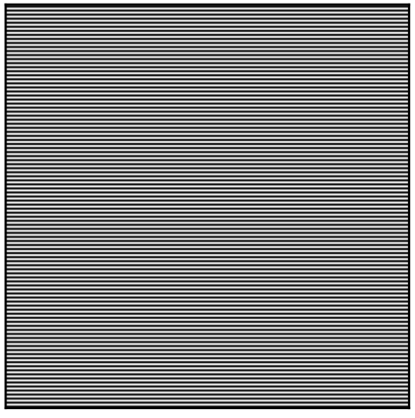
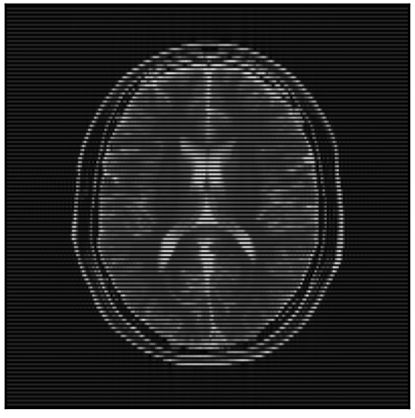
$g_1[r, c]$ 

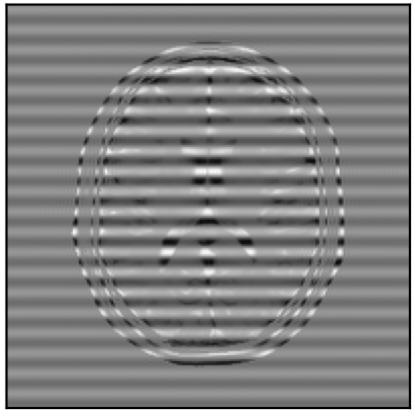
$g_2[r, c]$ 

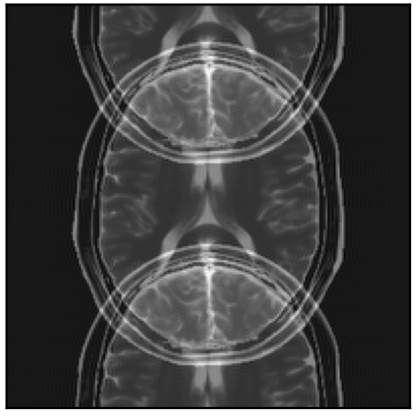
$g_3[r, c]$ 

$g_4[r, c]$ 

$g_5[r, c]$ 

$g_6[r, c]$ 

$g_7[r, c]$ 

$g_8[r, c]$ 

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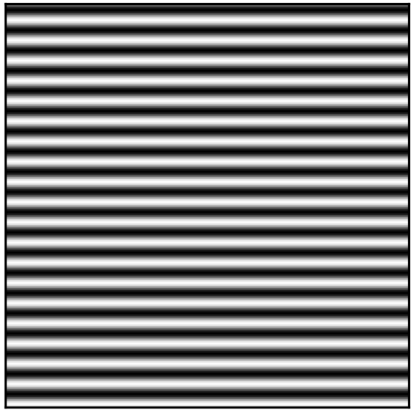
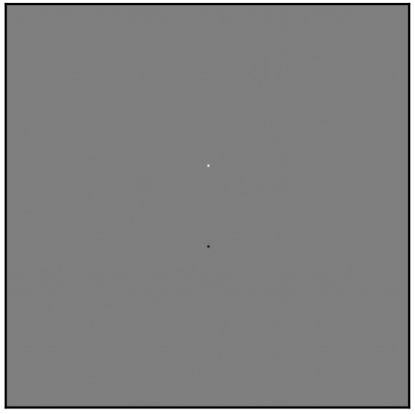
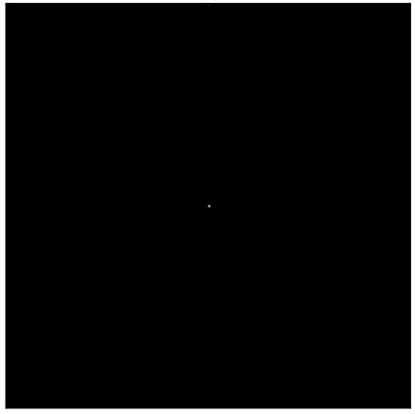
$g_1[r, c]$



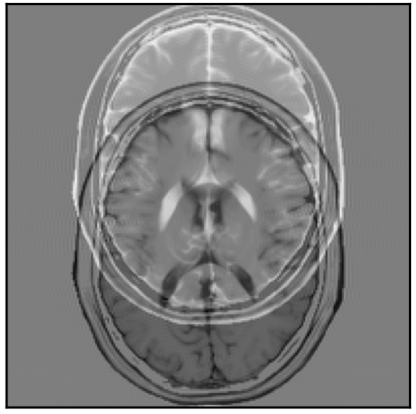
$g_2[r, c]$



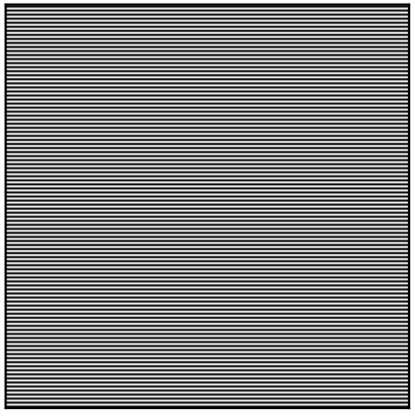
$g_3[r, c]$



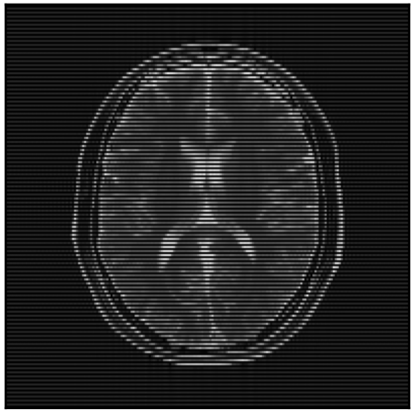
$g_4[r, c]$



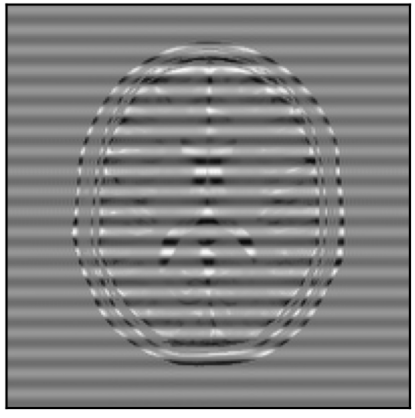
$g_5[r, c]$



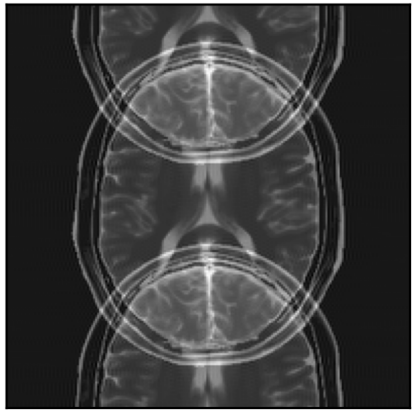
$g_6[r, c]$



$g_7[r, c]$



$g_8[r, c]$



Let $H_3[k_r, k_c] = j \sin\left(\frac{40\pi}{200} k_r\right)$. Which is $(f \circledast h_3)[r, c]$?

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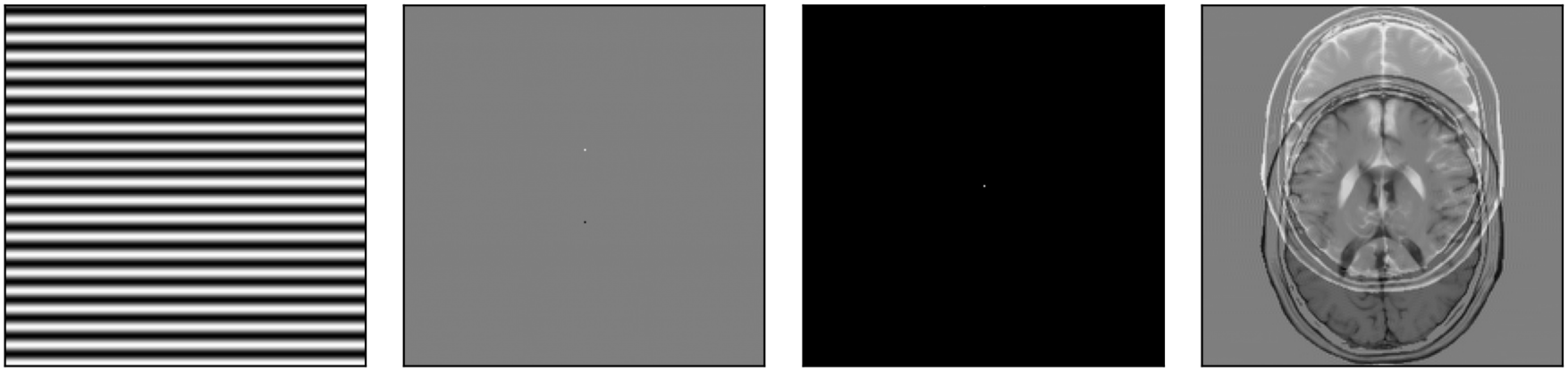
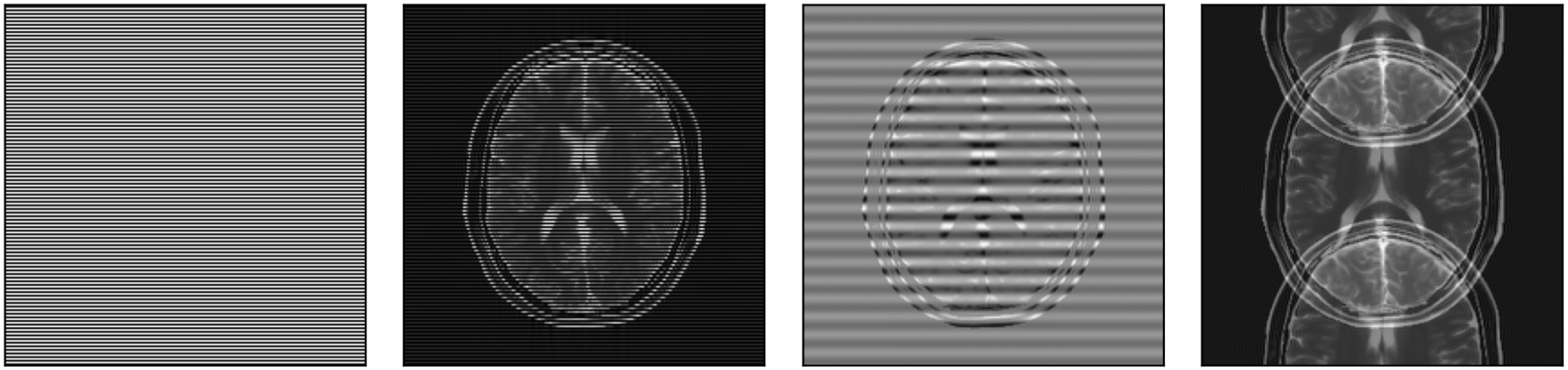
$g_1[r, c]$ $g_2[r, c]$ $g_3[r, c]$ $g_4[r, c]$



$g_5[r, c]$ $g_6[r, c]$ $g_7[r, c]$ $g_8[r, c]$



Let $H_3[k_r, k_c] = j \sin\left(\frac{40\pi}{200} k_r\right)$. Which is $(f \circledast h_3)[r, c]$? $g_4[r, c]$

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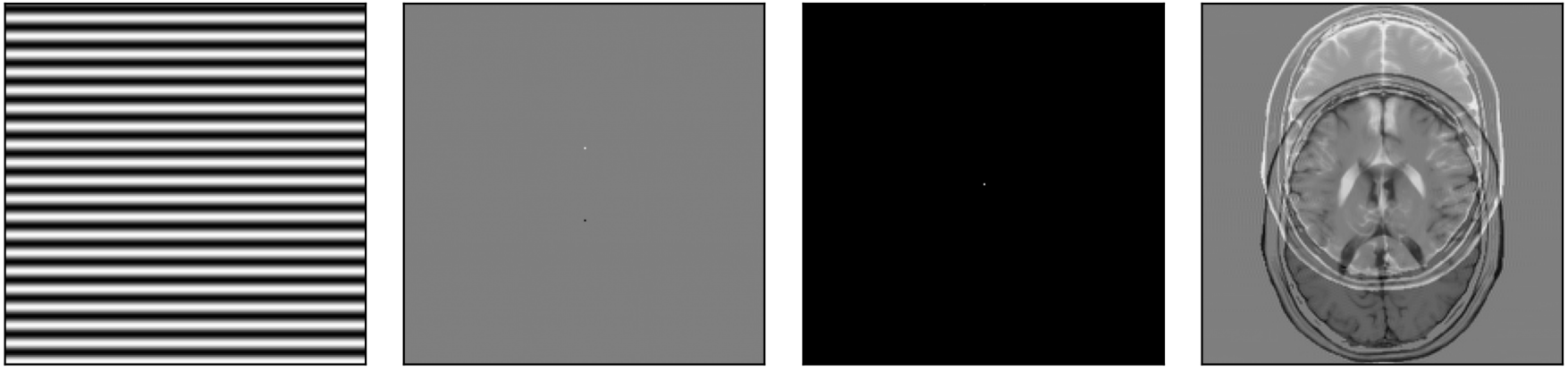
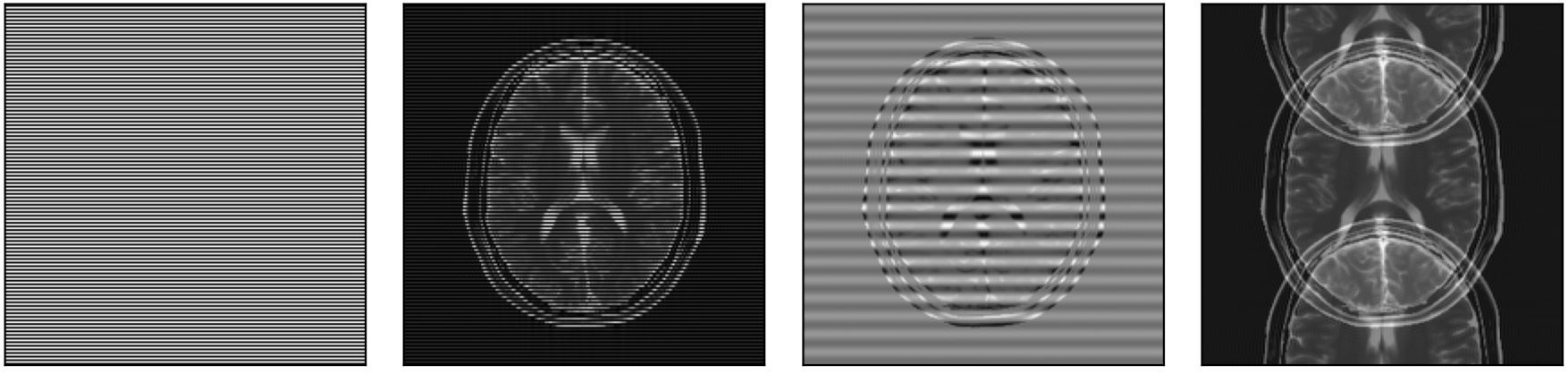
$g_1[r, c]$ $g_2[r, c]$ $g_3[r, c]$ $g_4[r, c]$



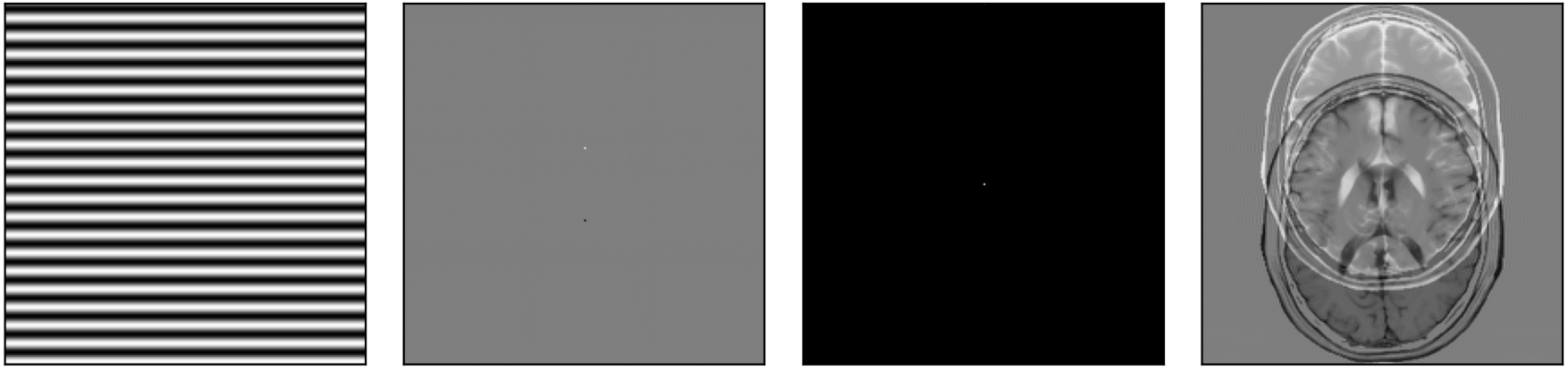
$g_5[r, c]$ $g_6[r, c]$ $g_7[r, c]$ $g_8[r, c]$



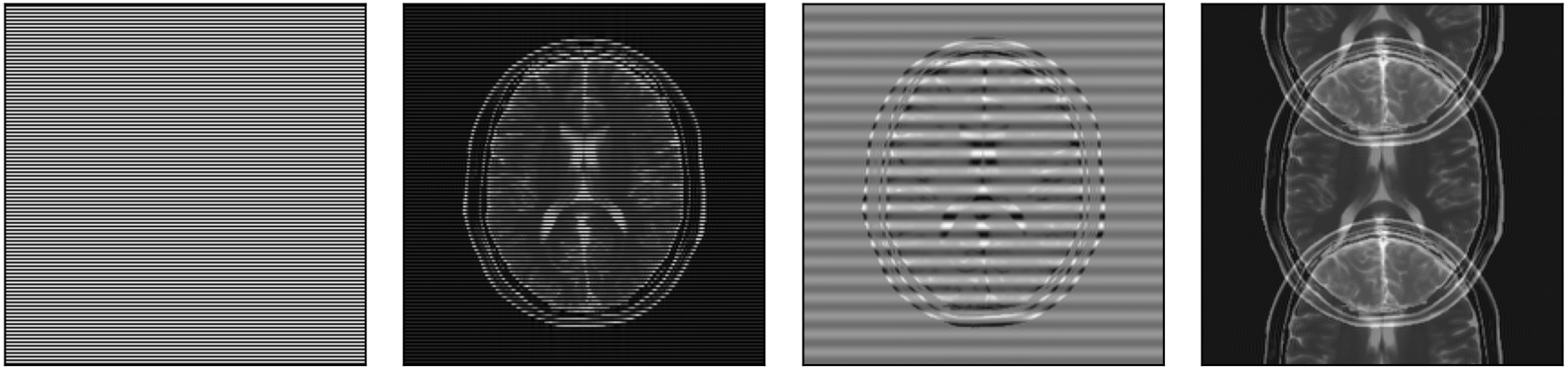
Let $H_4[k_r, k_c] = (-1)^{k_r} + 1$. Which is $(f \times h_4)[r, c]$?

This Is Your Brain on DSP

$g_1[r, c]$ $g_2[r, c]$ $g_3[r, c]$ $g_4[r, c]$



$g_5[r, c]$ $g_6[r, c]$ $g_7[r, c]$ $g_8[r, c]$



Let $H_4[k_r, k_c] = (-1)^{k_r} + 1$. Which is $(f \times h_4)[r, c]$? $g_3[r, c]$

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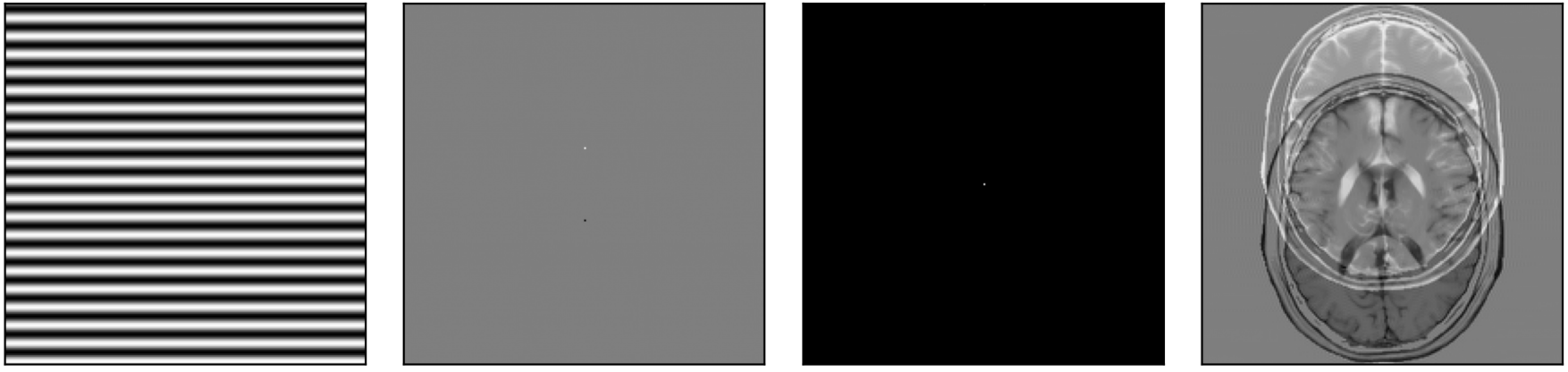
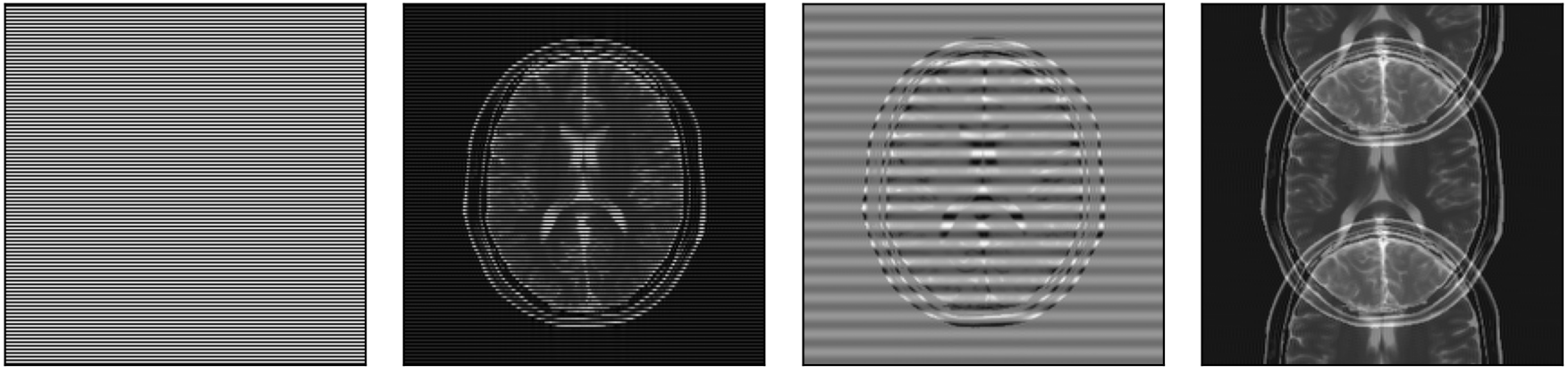
$g_1[r, c]$ $g_2[r, c]$ $g_3[r, c]$ $g_4[r, c]$



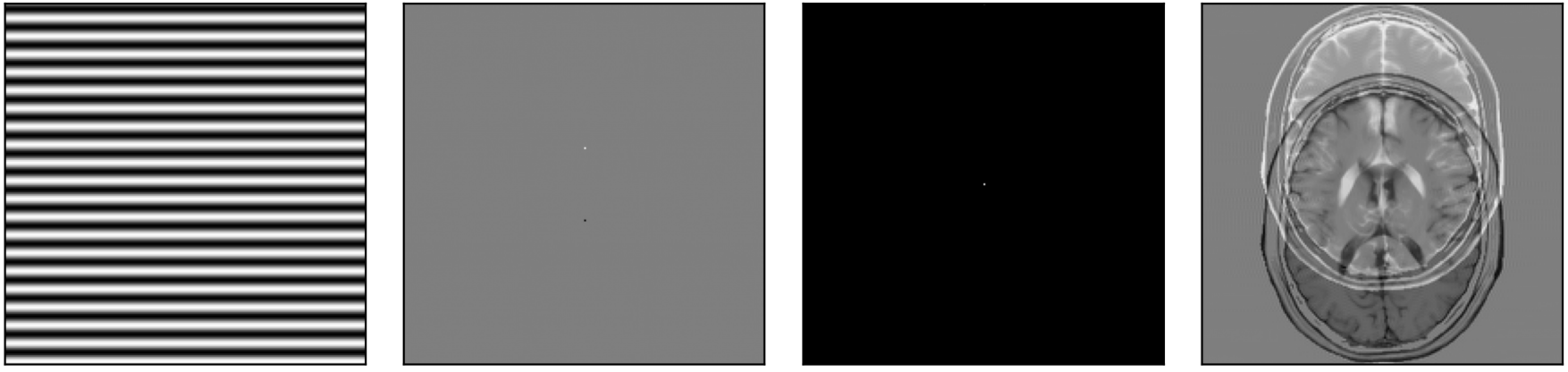
$g_5[r, c]$ $g_6[r, c]$ $g_7[r, c]$ $g_8[r, c]$



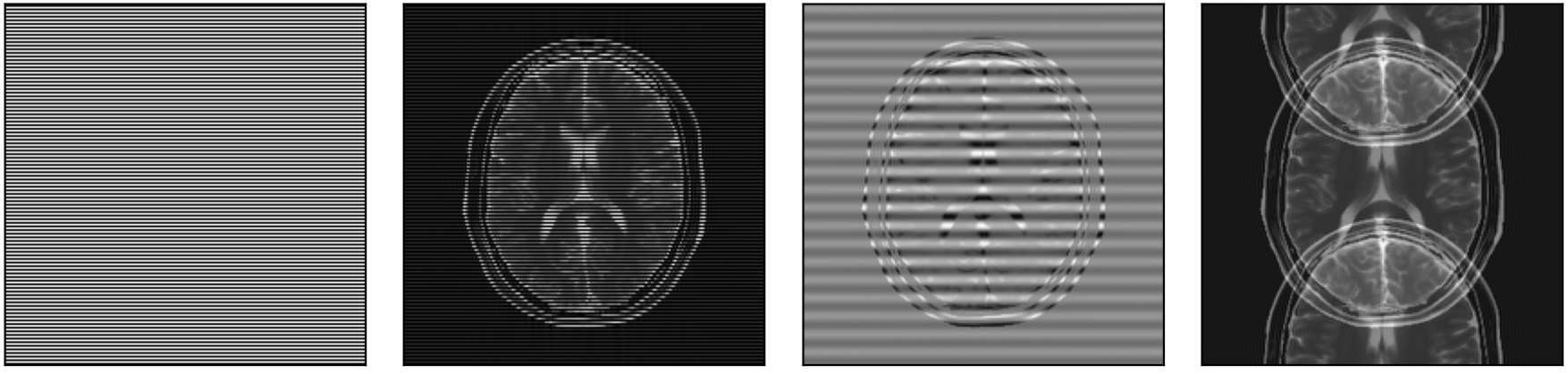
Let $H_4[k_r, k_c] = (-1)^{k_r} + 1$. Which is $(f \circledast h_4)[r, c]$?

This Is Your Brain on DSP

$g_1[r, c]$ $g_2[r, c]$ $g_3[r, c]$ $g_4[r, c]$



$g_5[r, c]$ $g_6[r, c]$ $g_7[r, c]$ $g_8[r, c]$



Let $H_4[k_r, k_c] = (-1)^{k_r} + 1$. Which is $(f \circledast h_4)[r, c]$? $g_8[r, c]$

Question of the Day

Let $f[r, c]$ and $g[r, c]$ denote monochrome images with $N = (1024 \times 1024)$ pixels. Determine the approximate number of operations it takes to compute $(f \circledast g)[r, c]$ **(#1)** directly and **(#2)** using fast Fourier transform (FFT) algorithms.

a.	$\mathcal{O}(1024)$	1 thousand operations
b.	$\mathcal{O}(1024 \log_2 1024)$	10 thousand operations
c.	$\mathcal{O}(1024^2)$	1 million operations
d.	$\mathcal{O}(1024^2 \log_2 1024^2)$	20 million operations
e.	$\mathcal{O}(1024^4)$	1 trillion operations
f.	$\mathcal{O}(1024^4 \log_2 1024^4)$	40 trillion operations
g.	none of the above	