6.300: Signal Processing

Final Exam Review

Final Exam

- on Friday, May 15
- from 9:00 a.m. to 12:00 p.m.
- in Walker Memorial (50-340)
- https://sigproc.mit.edu/spring26/q3_info

Please fill out a **subject evaluation** for this class at https://registrar.mit.edu/classes-grades-evaluations/subject-evaluation by Friday, May 15 at 9:00 a.m.

May 7, 2026

Tentative End-of-Term Office Hours

05/11 (M)	4:00 to 5:00 p.m. in 34-302 7:30 to 9:30 p.m. in 34-303
05/12 (T)	4:00 to 5:00 p.m. in 36-144 7:30 to 9:30 p.m. in 34-303
05/13 (W)	Review: 9:00 to 11:00 a.m. **(???)** 2:00 to 5:00 p.m. Room TBD. 7:00 to 9:00 p.m. Room TBD.
05/14 (R)	Review: 9:00 to 11:00 a.m. **(???)** 2:00 to 5:00 p.m. Room TBD. 7:00 to 9:00 p.m. Room TBD.

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Subject Evaluations

Please fill out a **subject evaluation** for this class at https://registrar.mit.edu/classes-grades-evaluations/subject-evaluation by Friday, May 15 at 9:00 a.m.

There are 101 registered students in 6.300 at this point.

If $> 50\%$ **(≥ 51 students)** submit the subject evaluation by 11:59 p.m. on Tuesday, May 12, we will hold an additional **review session** on Wednesday, May 13.

If $> 75\%$ **(≥ 76 students)** submit the subject evaluation by 11:59 p.m. on Tuesday, May 12, we will hold yet another additional **review session** on Thursday, May 14.

We will communicate details via the course website.

You can update your response to the subject evaluation any time before Friday, May 15 at 9:00 a.m.

Lecture and Recitation Retrospective

Fourier series: Frequency representations for periodic signals

- **02/03:** Fourier series for continuous-time signals
- **02/05:** Using symmetry to simplify Fourier series calculations
- **02/10:** Fourier series — now with complex exponentials!
- **02/12:** Sampling and aliasing (i.e., discretizing time)
- **02/17:** Presidents' Day
- **02/19:** Fourier series for discrete-time signals

Fourier transforms: Frequency representations for all signals[1]

- **02/24:** Blizzard of 2026
- **02/26:** Fourier transform for continuous-time signals
- **03/03:** Quiz #1
- **03/05:** Fourier transform for discrete-time signals

Systems: Analyzing and designing systems for processing signals

- **03/10:** Systems (e.g., linearity, time-invariance, diff. equations)
- **03/12:** Unit sample/impulse response and convolution
- **03/17:** Frequency response and filtering
- **03/19:** Communication systems (e.g., Fourier transform duality)

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[1] Not all, but a lot.

Lecture and Recitation Retrospective

Discrete Fourier transform: Good for numerical computation

- **03/31:** Relations among discrete-time Fourier representations
- **04/02:** Frequency resolution, circular convolution, impulse trains
- **04/07:** Fast Fourier transform (FFT) algorithms
- **04/09:** Short-time Fourier transform (i.e., sequence of transforms)
- **04/14:** Speech processing (e.g., source-filter model of speech)
- **04/16:** Quiz #2
- **04/21:** Quiz #2 retrospective

Multidimensional signal processing

- **04/23:** Two-dimensional Fourier transforms
- **04/28:** Two-dimensional Fourier transforms and convolution
- **04/30:** Image processing with the discrete Fourier transform
- **05/05:** Data compression with the discrete cosine transform
- **05/07:** Fourier transforms and magnetic resonance imaging
- **05/12:** Synthetic aperture optics (i.e., Fourier optics)
- **05/15:** Final examination

"**Fourier transforms** make the world go round."

System Identification

$$x(t) \rightarrow \boxed{\boxed{\mathcal{S}_1} \rightarrow (h_1 * x)(t) \rightarrow \boxed{\mathcal{S}_2}} \rightarrow y(t) = (h_1 * h_2 * x)(t)$$

Part a. Let $h_i(t)$ denote the impulse response of LTI system $\mathcal{S}_i$, and let $\mathcal{S}_{12}$ represent the LTI system that results from connecting systems $\mathcal{S}_1$ and $\mathcal{S}_2$ in series.

$$h_1(t) = e^{-t}u(t) \qquad h_2(t) = e^{-2t}u(t)$$

Determine a linear, constant-coefficient differential equation of the form

$$a_0 y(t) + a_1 y'(t) + a_2 y''(t) + \cdots = b_0 x(t) + b_1 x'(t) + b_2 x''(t) + \cdots$$

to relate $x(t)$ and $y(t)$. Assume initial rest conditions.

System Identification

$$x(t) \rightarrow \boxed{\boxed{\mathcal{S}_1} \rightarrow (h_1 * x)(t) \rightarrow \boxed{\mathcal{S}_2}} \rightarrow y(t) = (h_1 * h_2 * x)(t)$$

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to relate $x(t)$ and $y(t)$. Assume initial rest conditions.

$$\boxed{2y(t) + 3y'(t) + y''(t) = x(t)}$$

System Identification

Part b. Consider a discrete-time LTI system with unit sample response

$$h[n] = \sum_{i=0}^{\infty} \alpha^i \delta[n-4i].$$

Determine a linear, constant-coefficient differential equation of the form

$$a_0 y[n] + a_1 y[n-1] + a_2 y[n-2] + \cdots = b_0 x[n] + b_1 x[n-1] + \cdots$$

to relate $x[n]$ and $y[n]$. Assume initial rest conditions.

System Identification

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to relate $x[n]$ and $y[n]$. Assume initial rest conditions.

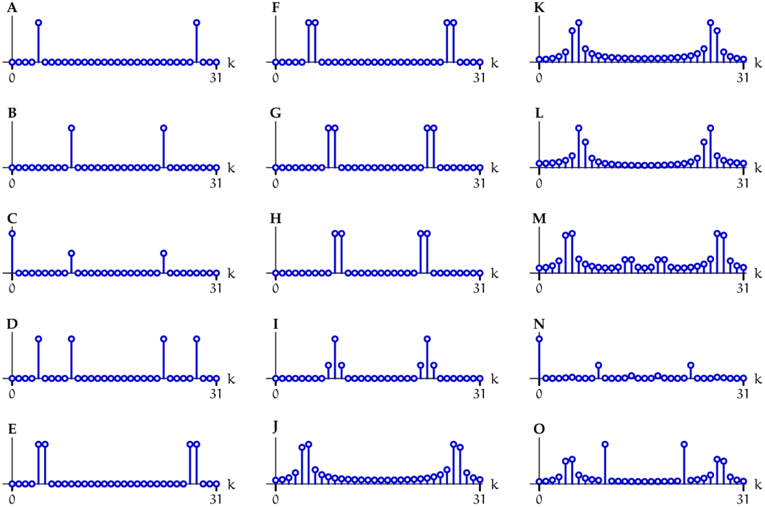
$$y[n] - \alpha y[n-4] = x[n]$$

Discrete Fourier Transform: Sinusoids

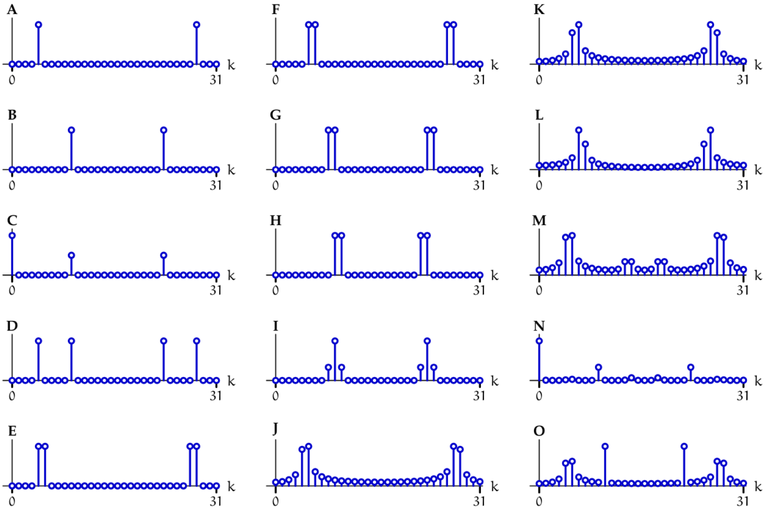
Consider the eight sinusoids defined below.

$\cos(9\pi n/32)$	$\cos(9\pi n/16)$	$\cos(9\pi n/26)$	$\cos(9\pi n/36)$
$\cos(2\pi n/5)$	$\lvert\cos(9\pi n/32)\rvert$	$\cos^2(9\pi n/32)$	$\cos^3(9\pi n/32)$

Match each expression above to the plot on the next slide that shows the magnitude of its discrete Fourier transform (DFT) computed with analysis length $N = 32$.



$\cos(9\pi n/32)$	$\cos(9\pi n/16)$	$\cos(9\pi n/26)$	$\cos(9\pi n/36)$
$\cos(2\pi n/5)$	$\lvert\cos(9\pi n/32)\rvert$	$\cos^2(9\pi n/32)$	$\cos^3(9\pi n/32)$



$\cos(9\pi n/32)$ **J**	$\cos(9\pi n/16)$ **B**	$\cos(9\pi n/26)$ **K**	$\cos(9\pi n/36)$ **A**
$\cos(2\pi n/5)$ **L**	$\lvert\cos(9\pi n/32)\rvert$ **N**	$\cos^2(9\pi n/32)$ **C**	$\cos^3(9\pi n/32)$ **M**

Sampling

Consider taking a continuous-time signal $x(t)$ that is periodic in $T = 1$ second and sampling at a rate of $f_s = 6$ samples per second to obtain a discrete-time signal $x[n]$ that is periodic in $N = 6$ samples. You know that . . .

- $x[n]$ is a symmetric function of n.
- $x[n]$ is positive for all values of n.
- $x[0] + x[1] + x[2] + x[3] + x[4] + x[5] = 3$
- $x[0] - x[1] + x[2] - x[3] + x[4] - x[5] = 1$
- Most of the Fourier series coefficients $X[k]$ are zero; only two out of every six coefficients are non-zero.

What are two distinct continuous-time signals that could have produced $x[n]$ as specified above?

$x_1(t) =$	$x_2(t) =$

Sampling

Consider taking a continuous-time signal $x(t)$ that is periodic in $T = 1$ second and sampling at a rate of $f_s = 6$ samples per second to obtain a discrete-time signal $x[n]$ that is periodic in $N = 6$ samples. You know that...

- $x[n]$ is a symmetric function of n.
- $x[n]$ is positive for all values of n.
- $x[0] + x[1] + x[2] + x[3] + x[4] + x[5] = 3$
- $x[0] - x[1] + x[2] - x[3] + x[4] - x[5] = 1$
- Most of the Fourier series coefficients $X[k]$ are zero; only two out of every six coefficients are non-zero.

What are two distinct continuous-time signals that could have produced $x[n]$ as specified above?

$x_1(t) = \frac{1}{2} + \frac{1}{6}\cos(6\pi t)$	$x_2(t) = \frac{1}{2} + \frac{1}{6}\cos(18\pi t)$

Convolutions

Part a. Define the signal $f_1[n]$ as

$$f_1[n] = \begin{cases} (-1)^n & 0 \leq n \leq 5 \\ 0 & \text{otherwise} \end{cases}$$

and let $f_2[n] = (f_1 * f_1)[n]$.

Determine values for $f_2[n]$ for $n \in \{0, 1, 2, \ldots, 14\}$.

$f_2[0] =$	$f_2[5] =$	$f_2[10] =$
$f_2[1] =$	$f_2[6] =$	$f_2[11] =$
$f_2[2] =$	$f_2[7] =$	$f_2[12] =$
$f_2[3] =$	$f_2[8] =$	$f_2[13] =$
$f_2[4] =$	$f_2[9] =$	$f_2[14] =$

Convolutions

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and let $f_2[n] = (f_1 * f_1)[n]$.

Determine values for $f_2[n]$ for $n \in \{0, 1, 2, \ldots, 14\}$.

$f_2[0] = 1$	$f_2[5] = -6$	$f_2[10] = 1$
$f_2[1] = -2$	$f_2[6] = 5$	$f_2[11] = 0$
$f_2[2] = 3$	$f_2[7] = -4$	$f_2[12] = 0$
$f_2[3] = -4$	$f_2[8] = 3$	$f_2[13] = 0$
$f_2[4] = 5$	$f_2[9] = -2$	$f_2[14] = 0$

Convolutions

Part b. Define the signal $g_1[n]$ as

$$g_1[n] = \begin{cases} 1 & n \text{ even} \\ 0 & n \text{ odd} \end{cases}$$

and let

$$G_1[k] = \mathrm{DFT}_6\{g_1[n]\}$$

denote the six-point DFT of $g_1[n]$. Define

$$g_2[n] = 6 \times \mathrm{DFT}_6^{-1}\{G_1[k] \times G_1[k]\}.$$

Determine values for $g_2[n]$ for $n \in \{0, 1, 2, 3, 4, 5\}$.

$g_2[0] =$	$g_2[2] =$	$g_2[4] =$
$g_2[1] =$	$g_2[3] =$	$g_2[5] =$

Convolutions

Part b. Define the signal $g_1[n]$ as

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$$g_2[n] = 6 \times \mathrm{DFT}_6^{-1}\{G_1[k] \times G_1[k]\}.$$

Determine values for $g_2[n]$ for $n \in \{0, 1, 2, 3, 4, 5\}$.

$g_2[0] = 3$	$g_2[2] = 3$	$g_2[4] = 3$
$g_2[1] = 0$	$g_2[3] = 0$	$g_2[5] = 0$

Systems

Part a. Let $\mathcal{S}$ denote a linear, time-invariant (LTI) system with unit sample response

$$h[n] = \begin{cases} \left(\frac{1}{2}\right)^n & n \ge 0 \text{ and } n \text{ even} \\ 0 & \text{otherwise.} \end{cases}$$

If the input to $\mathcal{S}$ is $x[n] = \cos\left(\frac{\pi}{4}n\right)$, then the output may be expressed as

$$y[n] = A\cos\left(\frac{\pi}{4}n\right) + B\sin\left(\frac{\pi}{4}n\right)$$

for real-valued constants A and B.

Determine values for A and B.

$A =$	$B =$

Systems

Part a. Let $\mathcal{S}$ denote a linear, time-invariant (LTI) system with unit sample response

$$h[n] = \begin{cases} \left(\frac{1}{2}\right)^n & n \ge 0 \text{ and } n \text{ even} \\ 0 & \text{otherwise.} \end{cases}$$

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Determine values for A and B.

$A = 16/17$	$B = 4/17$

Systems

$$x(t) \rightarrow \boxed{\boxed{\mathcal{S}_1} \rightarrow \boxed{\mathcal{S}_2} \rightarrow \boxed{\mathcal{S}_3}} \rightarrow y(t)$$

Part b. A system is constructed from three identical subsystems: $\mathcal{S}_1$, $\mathcal{S}_2$, and $\mathcal{S}_3$. The input signal $x_i(t)$ and output signal $y_i(t)$ of each subsystem are related by the following differential equation.

$$y_i(t) + \frac{dy_i(t)}{dt} = x_i(t)$$

If the input is $x(t) = \cos(t)$, then the output may be expressed as $y(t) = C\cos(t - \phi)$ for some real-valued constants C and ϕ.

Determine values for C and ϕ.

$C =$	$\phi =$

Systems

$$x(t) \rightarrow \boxed{\boxed{\mathcal{S}_1} \rightarrow \boxed{\mathcal{S}_2} \rightarrow \boxed{\mathcal{S}_3}} \rightarrow y(t)$$

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If the input is $x(t) = \cos(t)$, then the output may be expressed as $y(t) = C\cos(t - \phi)$ for some real-valued constants C and ϕ.

Determine values for C and ϕ.

$C = 1/(2\sqrt{2})$	$\phi = 3\pi/4$

Question of the Day

In a sentence or two, describe the role of the Fourier transform in magnetic resonance imaging.