

## 6.300: Signal Processing

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### Discrete-Time Fourier Transform (DTFT)

**Analysis:**  $X(\Omega) = \sum_{n=-\infty}^{\infty} x[n]e^{-j\Omega n}$

**Synthesis:**  $x[n] = \frac{1}{2\pi} \int_{2\pi} X(\Omega)e^{j\Omega n} d\Omega$

**Time Delay:**  $x[n - n_0] \iff e^{-j\Omega n_0} X(\Omega)$

**Frequency Derivative:**  $nx[n] \iff j \frac{d}{d\Omega} X(\Omega)$

**Periodic Signals:**  $X(\Omega) = \sum_k 2\pi X[k]\delta(\Omega - k\Omega_0)$

March 5, 2026

# Agenda for Recitation

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- Discrete-time Fourier transform (DTFT)

What questions do you have from lecture?

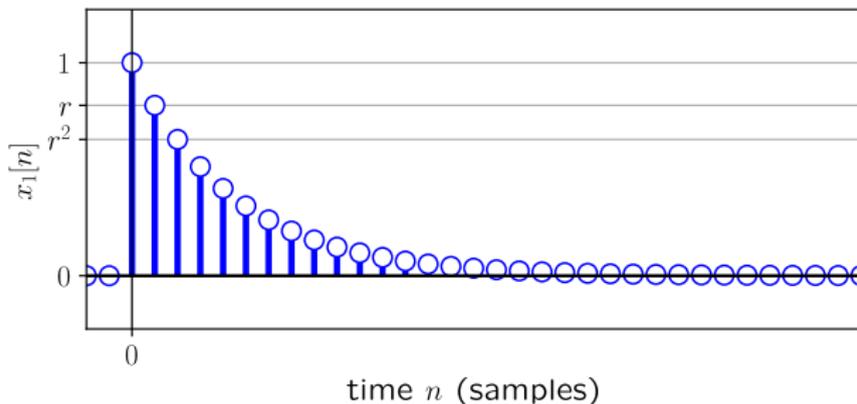
# Fourier Transforms

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Determine  $X_1(\Omega)$ , the Fourier transform of  $x_1[n]$ .

$$x_1[n] = \begin{cases} r^n & n \geq 0 \\ 0 & n < 0 \end{cases} \text{ where } |r| < 1$$

Sketch  $|X_1(\Omega)|$  and  $\angle X_1(\Omega)$ .

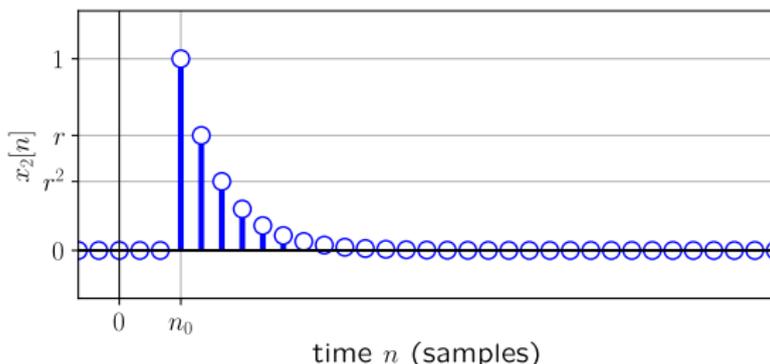


# Fourier Transforms

Determine  $X_2(\Omega)$ , the Fourier transform of  $x_2[n]$ .

$$x_2[n] = x_1[n - n_0] = \begin{cases} r^{n-n_0} & n \geq n_0 \\ 0 & n < n_0 \end{cases}$$

Sketch  $|X_2(\Omega)|$  and  $\angle X_2(\Omega)$ . How are  $|X_2(\Omega)|$  and  $|X_1(\Omega)|$  related? How are  $\angle X_2(\Omega)$  and  $\angle X_1(\Omega)$  related?



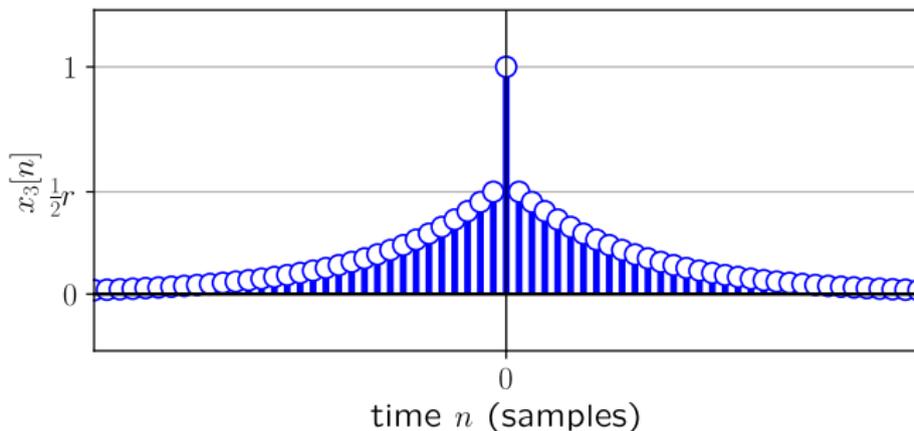
# Fourier Transforms

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Determine  $X_3(\Omega)$ , the Fourier transform of  $x_3[n]$ .

$$x_3[n] = \text{Symmetric}\{x_1[n]\}$$

If a time-domain signal is **real** and **symmetric**,  
what can you say about the Fourier transform?

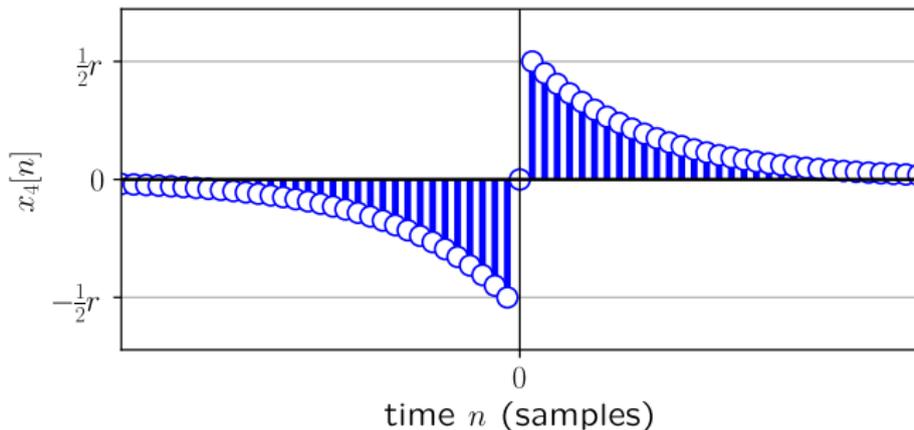


# Fourier Transforms

Determine  $X_4(\Omega)$ , the Fourier transform of  $x_4[n]$ .

$$x_4[n] = \text{Anti-symmetric}\{x_1[n]\}$$

If a time-domain signal is **real** and **anti-symmetric**, what can you say about the Fourier transform?



# Fourier Transforms

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Determine  $X_5(\Omega)$ , the Fourier transform of  $x_5[n]$ .

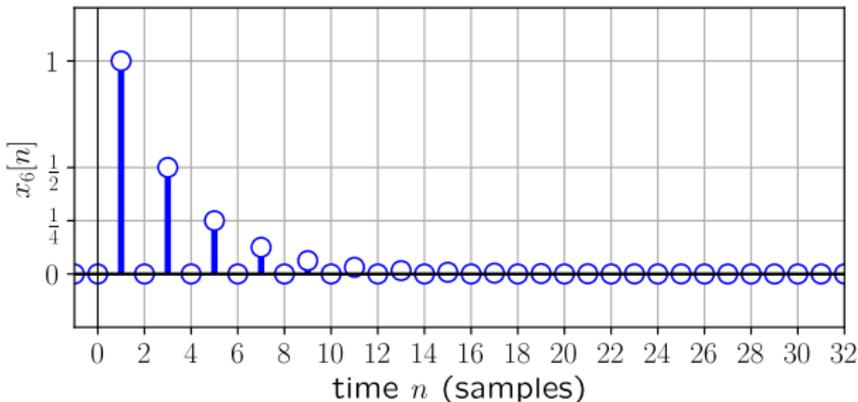
$$x_5[n] = nx_1[n]$$

# Fourier Transforms

Determine  $X_6(\Omega)$ , the Fourier transform of  $x_6[n]$ .

$$x_6[n] = \begin{cases} \left(\frac{1}{2}\right)^{\frac{n}{2}} & n \in \{0, 2, 4, 6, 8, 10, 12, \dots\} \\ 0 & \text{otherwise} \end{cases}$$

What does **“stretching in time”** do in frequency?



# Fourier Transforms

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$x_7[n]$  has Fourier transform  $X_7(\Omega)$ . Determine  $x_7[n]$ .

$$X_7(\Omega) = e^{-j3\Omega}$$

# Lessons Learned

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The **discrete-time Fourier transform (DTFT)** is a Fourier representation for aperiodic and periodic discrete-time signals. It has many useful properties.

**Analysis:**  $X(\Omega) = \sum_{n=-\infty}^{\infty} x[n]e^{-j\Omega n}$

**Synthesis:**  $x[n] = \frac{1}{2\pi} \int_{2\pi} X(\Omega)e^{j\Omega n} d\Omega$

**Time Delay:**  $x[n - n_0] \iff X(\Omega)e^{-j\Omega n_0}$

**Frequency Derivative:**  $nx[n] \iff j \frac{d}{d\Omega} X(\Omega)$

**Periodic Signals:**  $X(\Omega) = \sum_k 2\pi X[k]\delta(\Omega - k\Omega_0)$