6.300: Signal Processing

Discrete-Time Fourier Series (DTFS)

Analysis: $X[k] = \frac{1}{N} \sum_{n=\langle N \rangle} x[n] e^{-jk\frac{2\pi}{N}n}$

Synthesis: $x[n] = \sum_{k=\langle N \rangle} X[k] e^{jk\frac{2\pi}{N}n}$

Fourier Series Properties: We can derive a number of properties by writing out the analysis or synthesis equation, making a change of variables, and matching the resulting expression to an already-known result.

February 19, 2026

Agenda for Recitation

- Review
- Discrete-time Fourier series (DTFS)
- As time allows: Deriving Fourier series properties

What questions do you have from lecture?

Feb. 03: Fourier series — a sum of sinusoids

$$f(t) = f(t+T) = c_0 + \sum_{k=0}^{\infty} c_k \cos\left(k\tfrac{2\pi}{T}t\right) + \sum_{k=0}^{\infty} d_k \sin\left(k\tfrac{2\pi}{T}t\right)$$

Feb. 05: Fourier series and symmetry

$$f(t) = \underbrace{\Big(f_{\text{symmetric}}(t)\Big)}_{\text{cosines}} + \underbrace{\Big(f_{\text{anti-symmetric}}(t)\Big)}_{\text{sines}}$$

Feb. 10: Complex exponentials and Fourier series

$$f(t) = f(t+T) = \sum_{k=-\infty}^{\infty} a_k e^{jk\frac{2\pi}{T}t}$$

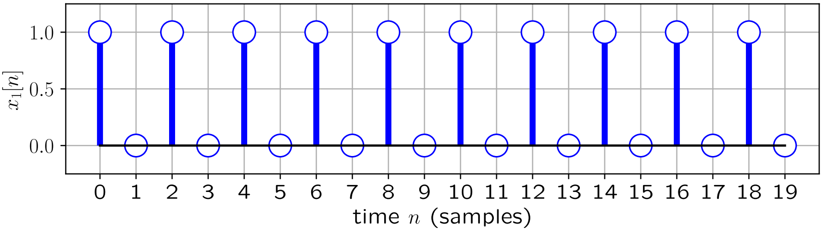
Feb. 12: Sampling — discretizing time

$$x[n] \triangleq x(n\Delta) = x\left(\frac{n}{f_s}\right)$$

Review: Aliasing

We sample the CT waveform $x(t) = \cos(1200\pi t)$ using a sampling rate of $f_s = 1$ kHz. We listen to the resulting DT samples using the same sampling rate. What is the fundamental frequency (Hz) of the baseband CT waveform that we'd hear?

Discrete-Time Fourier Series

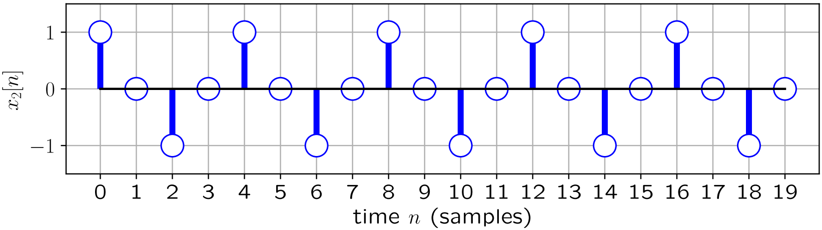


Determine the Fourier series coefficients for $x_1[n]$.

$$x_1[n] = \frac{1+(-1)^n}{2} = \begin{cases} 1 & n \text{ even} \\ 0 & n \text{ odd} \end{cases}$$

Choose N to be the fundamental period of $x_1[n]$.

Discrete-Time Fourier Series

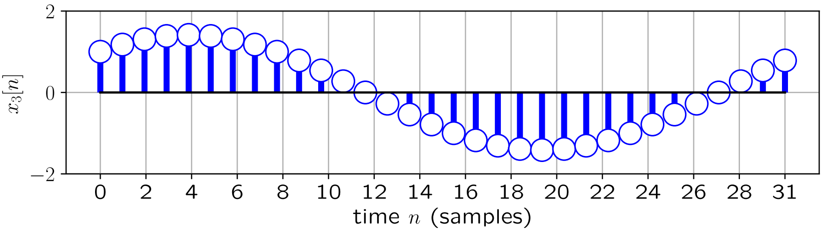


Determine the Fourier series coefficients for $x_2[n]$.

$$x_2[n] = \cos\left(\frac{\pi}{2}n\right)$$

Choose N to be the fundamental period of $x_2[n]$.

Discrete-Time Fourier Series



Determine the Fourier series coefficients for $x_3[n]$.

$$x_3[n] = \cos\left(\tfrac{2\pi}{32}n\right) + \sin\left(\tfrac{2\pi}{32}n\right)$$

Choose N to be the fundamental period of $x_3[n]$.

Deriving Fourier Series Properties

Let $f[n]$ denote an arbitrary N-periodic discrete-time signal. Determine Fourier series coefficients for $g[n]$.

$$g[n] = 9 - 3f[n-1]$$

Deriving Fourier Series Properties

Let $f[n]$ denote an arbitrary discrete-time signal with fundamental period N, and let $F[k]$ denote the Fourier series coefficients of $f[n]$ computed with period N.

$$F[k] = \frac{1}{N} \sum_{n=0}^{N-1} f[n] e^{-jk\frac{2\pi}{N}n}$$

Let $G[k]$ denote the Fourier series coefficients of $f[n]$ computed with period $2N$. Determine a closed-form expression for $G[k]$ in terms of $F[k]$.

$$G[k] = \frac{1}{2N} \sum_{n=0}^{2N-1} f[n] e^{-jk\frac{2\pi}{2N}n}$$

Deriving Fourier Series Properties

Let $f[n]$ denote an arbitrary N-periodic discrete-time signal. Determine Fourier series coefficients for $g[n]$.

$$g[n] = \begin{cases} f\left[\frac{n}{2}\right] & n \text{ even} \\ 0 & \text{otherwise} \end{cases}$$

Lessons Learned

A **discrete-time Fourier series (DTFS)** is a Fourier representation for periodic discrete-time signals.

Analysis: $X[k] = \frac{1}{N} \sum_{n=\langle N \rangle} x[n] e^{-jk\frac{2\pi}{N}n}$

Synthesis: $x[n] = \sum_{k=\langle N \rangle} X[k] e^{jk\frac{2\pi}{N}n}$

Fourier Series Properties: We can derive a number of properties by writing out the analysis or synthesis equation, making a change of variables, and matching the resulting expression to an already-known result.