6.300: Signal Processing

Continuous-Time Fourier Transform (CTFT)

Analysis: $X(\omega) = \int_{-\infty}^{\infty} x(t)e^{-j\omega t}dt$

Synthesis: $x(t) = \frac{1}{2\pi}\int_{-\infty}^{\infty} X(\omega)e^{j\omega t}d\omega$

Time Delay: $x(t-t_0) \iff X(\omega)e^{-j\omega t_0}$

Time Derivative: $\frac{d}{dt}x(t) \iff j\omega X(\omega)$

Periodic Signals: $X(\omega) = \sum_k 2\pi X[k]\delta(\omega - k\omega_0)$

February 26, 2026

Quiz #1 Information

Quiz #1 takes place in **Walker Memorial (50-340)** this **Tuesday, March 3** from **2:00 to 4:00 p.m.** See the **Quiz #1 Information** page on the website.

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Quiz 1 Information

1) Logistics

Quiz 1 will take place on Tuesday, March 3 from 2:05 to 3:55 p.m. (i.e., the regularly-scheduled class hours) in 50-340, which encompasses the majority of Walker Memorial's third floor.

- The quiz covers content from Homework 3 and from lectures and recitations up to (and including) February 19.
- The quiz will be administered on paper, so be sure to bring a pencil. (We'll have spare pencils on hand, but probably not enough for everyone.)
- You may use one 8.5"-by-11.0" page (two sides) of **handwritten** notes.
- To prepare for the quiz, we recommend that you review content from the relevant lectures, recitations, and homeworks; prepare your sheet of **handwritten** notes; and take the practice quizzes under authentic quiz conditions. (Print out a practice quiz and take it in a quiet environment where you can focus. Time yourself. Use only your **handwritten** notes as a reference.)

2) Problem-Solving Session

Our lab assistants will run a **problem-solving session** from 1:00 to 3:00 p.m. on Sunday, March 1 in 34-101. The problem-solving session is more like a two-

Agenda for Recitation

- Continuous-time Fourier transform (CTFT)

What questions do you have from lecture?

Fourier Transform Conventions

There are many competing conventions for how to define the Fourier transform. **You can refer to any source you want, but you must use our conventions in 6.300!**

$$X(\omega) \triangleq \int_{-\infty}^{\infty} x(t)e^{-j\omega t}dt \qquad x(t) = \frac{1}{2\pi}\int_{-\infty}^{\infty} X(\omega)e^{j\omega t}d\omega$$

Don't use these conventions — at least for now!

$X(j\omega) \triangleq \int_{-\infty}^{\infty} x(t)e^{-j\omega t}dt$ Why call it a function of $j\omega$?

$X(f) \triangleq \int_{-\infty}^{\infty} x(t)e^{-j2\pi ft}dt$ Function of f, not $\omega = 2\pi f$.

$\widehat{x}(\omega) \triangleq \frac{1}{\sqrt{2\pi}}\int_{-\infty}^{\infty} x(t)e^{-j\omega t}dt$ Too many $\sqrt{2\pi}$ factors.

While we're all still learning the basics, please stick to our conventions. We want to minimize confusion.

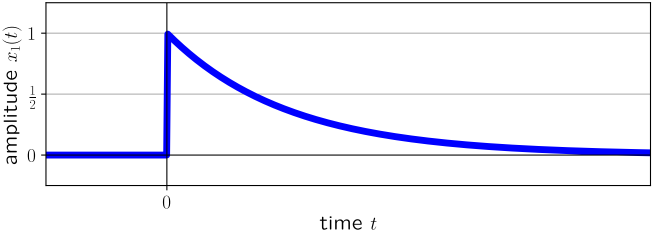
Fourier Transforms

Determine $X_1(\omega)$, the Fourier transform of $x_1(t)$.

$$x_1(t) = \begin{cases} e^{-t} & t \geq 0 \\ 0 & t < 0 \end{cases}$$

Next, sketch $\text{Re}\{X_1(\omega)\}$ and $\text{Im}\{X_1(\omega)\}$.

Finally, sketch $|X_1(\omega)|$ and $\angle X_1(\omega)$.



Fourier Transforms

Determine $X_1(\omega)$, the Fourier transform of $x_1(t)$.

$$x_1(t) = \begin{cases} e^{-t} & t \geq 0 \\ 0 & t < 0 \end{cases}$$

Next, sketch $\text{Re}\{X_1(\omega)\}$ and $\text{Im}\{X_1(\omega)\}$.

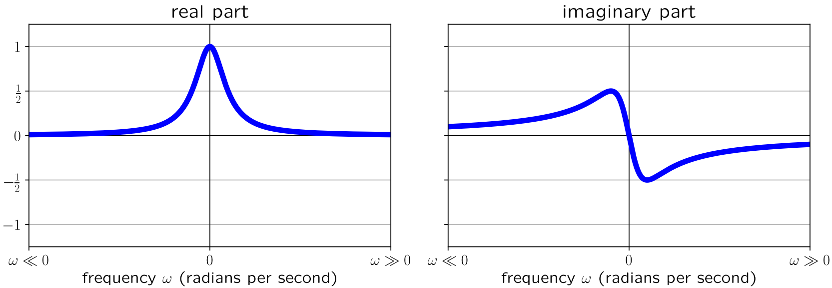
Finally, sketch $|X_1(\omega)|$ and $\angle X_1(\omega)$.

Directly compute the Fourier transform.

$$X_1(\omega) = \int_0^\infty e^{-t} e^{-j\omega t} dt = \frac{1}{1+j\omega} = \underbrace{\left(\frac{1}{1+\omega^2}\right)}_{\text{Re}\{X_1(\omega)\}} + j \underbrace{\left(\frac{-\omega}{1+\omega^2}\right)}_{\text{Im}\{X_1(\omega)\}}$$

Now, let's sketch $\text{Re}\{X_1(\omega)\}$ and $\text{Im}\{X_1(\omega)\}$.

Fourier Transforms



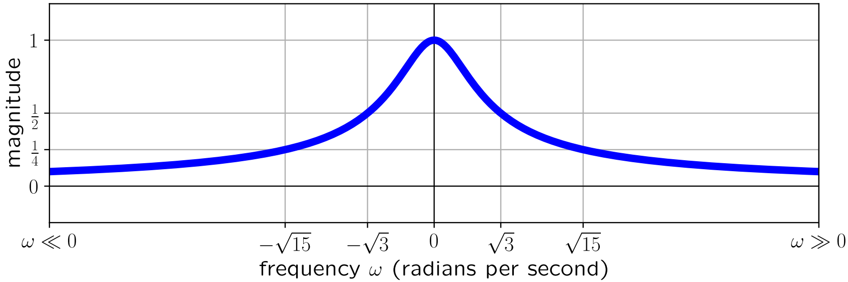
Think of the asymptotic behavior to make a sketch of

$$X_1(\omega) = \frac{1}{1+\omega^2} - j\frac{\omega}{1+\omega^2} = \mathrm{Re}\{X_1(\omega)\} + j\,\mathrm{Im}\{X_1(\omega)\}.$$

As $\omega \to 0$, $X_1(\omega) \to 1$. As $\omega \to \pm\infty$, $X_1(\omega) \to 0$.

You don't need to be Rembrandt or Rubens to sketch these plots — a rough idea is fine.

Fourier Transforms

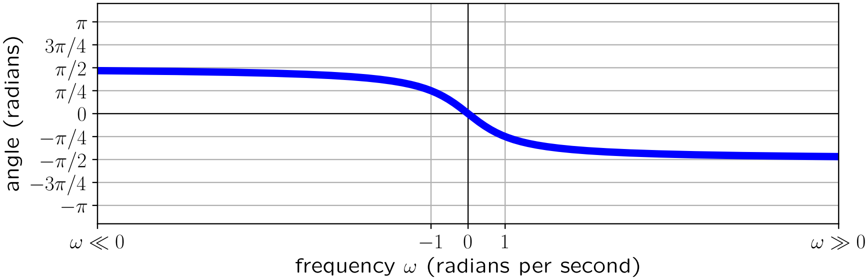


Sketch the magnitude of

$$X_1(\omega) = \frac{1}{1 + j\omega}$$

using a graphical method: Sketch $1 + j\omega$ (as a parametric function of ω) in the complex plane. When $|1 + j\omega|$ is small, $|X_1(\omega)|$ is big. When $|1 + j\omega|$ is big, $|X_1(\omega)|$ is small.

Fourier Transforms



Sketch the phase of

$$X_1(\omega) = \frac{1}{1 + j\omega}$$

using the same graphical method.

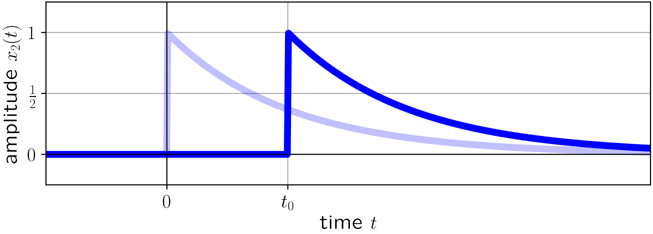
$$\angle X_1(\omega) = \angle(1) - \angle(1 + j\omega) = 0 - \tan^{-1}(\omega) = -\tan^{-1}(\omega)$$

Fourier Transforms

Determine $X_2(\omega)$, the Fourier transform of $x_2(t)$.

$$x_2(t) = x_1(t-t_0) = \begin{cases} e^{-(t-t_0)} & t \geq t_0 \\ 0 & t < t_0 \end{cases}$$

Sketch $|X_2(\omega)|$ and $\angle X_2(\omega)$. How are $|X_2(\omega)|$ and $|X_1(\omega)|$ related? How are $\angle X_2(\omega)$ and $\angle X_1(\omega)$ related?



Fourier Transforms

Determine $X_2(\omega)$, the Fourier transform of $x_2(t)$.

$$x_2(t) = x_1(t - t_0) = \begin{cases} e^{-(t-t_0)} & t \geq t_0 \\ 0 & t < t_0 \end{cases}$$

Sketch $|X_2(\omega)|$ and $\angle X_2(\omega)$. How are $|X_2(\omega)|$ and $|X_1(\omega)|$ related? How are $\angle X_2(\omega)$ and $\angle X_1(\omega)$ related?

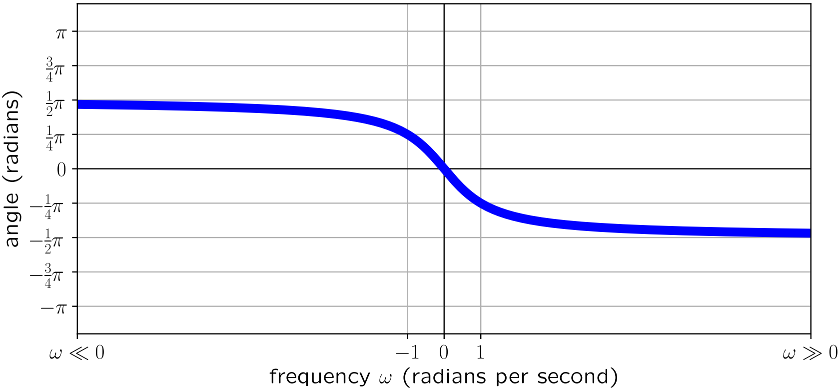
Delay property: $x_2(t) = x_1(t - t_0) \iff X_2(\omega) = e^{-j\omega t_0} X_1(\omega)$.

Magnitudes multiply: $|X_2(\omega)| = |e^{-j\omega t_0}||X_1(\omega)| = |X_1(\omega)|$.

Angles add: $\angle X_2(\omega) = \angle e^{-j\omega t_0} + \angle X_1(\omega) = -\omega t_0 + \angle X_1(\omega)$.

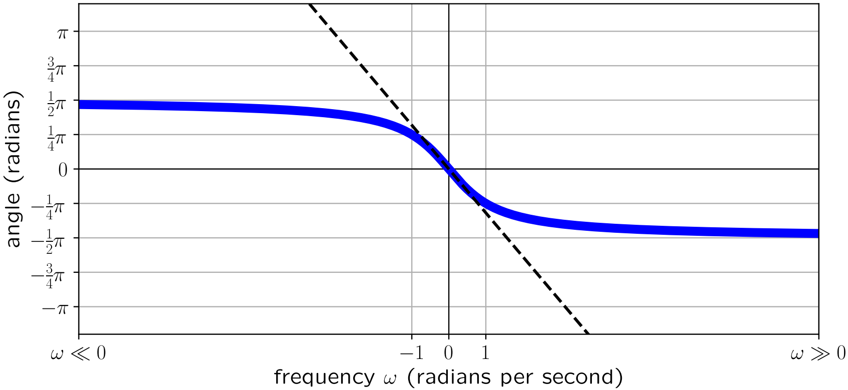
A time delay alters phase — but not magnitude. For a higher $|\omega|$, the same delay t_0 corresponds to a larger phase shift.

Fourier Transforms



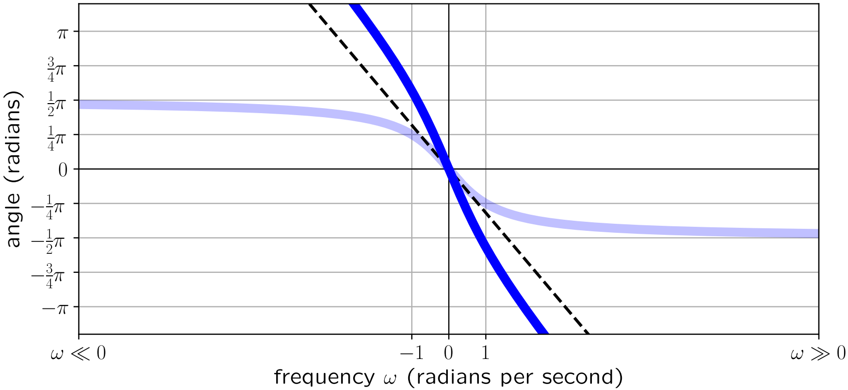
To sketch $\angle X_2(\omega)$, start by sketching $\angle X_1(\omega)$.

Fourier Transforms



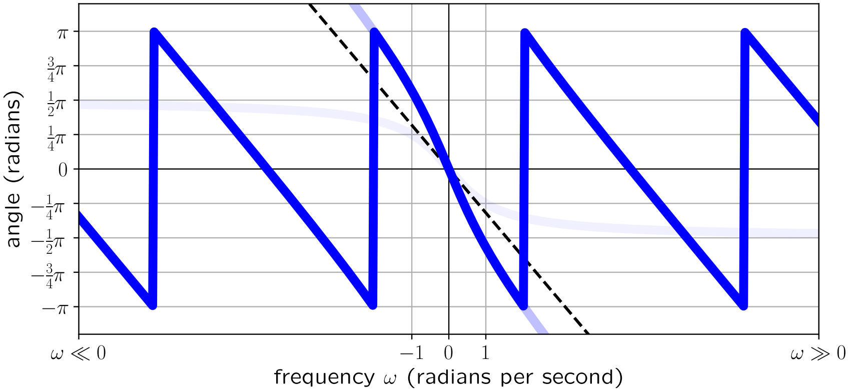
Sketch the $-\omega t_0$ term.

Fourier Transforms



Angles add: $\angle X_2(\omega) = -\omega t_0 + \angle X_1(\omega)$.

Fourier Transforms



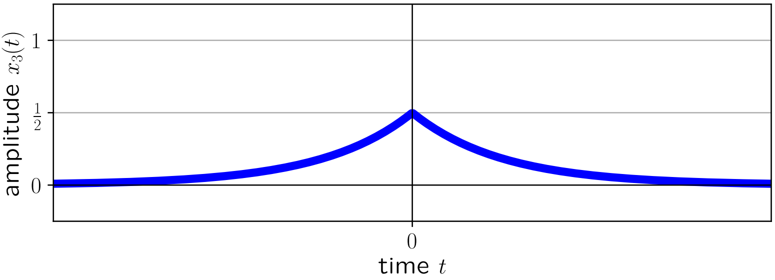
Finally, wrap $\angle X_2(\omega)$ into the $[-\pi, \pi]$ range.

Fourier Transforms

Determine $X_3(\omega)$, the Fourier transform of $x_3(t)$.

$$x_3(t) = \text{Symmetric}\{x_1(t)\}$$

If a time-domain signal is **real** and **symmetric**, what can you say about the Fourier transform?



Fourier Transforms

Determine $X_3(\omega)$, the Fourier transform of $x_3(t)$.

$$x_3(t) = \text{Symmetric}\{x_1(t)\}$$

If a time-domain signal is real and symmetric, then the Fourier transform is real and symmetric, too.

$$x_3(t) = \tfrac{1}{2}x_1(t) + \tfrac{1}{2}x_1(-t)$$

$$X_3(\omega) = \tfrac{1}{2}X_1(\omega) + \tfrac{1}{2}X_1(-\omega) = \frac{1}{1+\omega^2} = \mathrm{Re}\{X_1(\omega)\}$$

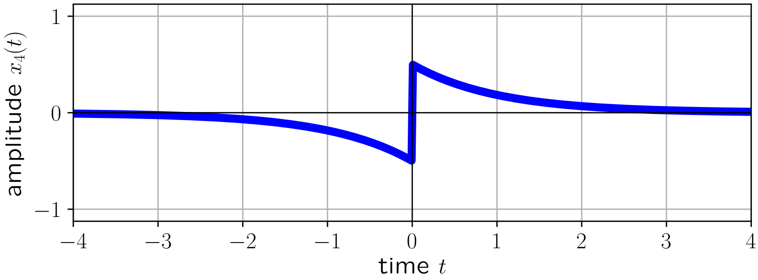
$X_3(\omega)$ is real and non-negative, so $\angle X_3(\omega) = 0$ for all ω. In general, real-valued signals either have angle 0 (when non-negative) or angle $\pm\pi$ (when negative) — they lie flat along the real axis.

Fourier Transforms

Determine $X_4(\omega)$, the Fourier transform of $x_4(t)$.

$$x_4(t) = \text{Anti-symmetric}\{x_1(t)\}$$

If a time-domain signal is **real** and **anti-symmetric**, what can you say about the Fourier transform?



Fourier Transforms

Determine $X_4(\omega)$, the Fourier transform of $x_4(t)$.

$$x_4(t) = \text{Anti-symmetric}\{x_1(t)\}$$

If a time-domain signal is **real** and **anti-symmetric**, what can you say about the Fourier transform?

If a time-domain signal is real and anti-symmetric, then the Fourier transform is imaginary and anti-symmetric.

$$x_4(t) = \tfrac{1}{2}x_1(t) - \tfrac{1}{2}x_1(-t)$$

$$X_4(\omega) = \tfrac{1}{2}X_1(\omega) - \tfrac{1}{2}X_1(-\omega) = j\frac{-\omega}{1+\omega^2} = j\,\mathrm{Im}\{X_1(\omega)\}$$

Remember: The imaginary part $\mathrm{Im}\{X_1(\omega)\}$ is real.

Fourier Transforms

Determine $X_5(\omega)$, the Fourier transform of $x_5(t)$.

$$x_5(t) = \frac{d}{dt} x_4(t)$$

Hint: Start from the **synthesis equation**.

What does **differentiating with respect to time** do the Fourier transform of a signal?

Fourier Transforms

Determine $X_5(\omega)$, the Fourier transform of $x_5(t)$.

$$x_5(t) = \frac{d}{dt}x_4(t)$$

Hint: Start from the **synthesis equation**.

What does **differentiating with respect to time** do the Fourier transform of a signal?

Derivative in time? Multiply by $j\omega$ in frequency.

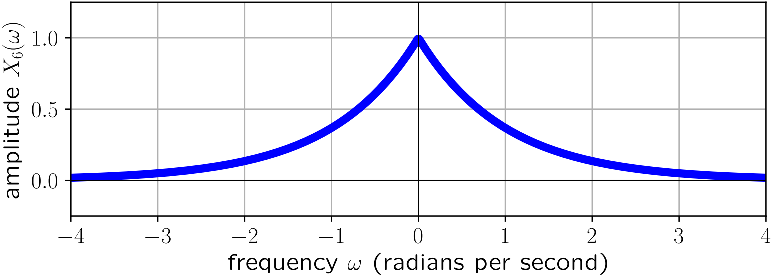
$$x_5(t) = \frac{d}{dt}x_4(t) = \frac{d}{dt}\left(\frac{1}{2\pi}\int_{-\infty}^{\infty} X_4(\omega)e^{j\omega t}d\omega\right)$$

$$= \frac{1}{2\pi}\int_{-\infty}^{\infty} \underbrace{\left(j\omega X_4(\omega)\right)}_{X_5(\omega)} e^{j\omega t}d\omega$$

Fourier Transforms

$x_6(t)$ has Fourier transform $X_6(\omega)$. Determine $x_6(t)$.

$$X_6(\omega) = e^{-|\omega|} = \begin{cases} e^{-\omega} & \omega \ge 0 \\ e^{\omega} & \omega < 0 \end{cases}$$



Fourier Transforms

$x_6(t)$ has Fourier transform $X_6(\omega)$. Determine $x_6(t)$.

$$X_6(\omega) = e^{-|\omega|} = \begin{cases} e^{-\omega} & \omega \geq 0 \\ e^{\omega} & \omega < 0 \end{cases}$$

Split into two intervals: $\omega \geq 0$ and $\omega < 0$.

$$x_6(t) = \frac{1}{2\pi}\int_{-\infty}^{0} e^{\omega} e^{j\omega t} d\omega + \frac{1}{2\pi}\int_{0}^{\infty} e^{-\omega} e^{j\omega t} d\omega = \frac{1}{\pi}\left(\frac{1}{1+t^2}\right)$$

The result is proportional to the Fourier transform of Symmetric$\{x_1(t)\}$ — with time t in place of frequency ω.

$$x(t) \iff X(\omega)$$
$$X(t) \iff 2\pi x(-\omega)$$

Lessons Learned

The **continuous-time Fourier transform (CTFT)** is a Fourier representation for aperiodic and periodic continuous-time signals. It has many useful properties.

Analysis: $X(\omega) = \int_{-\infty}^{\infty} x(t) e^{-j\omega t} dt$

Synthesis: $x(t) = \frac{1}{2\pi} \int_{-\infty}^{\infty} X(\omega) e^{j\omega t} d\omega$

Time Delay: $x(t - t_0) \iff X(\omega) e^{-j\omega t_0}$

Time Derivative: $\frac{d}{dt} x(t) \iff j\omega X(\omega)$

Periodic Signals: $X(\omega) = \sum_k 2\pi X[k] \delta(\omega - k\omega_0)$

Question of the Day

Suppose that $x(t)$ is periodic in $T = 2\pi$.

The **Fourier series coefficients** of $x(t)$ are

$$X[k] = \begin{cases} \frac{1}{2\pi} & k = \pm 1 \\ 0 & \text{otherwise} \end{cases}$$

Make a rough sketch of the **Fourier transform** $X(\omega)$.