6.300: Signal Processing

2D Convolution

Suppose that $f[r,c]$ is an $R_f \times C_f$ image and that $g[r,c]$ is an $R_g \times C_g$ image. The convolution $(f * g)[r,c]$ is an image of size $(R_f + R_g - 1) \times (C_f + C_g - 1)$. By contrast, the circular convolution $(f \circledast g)[r,c]$ is an image of size $R \times C$, where R is the length of the 2D DFT along the row dimension and C is the length of the 2D DFT along the column dimension.

$$f[r,c] \rightarrow \boxed{h[r,c]} \rightarrow g[r,c] = \frac{1}{RC}(f \circledast h)[r,c]$$

$$F[k_r,k_c] \rightarrow \boxed{H[k_r,k_c]} \rightarrow G[k_r,k_c] = F[k_r,k_c]H[k_r,k_c]$$

April 28, 2026

Agenda for Recitation

- Convolution, circular convolution, and filtering in 2D

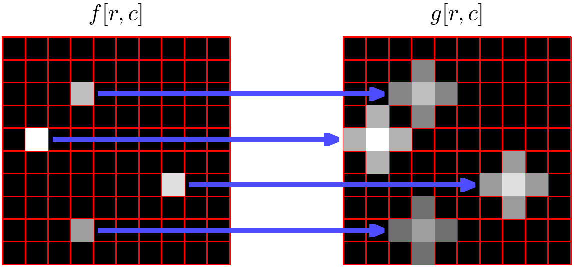
What questions do you have from lecture?

2D Convolution

We can represent a system that is linear and shift-invariant by its unit-sample response (its response to a unit-sample signal):

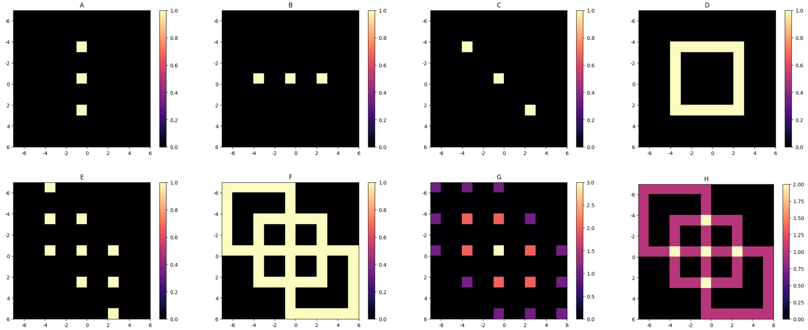
$$\delta[r, c] \rightarrow h[r, c]$$

The response of such a system to an input $f[r, c]$ is the superposition of shifted and scaled versions of the unit-sample response.



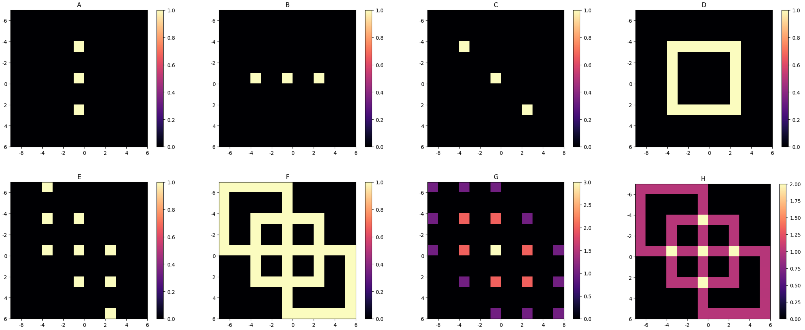
2D Convolution

Which of the following images can be constructed by convolving two of the other images?



2D Convolution

Which of the following images can be constructed by convolving two of the other images?



$$E = A * C$$

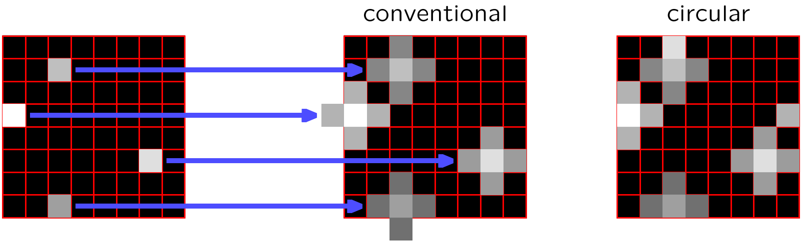
$$G = B * E$$

$$H = C * D$$

2D Circular Convolution

Convolution in space is equivalent to multiplication of DTFT's.

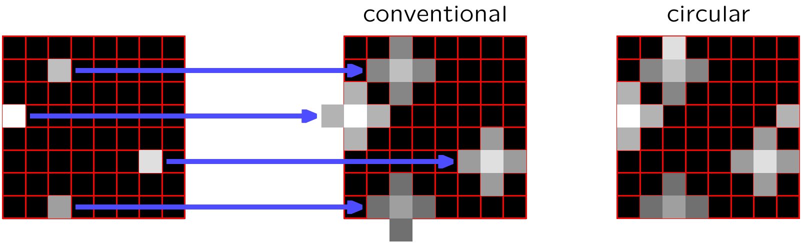
However, multiplication of DFT's (or DTFS's) is equivalent to **circular convolution** in space.



The domains of the input and output signals are limited by the dimensions of the DFTs.

2D Circular Convolution

Two perspectives.

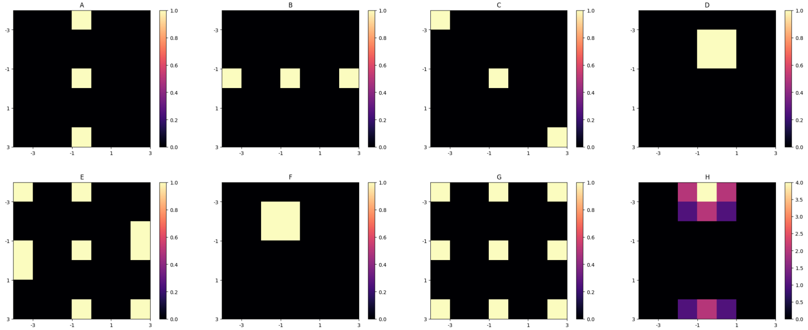


Focusing on the output: If part of the output image falls outside the region, move it back into the region by shifting that part by an integer number of widths or heights.

Focusing on the input: Start by periodically extending the input by repeating the region of interest to tile the entire plane. Then do conventional convolution.

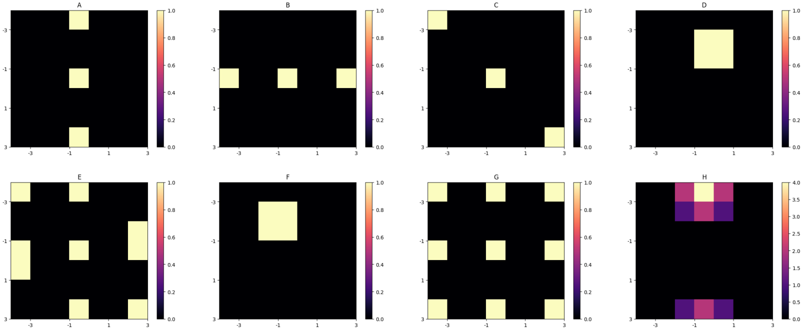
2D Circular Convolution

Which of the following images can be constructed by circularly convolving two of the other images?



2D Circular Convolution

Which of the following images can be constructed by circularly convolving two of the other images?



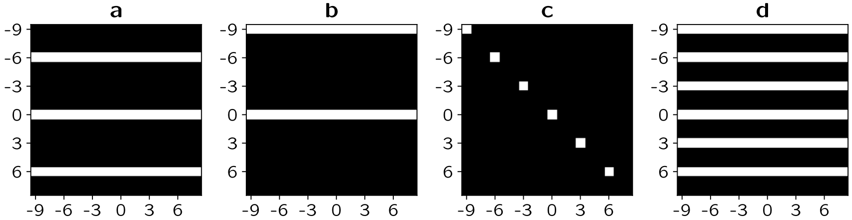
$$E = A \circledast C$$

$$G = A \circledast B$$

$$H = D \circledast F$$

Convolution and Filtering in 2D

Consider the four images shown below.[1]

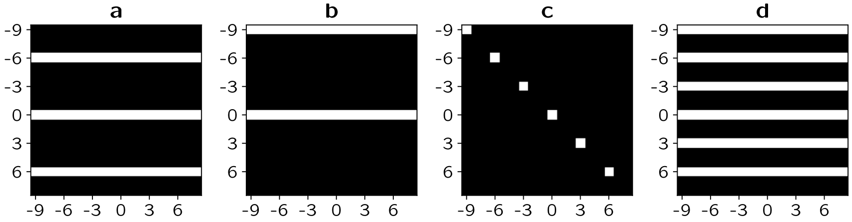


Which image(s) may be constructed by convolving ($\circledast$) two of the other images?

[1] The brightness of a black pixel is zero. The brightness of a white pixel is greater than zero and may differ between images.

Convolution and Filtering in 2D

Consider the four images shown below.



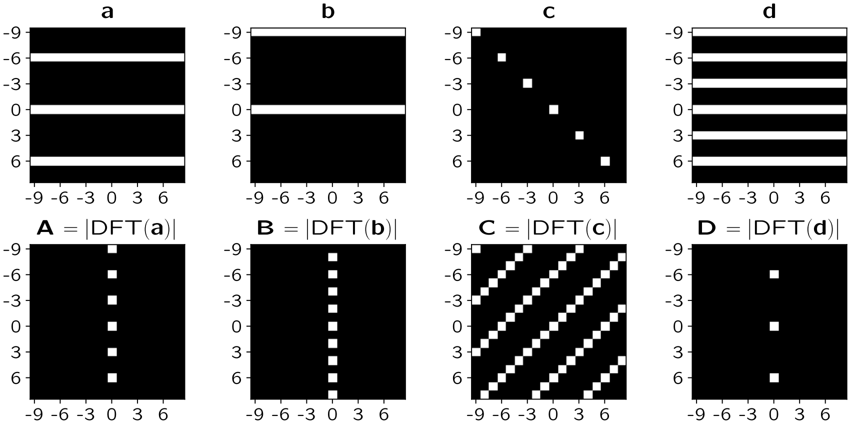
Which image(s) may be constructed by convolving ($\circledast$) two of the other images?

$\mathbf{d} \propto \mathbf{a} \circledast \mathbf{b}$

$\mathbf{d} \propto \mathbf{a} \circledast \mathbf{c}$

$\mathbf{d} \propto \mathbf{b} \circledast \mathbf{c}$

Convolution and Filtering in 2D



Which image(s) may be constructed by multiplying ($\times$) the transforms of two other images?

Convolution and Filtering in 2D

Convolution ($\circledast$) in space corresponds to multiplication ($\times$) in frequency. We should be able to do the problem both ways.

$$f[r,c] \rightarrow \boxed{h[r,c]} \rightarrow g[r,c] = \frac{1}{RC}(f \circledast h)[r,c]$$

$$F[k_r,k_c] \rightarrow \boxed{H[k_r,k_c]} \rightarrow G[k_r,k_c] = F[k_r,k_c]H[k_r,k_c]$$

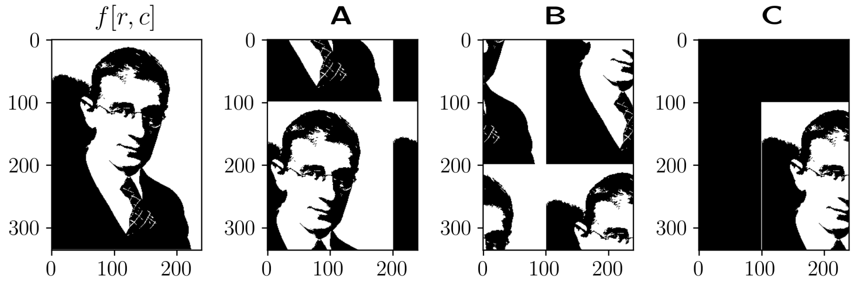
Convolution in space:

$\mathbf{d} \propto \mathbf{a} \circledast \mathbf{b}$
$\mathbf{d} \propto \mathbf{a} \circledast \mathbf{c}$
$\mathbf{d} \propto \mathbf{b} \circledast \mathbf{c}$

Multiplication in frequency:

$\mathbf{D} \propto \mathbf{A} \times \mathbf{B}$
$\mathbf{D} \propto \mathbf{A} \times \mathbf{C}$
$\mathbf{D} \propto \mathbf{B} \times \mathbf{C}$

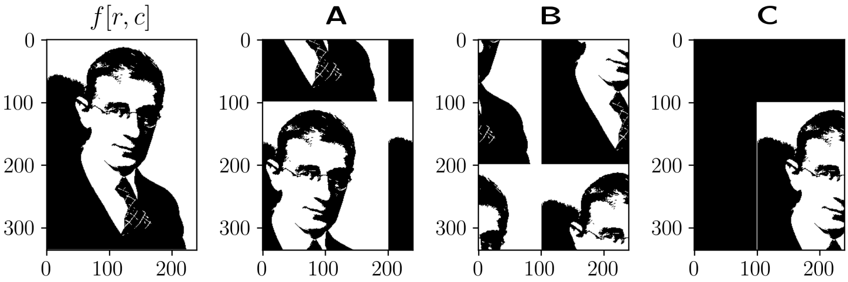
Convolution vs. Circular Convolution



Match an expression to each image above.

$$g_1[r,c] = f[r,c] * \delta[r-100, c-100]$$
$$g_2[r,c] = f[r,c] * \delta[r-100, c-200]$$
$$g_3[r,c] = f[r,c] * \delta[r-200, c-100]$$
$$g_4[r,c] = f[r,c] \circledast \delta[r-100, c-100]$$
$$g_5[r,c] = f[r,c] \circledast \delta[r-100, c-200]$$
$$g_6[r,c] = f[r,c] \circledast \delta[r-200, c-100]$$

Convolution vs. Circular Convolution



Match an expression to each image above.

$$g_1[r,c] = f[r,c] * \delta[r-100, c-100] \Longrightarrow \textbf{C}$$

$$g_2[r,c] = f[r,c] * \delta[r-100, c-200]$$

$$g_3[r,c] = f[r,c] * \delta[r-200, c-100]$$

$$g_4[r,c] = f[r,c] \circledast \delta[r-100, c-100]$$

$$g_5[r,c] = f[r,c] \circledast \delta[r-100, c-200] \Longrightarrow \textbf{A}$$

$$g_6[r,c] = f[r,c] \circledast \delta[r-200, c-100] \Longrightarrow \textbf{B}$$

Question of the Day

Let $f[r, c]$ denote the $R \times C$ image of Prof. Vannevar Bush above at left. Define $H[k_r, k_c] = \cos(\pi k_r)\cos(\pi k_c)$. Describe what the image $(f \circledast h)[r, c]$ would look like.