

Name:

Kerberos (Athena) Username:

Please WAIT until we tell you to begin.

This exam is closed book, but you may use three 8.5×11 sheets of notes (six sides).

You may NOT use any electronic devices (such as calculators and phones).

If you have questions, please **come to us** at the front of the room to ask.

Please enter all solutions in the boxes provided.

Work on other pages with QR codes will be considered for partial credit.

Please provide a note if you continue work on worksheets at the end of the exam.

Please do not write on the QR codes at the bottom of each page.

We use those codes to identify which pages belong to each student.

Trigonometric Identities Reference

$$\cos(a+b) = \cos(a)\cos(b) - \sin(a)\sin(b)$$

$$\sin(a+b) = \sin(a)\cos(b) + \cos(a)\sin(b)$$

$$\cos(a) + \cos(b) = 2\cos\left(\frac{a+b}{2}\right)\cos\left(\frac{a-b}{2}\right)$$

$$\sin(a) + \sin(b) = 2\sin\left(\frac{a+b}{2}\right)\cos\left(\frac{a-b}{2}\right)$$

$$\cos(a+b) + \cos(a-b) = 2\cos(a)\cos(b)$$

$$\sin(a+b) + \sin(a-b) = 2\sin(a)\cos(b)$$

$$2\cos(a)\cos(b) = \cos(a-b) + \cos(a+b)$$

$$2\sin(a)\cos(b) = \sin(a+b) + \sin(a-b)$$

$$\cos(a-b) = \cos(a)\cos(b) + \sin(a)\sin(b)$$

$$\sin(a-b) = \sin(a)\cos(b) - \cos(a)\sin(b)$$

$$\cos(a) - \cos(b) = -2\sin\left(\frac{a+b}{2}\right)\sin\left(\frac{a-b}{2}\right)$$

$$\sin(a) - \sin(b) = 2\cos\left(\frac{a+b}{2}\right)\sin\left(\frac{a-b}{2}\right)$$

$$\cos(a+b) - \cos(a-b) = -2\sin(a)\sin(b)$$

$$\sin(a+b) - \sin(a-b) = 2\cos(a)\sin(b)$$

$$2\sin(a)\sin(b) = \cos(a-b) - \cos(a+b)$$

$$2\cos(a)\sin(b) = \sin(a+b) - \sin(a-b)$$

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Page 1

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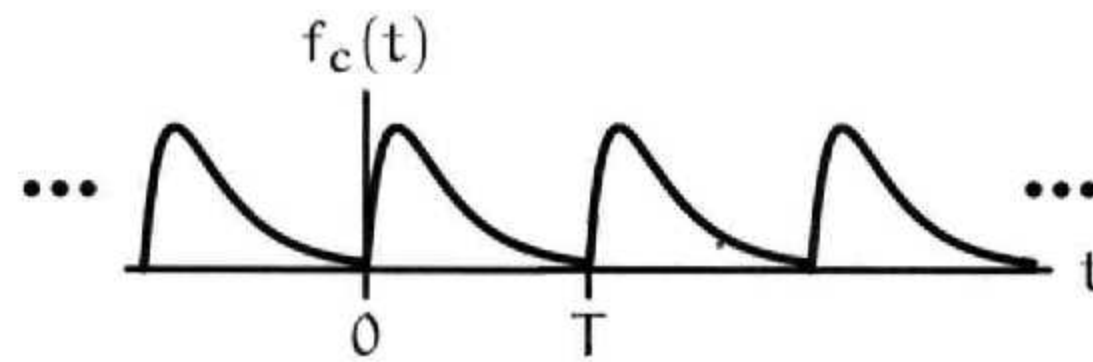
exam #0
page #1



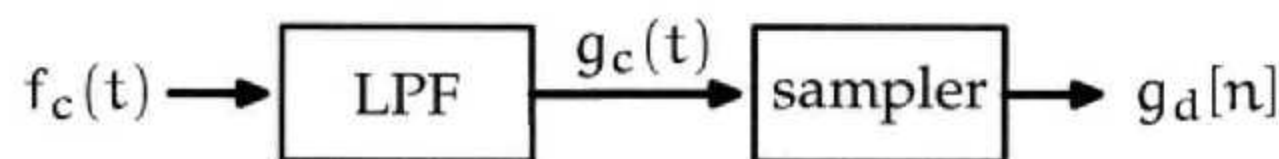
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1 Sampling Harmonics (16 points)

Let $f_c(t)$ represent a continuous-time signal with period $T = 1/440$ seconds, as shown below.



Suppose $f_c(t)$ is the input to an ideal low-pass filter (LPF) with cut-off frequency $\omega_0 = 11\pi/T$ radians per second. The output of the low-pass filter, $g_c(t)$, is subsequently sampled at times $t = n\Delta$, where $\Delta = 1/1000$ seconds. The result is the sequence $g_d[n] = g_c(n\Delta)$.



Let $G_d(\Omega)$ represent the discrete-time Fourier transform (DTFT) of $g_d[n]$. $G_d(\Omega)$ is non-zero only for countably many frequencies $\Omega \in [0, \pi]$. Determine the number of frequencies Ω for which $G_d(\Omega)$ is non-zero, and write your answer in the box below.

Number of frequencies Ω for which $G_d(\Omega)$ is non-zero:

List all these frequencies in the boxes provided below. You may list the frequencies as real numbers or as a fraction of π (such as 0.1π or $\pi/10$), and you may list the frequencies in any order. Use as many boxes as needed. Leave unneeded boxes blank.

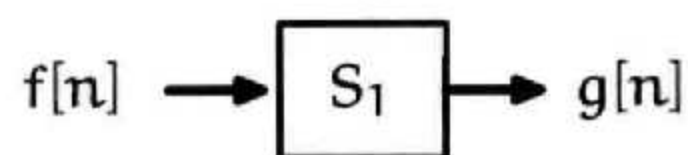
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2 System Identification and Inversion (22 points)

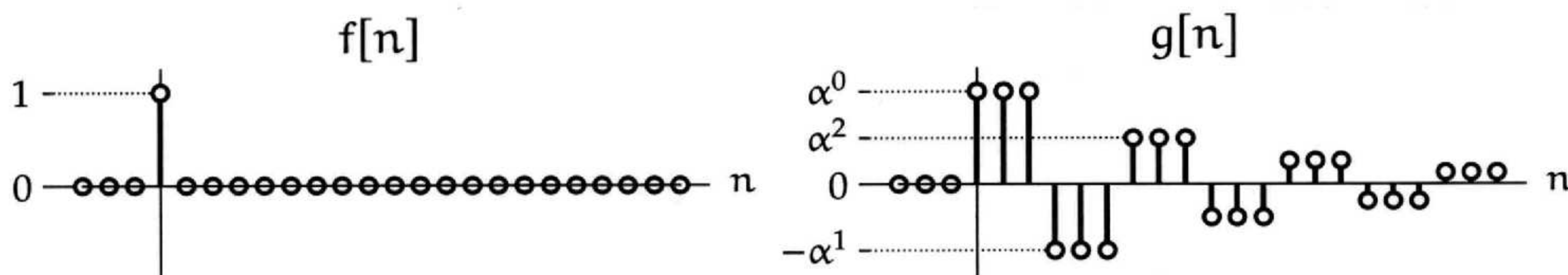
Part a. Let S_1 represent a linear, time-invariant (LTI) system.



Let α denote a real number in the interval $(0, 1)$. When the input is $f[n] = \delta[n]$, the response is

$$g[n] = \begin{cases} 0 & n < 0 \\ \alpha^0 & 0 \leq n \leq 2 \\ -\alpha^1 & 3 \leq n \leq 5 \\ \alpha^2 & 6 \leq n \leq 8 \\ -\alpha^3 & 9 \leq n \leq 11 \\ \dots & \dots \end{cases}$$

as shown below for $n \in [-3, 20]$.



Determine a linear, constant-coefficient difference equation that relates $f[n]$ and $g[n]$. Assume initial rest conditions. Your answer may include α as well as familiar constants like $\pi \approx 3.14159$ and $e \approx 2.718$.

Part b. Let $H_1(\Omega)$ denote the frequency response of system S_1 . Determine a closed-form expression for $H_1(\Omega)$. Your answer may include α as well as familiar constants like $\pi \approx 3.14159$ and $e \approx 2.718$.

For which values of $\Omega \in [-\pi, \pi]$ (if any) is $H_1(\Omega) = 0$? Write X if there are no such values of Ω .

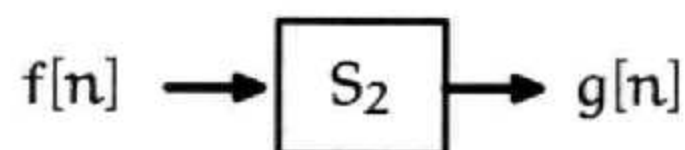
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Part c. Let S_2 represent a linear, time-invariant (LTI) system.



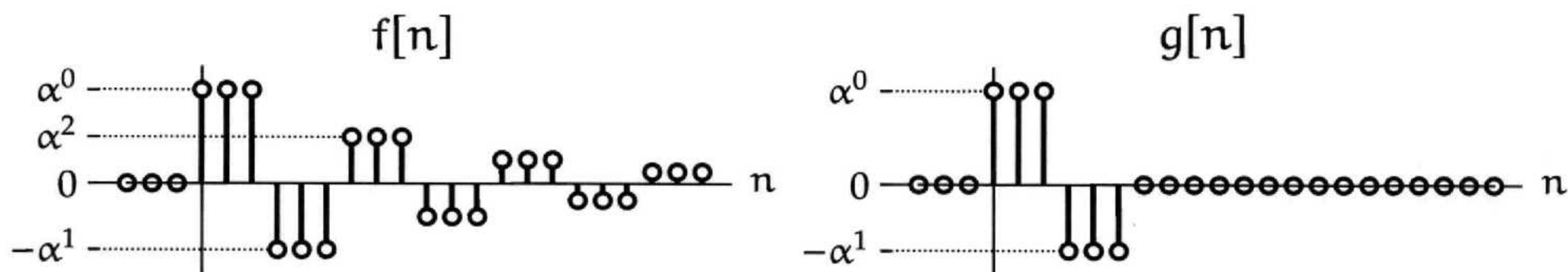
Let α denote a real number in the interval $(0, 1)$. When the input is

$$f[n] = \begin{cases} 0 & n < 0 \\ \alpha^0 & 0 \leq n \leq 2 \\ -\alpha^1 & 3 \leq n \leq 5 \\ \alpha^2 & 6 \leq n \leq 8 \\ -\alpha^3 & 9 \leq n \leq 11 \\ \dots & \dots \end{cases}$$

the response is

$$g[n] = \begin{cases} 0 & n < 0 \\ \alpha^0 & 0 \leq n \leq 2 \\ -\alpha^1 & 3 \leq n \leq 5 \\ 0 & 6 \leq n \end{cases}$$

as shown below for $n \in [-3, 20]$.



Let $h_2[n]$ denote the unit-sample response of system S_2 . Determine a closed-form expression for $h_2[n]$. Your answer may include α as well as familiar constants like $\pi \approx 3.14159$ and $e \approx 2.718$.

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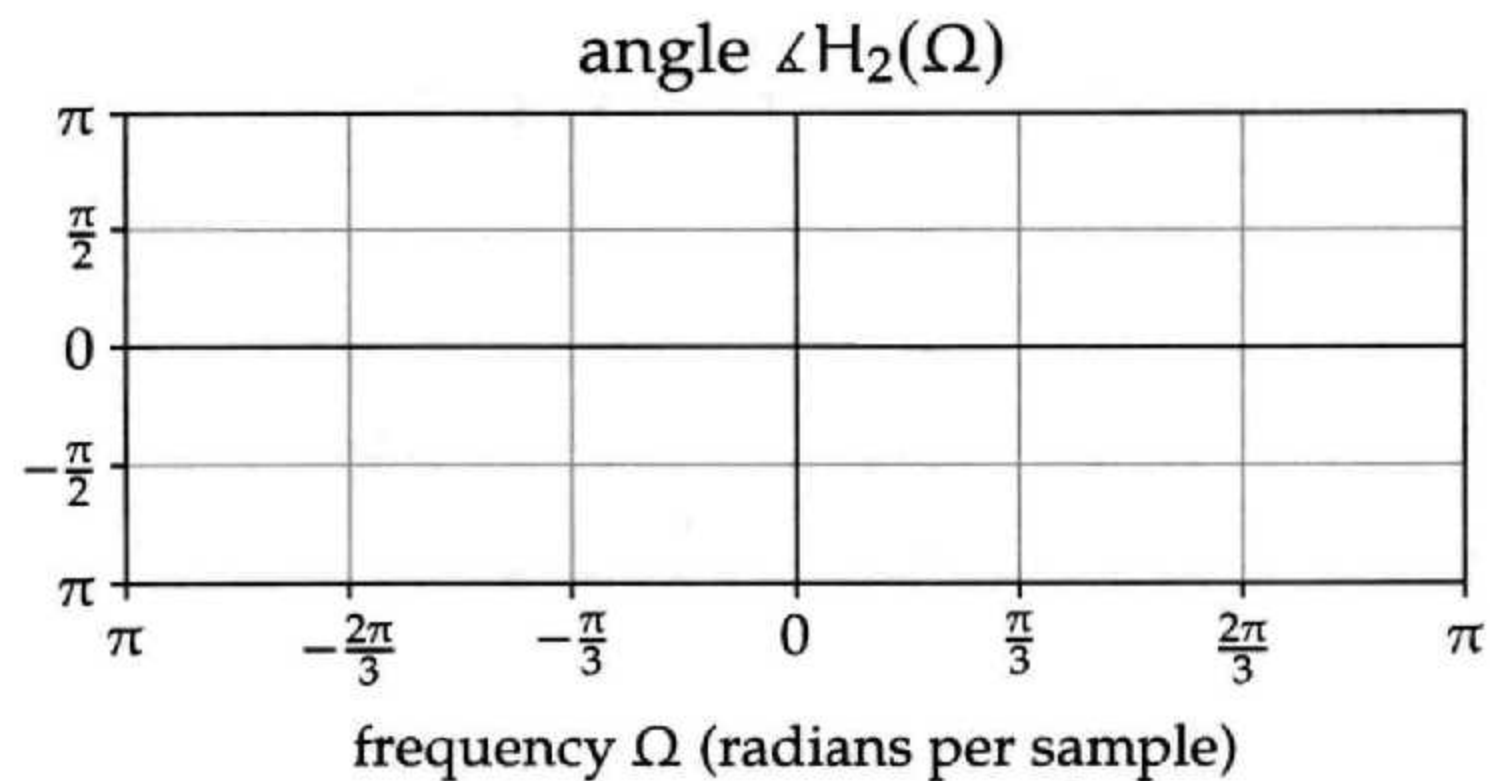
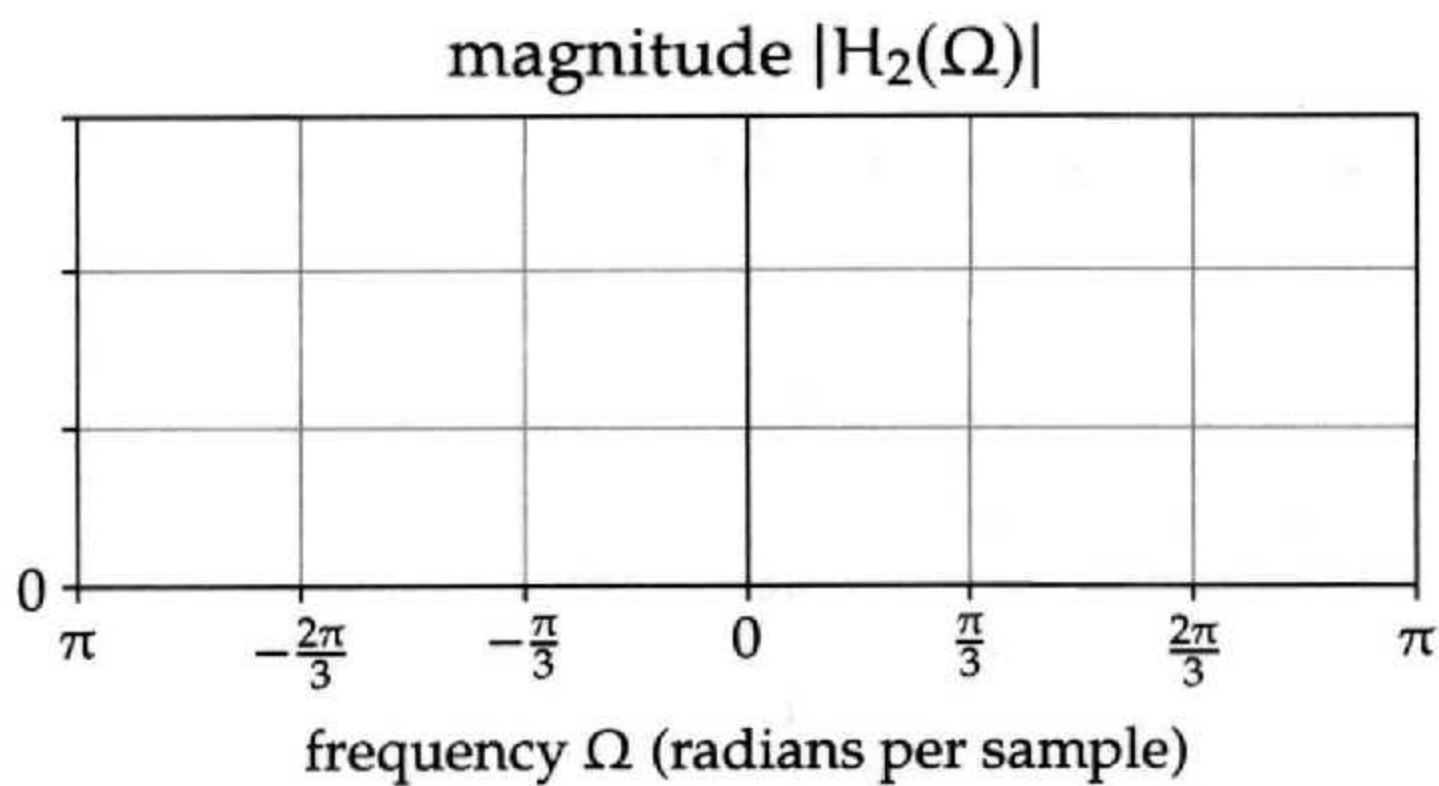
Page 4

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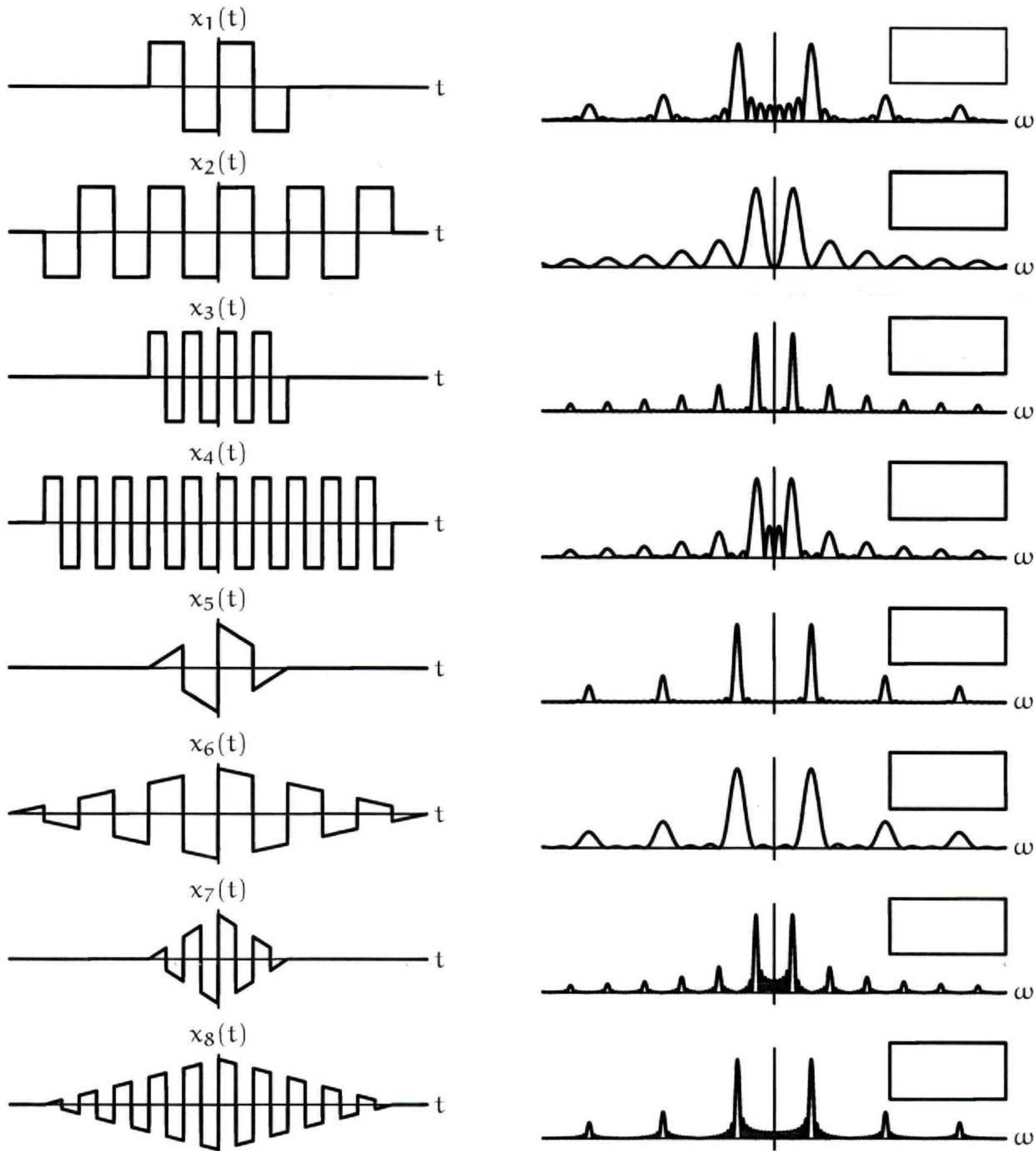
Part d. Let $H_2(\Omega)$ denote the frequency response of system S_2 . Determine a closed-form expression for $H_2(\Omega)$. Your answer may include α as well as familiar constants like $\pi \approx 3.14159$ and $e \approx 2.718$.

Sketch $|H_2(\Omega)|$ and $\angle H_2(\Omega)$ on the axes below for $\alpha = 1$.



3 Time-Frequency Patterns (16 points)

Eight time-domain signals are shown below at left. Eight Fourier transform magnitudes are shown below at right. For each transform shown on the right, determine the corresponding time-domain signal on the left and enter its number in the box provided. Each time-domain signal is plotted on the same time scale, and each Fourier transform is plotted on the same frequency scale. Time-domain signals are zero outside the range shown.



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4 Two-Dimensional Patterns (16 points)

Eight two-dimensional, discrete-space signals ($f_1[r, c], f_2[r, c], \dots, f_8[r, c]$) are shown below, where pixels are indexed by row and column numbers (r and c) that are each in the range $[-4, 4]$. White and black pixels represent values of 1 and -1 respectively, and grey pixels represent values of 0.

Determine which of the magnitude plots on the next page (A,B,...,H) shows the magnitude of the two-dimensional discrete Fourier transform of each of the discrete-space signals, and record your answers by entering the appropriate labels in the corresponding boxes below.

Similarly, determine which of the phase plots on the next page (a,b,...,h) shows the phase of the two-dimensional discrete Fourier transform of each of the discrete-space signals, and record your answers by entering the appropriate labels in the corresponding boxes below.

$f_1[r, c]$ 	$ F_1[k_r, k_c] $ <input type="text"/>	$\angle F_1[k_r, k_c]$ <input type="text"/>	$f_5[r, c]$ 	$ F_5[k_r, k_c] $ <input type="text"/>	$\angle F_5[k_r, k_c]$ <input type="text"/>
$f_2[r, c]$ 	$ F_2[k_r, k_c] $ <input type="text"/>	$\angle F_2[k_r, k_c]$ <input type="text"/>	$f_6[r, c]$ 	$ F_6[k_r, k_c] $ <input type="text"/>	$\angle F_6[k_r, k_c]$ <input type="text"/>
$f_3[r, c]$ 	$ F_3[k_r, k_c] $ <input type="text"/>	$\angle F_3[k_r, k_c]$ <input type="text"/>	$f_7[r, c]$ 	$ F_7[k_r, k_c] $ <input type="text"/>	$\angle F_7[k_r, k_c]$ <input type="text"/>
$f_4[r, c]$ 	$ F_4[k_r, k_c] $ <input type="text"/>	$\angle F_4[k_r, k_c]$ <input type="text"/>	$f_8[r, c]$ 	$ F_8[k_r, k_c] $ <input type="text"/>	$\angle F_8[k_r, k_c]$ <input type="text"/>

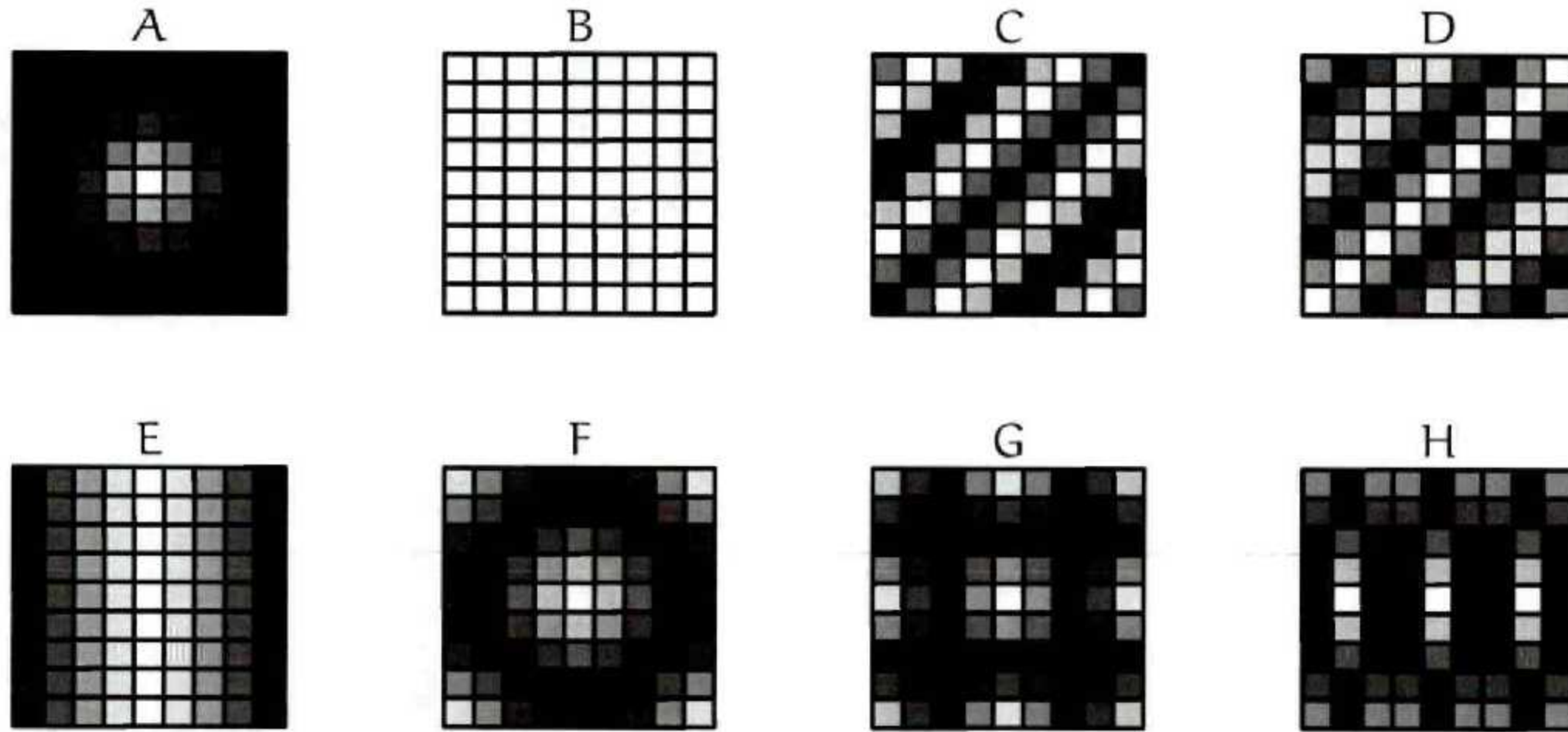
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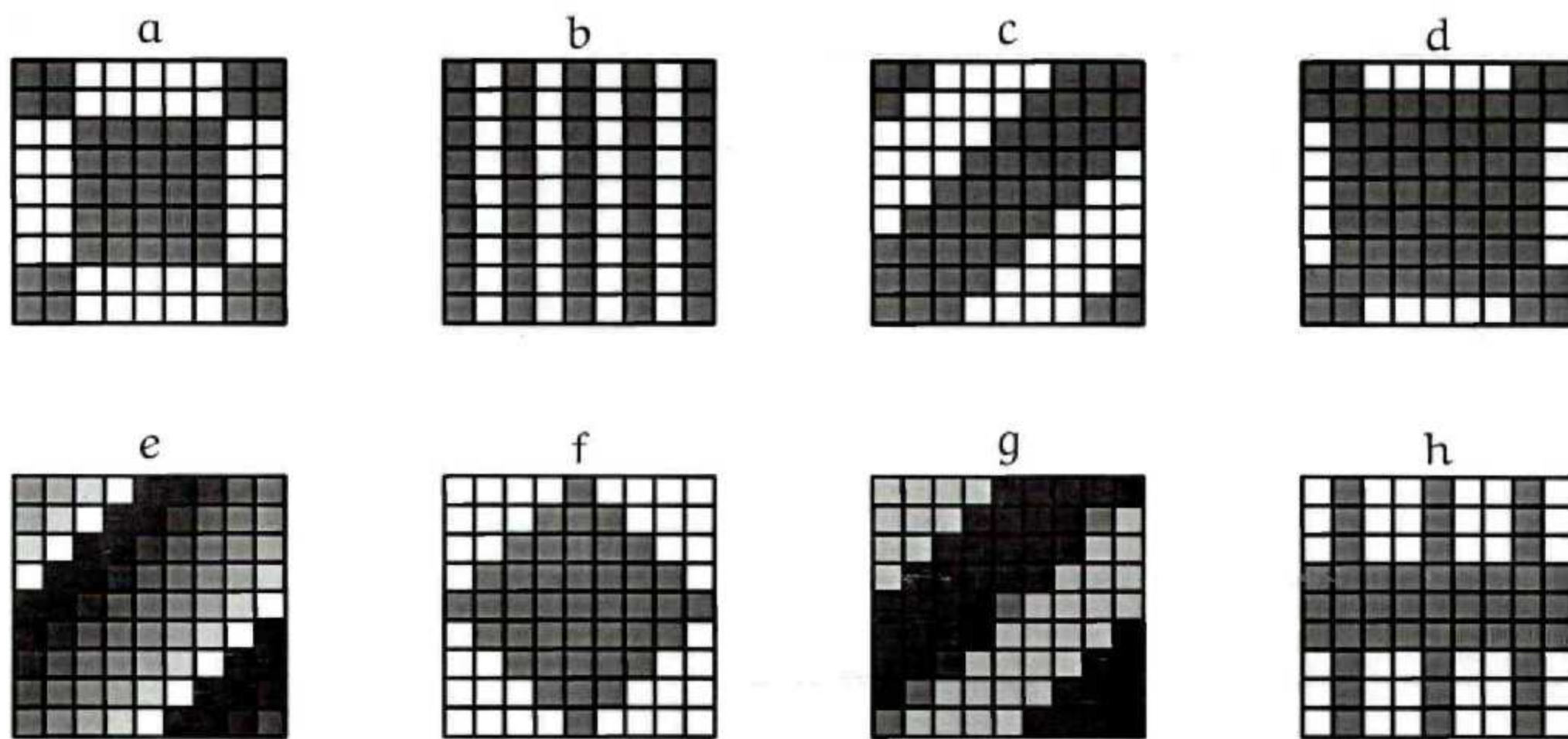
Magnitude plots

grayscale: from 0 (black) to the maximum magnitude in each image (white)



Phase plots

grayscale: from $-\pi$ (black) to π (white)



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5 Convolution Signals (12 points)

Part a. Two 5x5 pixel images are circularly convolved (using 5x5 DFTs and a 5x5 inverse DFT) to produce a new 5x5 image, where black pixels represent values of 0 and white pixels represent values of 1.

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Identify which of the following images (A-J) results for each of the previous circular convolutions, and enter that image label (A-J) in the corresponding box above. If none of the images match, enter X.

A 	B 	C 	D 	E
F 	G 	H 	I 	J

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Part b. Two 13×9 pixel images are circularly convolved (using 13×9 DFTs and a 13×9 inverse DFT) to produce a new 13×9 image, where positive and negative values are represented by white and black pixels, respectively. Gray pixels represent values of 0.

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Identify which of the following images (a-l) results for each of the circular convolutions shown above, and enter that image label (a-l) in the corresponding box above. If none of the images match, enter X.

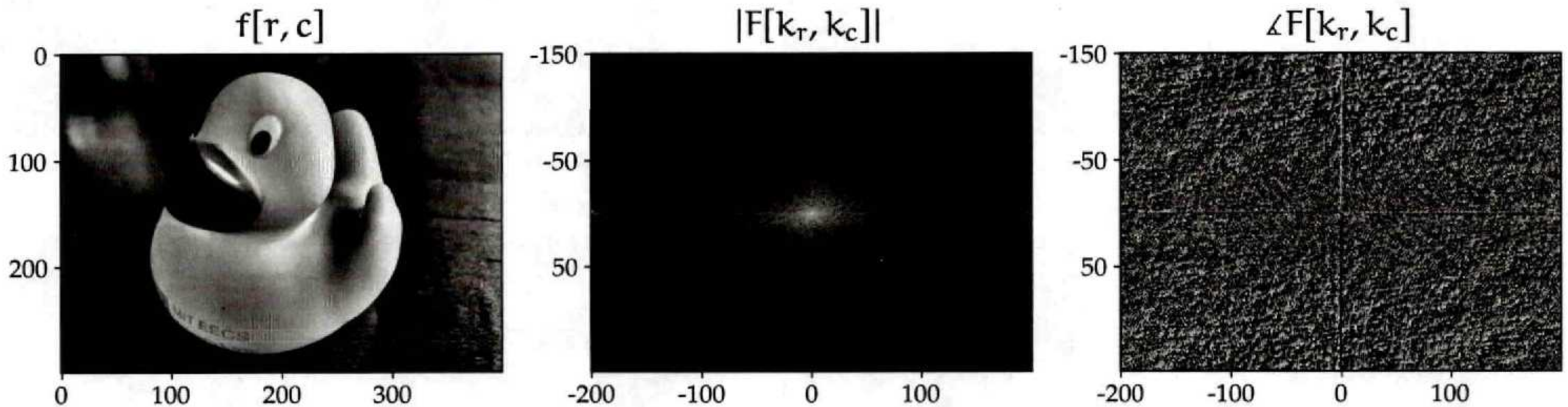
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6 Ducks (18 points)

Let $f[r, c]$ represent the image of a rubber duck shown below at left. The pixels are indexed by row r and column c , where $0 \leq r < 300$ and $0 \leq c < 400$. Furthermore, let $F[k_r, k_c]$ denote the two-dimensional discrete Fourier transform (DFT) of $f[r, c]$ computed with $R = 300$ and $C = 400$. $|F[k_r, k_c]|$ and $\angle F[k_r, k_c]$ denote the magnitude and phase, respectively, of $F[k_r, k_c]$ and are shown below. In each image, black represents the minimum value, and white represents the maximum value. Note that the minima and maxima may differ between images.



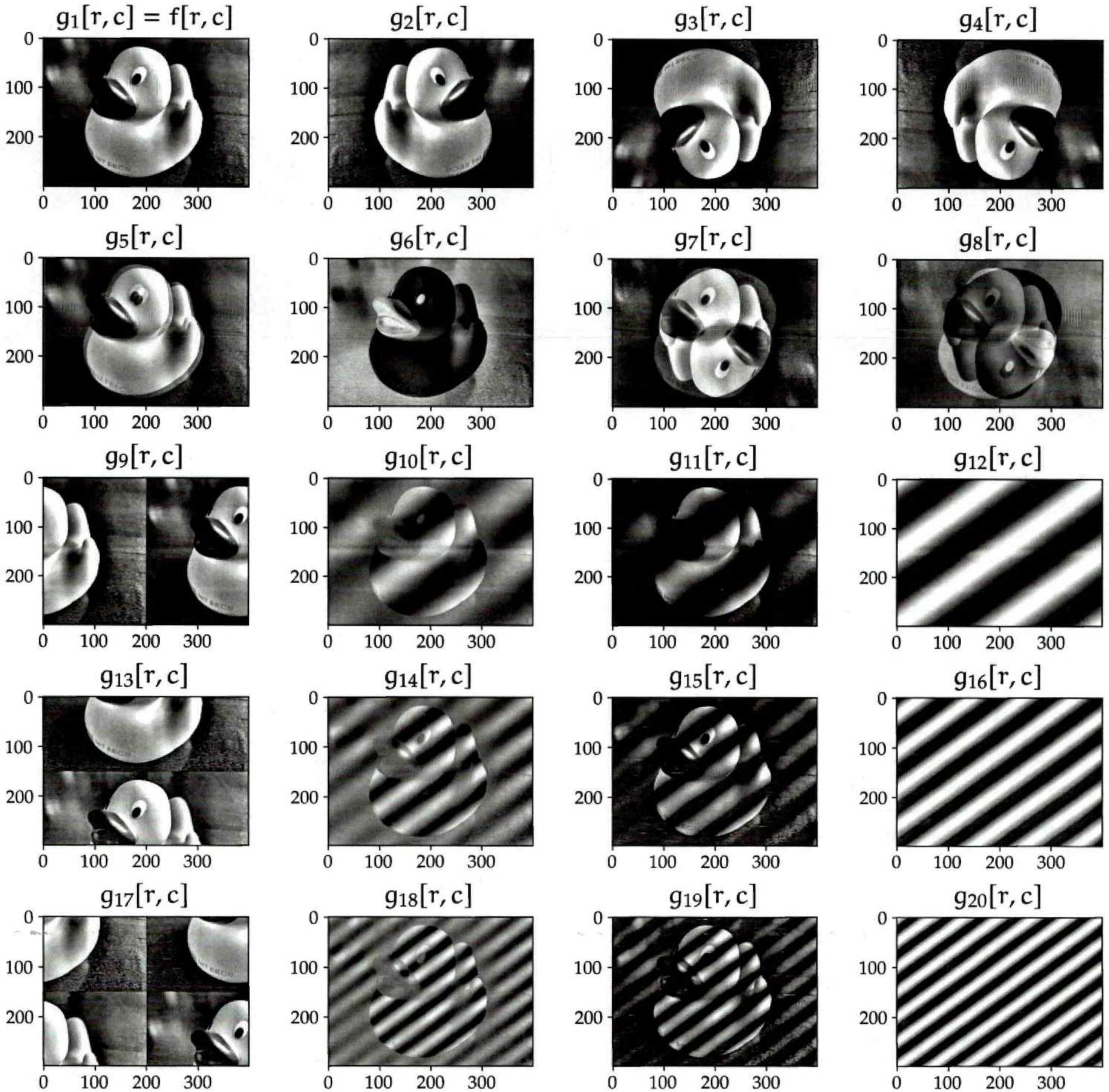
The table below lists operations which are performed either on $f[r, c]$ or on $F[k_r, k_c]$. Determine which (if any) of the images on the following page shows the image that results from the operation specified. In each image, black represents the minimum value and white represents the maximum value. The minima and maxima may differ between images. If there is no match, write **X** in the corresponding box.

operation	resulting image (g_1, g_2, \dots, g_{20} or X)
multiply $f[r, c]$ by $\cos\left(\frac{8\pi}{300}r + \frac{8\pi}{400}c\right)$	
convolve $f[r, c]$ with $\cos\left(\frac{8\pi}{300}r + \frac{8\pi}{400}c\right)$	
multiply $F[k_r, k_c]$ by $\cos\left(\frac{8\pi}{300}k_r + \frac{8\pi}{400}k_c\right)$	
convolve $F[k_r, k_c]$ with $\cos\left(\frac{8\pi}{300}k_r + \frac{8\pi}{400}k_c\right)$	
multiply $F[k_r, k_c]$ by $\cos(\pi k_r) \cos(\pi k_c)$	
convolve $F[k_r, k_c]$ with $\cos(\pi k_r) \cos(\pi k_c)$	
compute the inverse DFT of $\text{Re}\{F[k_r, k_c]\}$	
compute the inverse DFT of $\text{Im}\{F[k_r, k_c]\}$	
compute the inverse DFT of $ F[k_r, k_c] e^{-j\angle F[k_r, k_c]}$	



Images for "Ducks" Problem

In each image, black represents the minimum value and white represents the maximum value. The minima and maxima may differ between images. If there is no match, write X in the corresponding box on the previous page.



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Page 13

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Page 14

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Page 15

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page #15



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Page 16

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page #16



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