

Name:

Kerberos (Athena) username:

Please WAIT until we tell you to begin.

This quiz is closed book, but you may use two 8.5×11 sheets of notes (four sides).

You may NOT use any electronic devices (such as calculators and phones).

If you have questions, please **come to us** at the front of the room to ask.

Please enter all solutions in the boxes provided.

Work on other pages with QR codes will be considered for partial credit.

Please provide a note if you continue work on worksheets at the end of the exam.

Please do not write on the QR codes at the bottom of each page.

We use those codes to identify which pages belong to each student.

Trigonometric Identities Reference

$$\cos(a+b) = \cos(a)\cos(b) - \sin(a)\sin(b)$$

$$\sin(a+b) = \sin(a)\cos(b) + \cos(a)\sin(b)$$

$$\cos(a) + \cos(b) = 2\cos\left(\frac{a+b}{2}\right)\cos\left(\frac{a-b}{2}\right)$$

$$\sin(a) + \sin(b) = 2\sin\left(\frac{a+b}{2}\right)\cos\left(\frac{a-b}{2}\right)$$

$$\cos(a+b) + \cos(a-b) = 2\cos(a)\cos(b)$$

$$\sin(a+b) + \sin(a-b) = 2\sin(a)\cos(b)$$

$$2\cos(a)\cos(b) = \cos(a-b) + \cos(a+b)$$

$$2\sin(a)\cos(b) = \sin(a+b) + \sin(a-b)$$

$$\cos(a-b) = \cos(a)\cos(b) + \sin(a)\sin(b)$$

$$\sin(a-b) = \sin(a)\cos(b) - \cos(a)\sin(b)$$

$$\cos(a) - \cos(b) = -2\sin\left(\frac{a+b}{2}\right)\sin\left(\frac{a-b}{2}\right)$$

$$\sin(a) - \sin(b) = 2\cos\left(\frac{a+b}{2}\right)\sin\left(\frac{a-b}{2}\right)$$

$$\cos(a+b) - \cos(a-b) = -2\sin(a)\sin(b)$$

$$\sin(a+b) - \sin(a-b) = 2\cos(a)\sin(b)$$

$$2\sin(a)\sin(b) = \cos(a-b) - \cos(a+b)$$

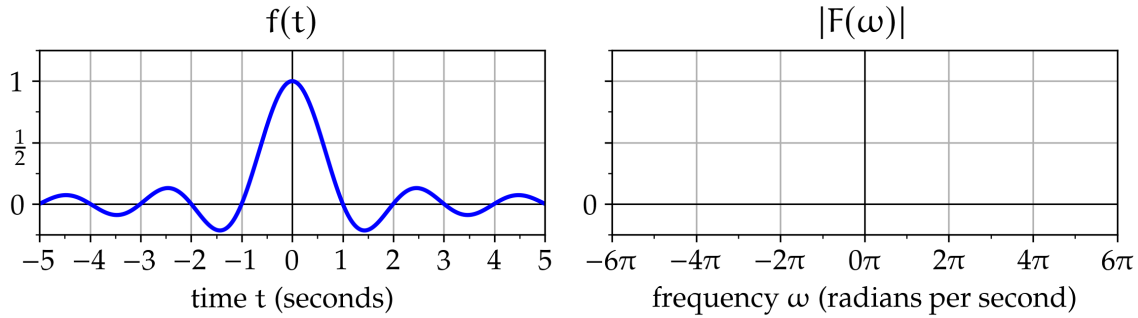
$$2\cos(a)\sin(b) = \sin(a+b) - \sin(a-b)$$

1 Sampling in Time and Frequency (24 points)

Part a. Consider a continuous-time LTI system with impulse response $f(t)$, as follows:

$$f(t) = \frac{\sin(\pi t)}{\pi t}$$

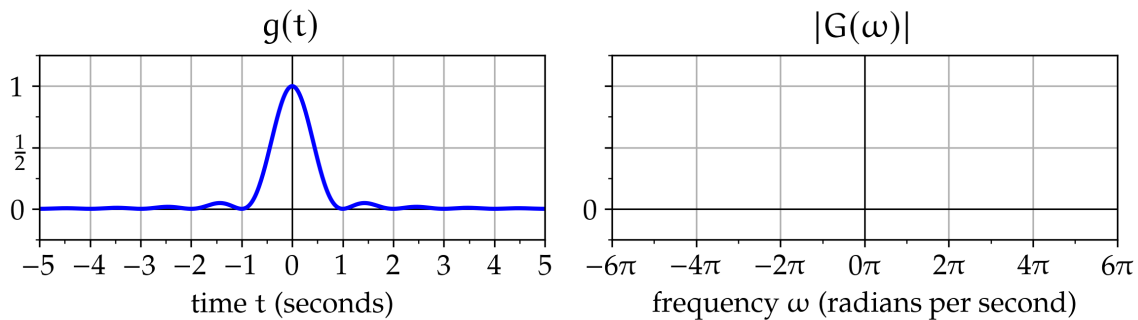
Let $F(\omega)$ denote the frequency response. Sketch $|F(\omega)|$ for $\omega \in [-6\pi, 6\pi]$. Label the vertical axis.



Part b. Consider a continuous-time LTI system with impulse response $g(t)$, as follows:

$$g(t) = f^2(t)$$

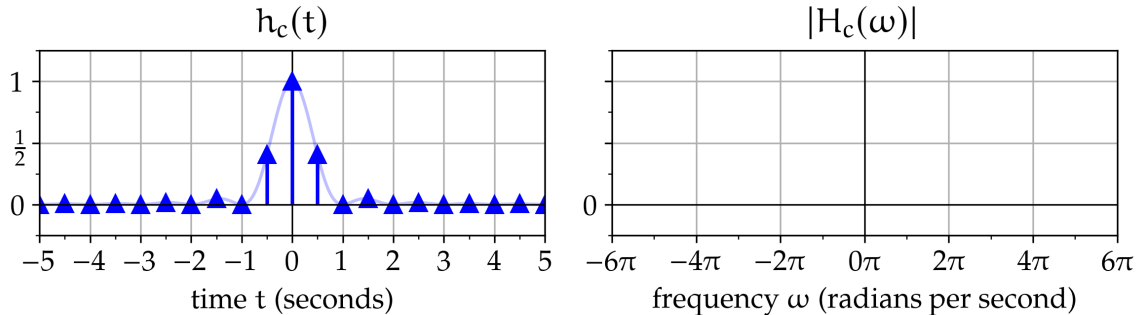
Let $G(\omega)$ denote the frequency response. Sketch $|G(\omega)|$ for $\omega \in [-6\pi, 6\pi]$. Label the vertical axis.



Part c. Consider a continuous-time LTI system with impulse response $h_c(t)$, as follows:

$$h_c(t) = g(t) \sum_{m=-\infty}^{\infty} \delta(t-0.5m)$$

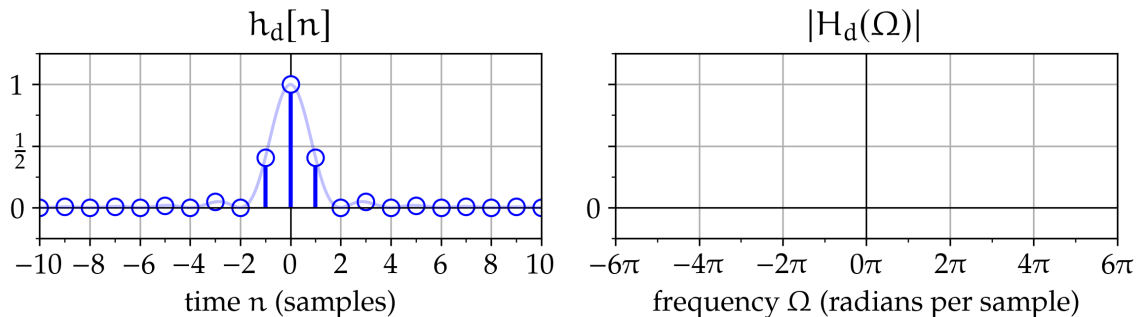
Let $H_c(\omega)$ denote the frequency response. Sketch $|H_c(\omega)|$ for $\omega \in [-6\pi, 6\pi]$. Label the vertical axis.



Part d. Consider a discrete-time LTI system with unit-sample response $h_d[n]$, as follows:

$$h_d[n] = h_c(0.5n)$$

Let $H_d(\Omega)$ denote the frequency response. Sketch $|H_d(\Omega)|$ for $\Omega \in [-6\pi, 6\pi]$. Label the vertical axis.



2 Difference Equations (28 points)

Let $\alpha > 0$ denote a real-valued constant. Consider eight discrete-time LTI systems:

$$x[n] \rightarrow \boxed{S_0} \rightarrow y[n] = x[n] + \alpha y[n-1]$$

$$x[n] \rightarrow \boxed{S_1} \rightarrow y[n] = x[n] + \alpha y[n-2]$$

$$x[n] \rightarrow \boxed{S_2} \rightarrow y[n] = x[n] - \alpha y[n-1]$$

$$x[n] \rightarrow \boxed{S_3} \rightarrow y[n] = x[n] - \alpha y[n-2]$$

$$x[n] \rightarrow \boxed{S_4} \rightarrow y[n] = x[n] + x[n-2] - \alpha y[n-4]$$

$$x[n] \rightarrow \boxed{S_5} \rightarrow y[n] = x[n] + x[n-1] + x[n-2] + x[n-3] - \alpha y[n-4]$$

$$x[n] \rightarrow \boxed{S_6} \rightarrow y[n] = x[n] + x[n-1] + x[n-2] + x[n-3] + x[n-4]$$

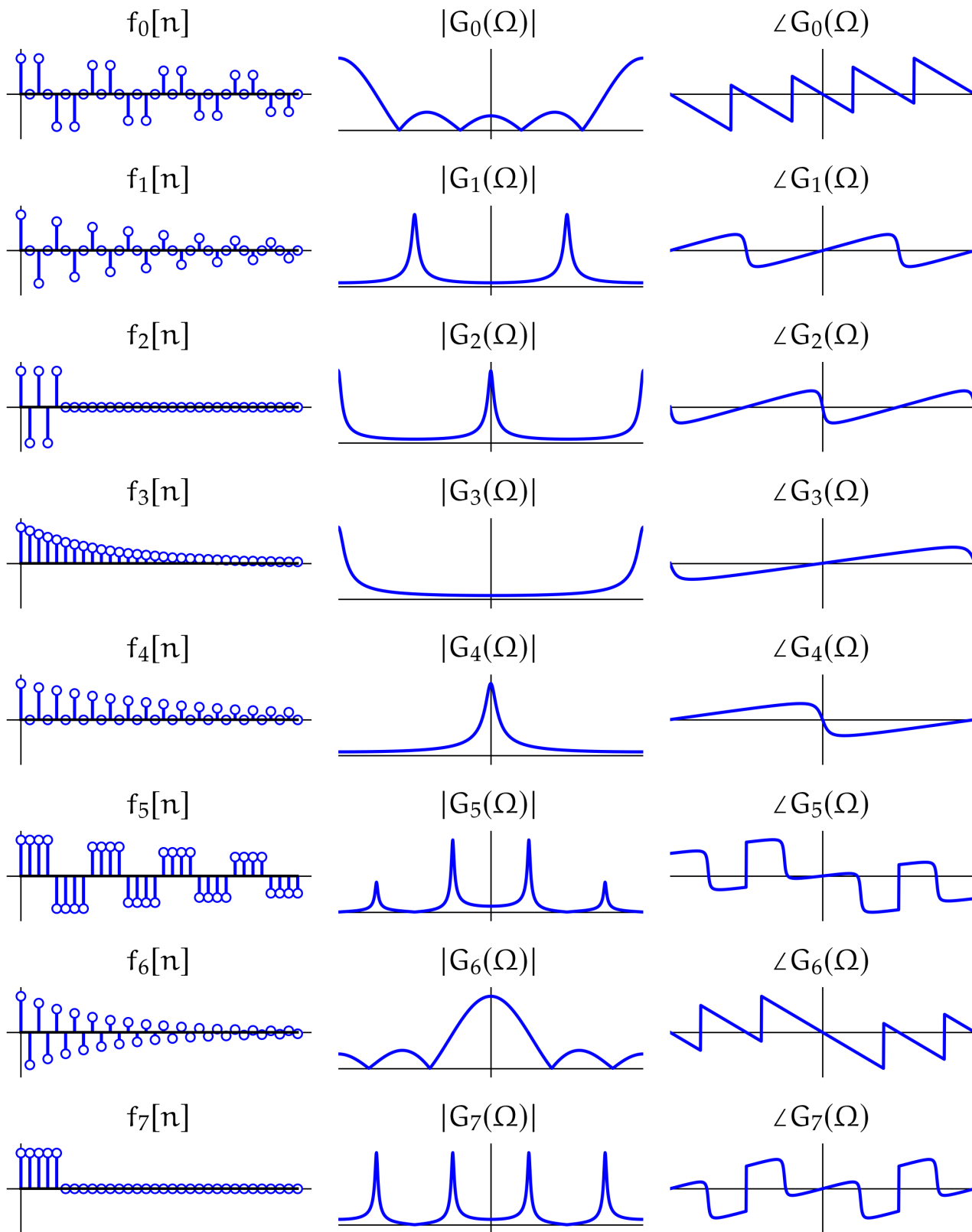
$$x[n] \rightarrow \boxed{S_7} \rightarrow y[n] = x[n] - x[n-1] + x[n-2] - x[n-3] + x[n-4]$$

Twenty-four plots are shown on the next page.

- Plots in the left column show (for $0 \leq n \leq 31$) a unit-sample response $f_k[n]$.
- Plots in the middle and right columns show (for $-\pi \leq \Omega \leq \pi$) the magnitude $|G_\ell(\Omega)|$ and angle $\angle G_\ell(\Omega)$, respectively, of a frequency response $G_\ell(\Omega)$.

For each system, determine which plots on the next page show the system's unit-sample response and frequency response. If the unit-sample response or frequency response of a system is not shown in any of the plots given, write an **X** in the corresponding box.

system	unit-sample response (f_0, f_1, \dots, f_7)	frequency response (G_0, G_1, \dots, G_7)
S_0		
S_1		
S_2		
S_3		
S_4		
S_5		
S_6		
S_7		

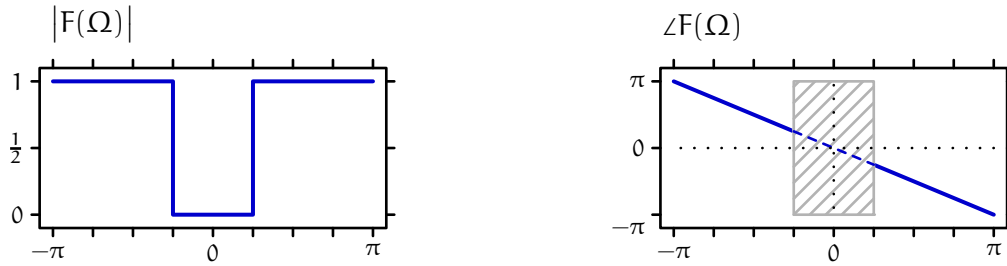


do not write below

do not write below

3 Fourier Matching (28 points)

The following plots show one period of the magnitude (left) and angle (right) of $F(\Omega)$, which is the Discrete-Time Fourier Transform of a discrete-time signal $f[n]$.

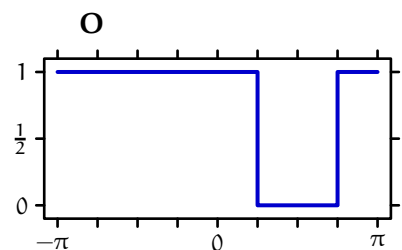
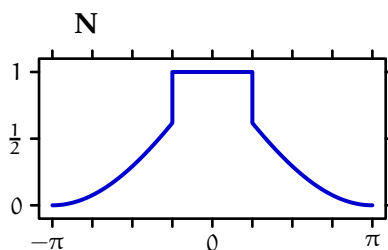
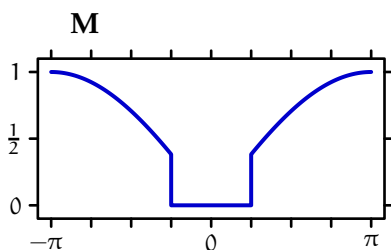
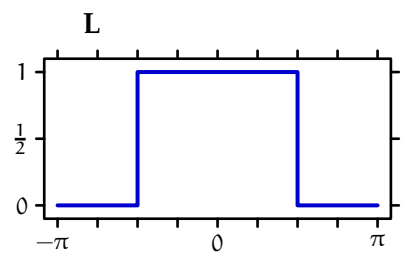
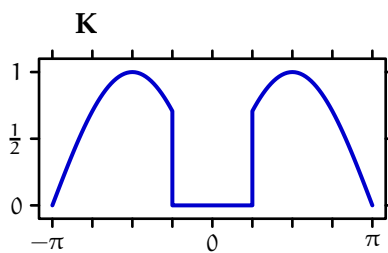
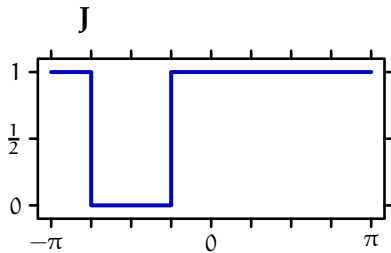
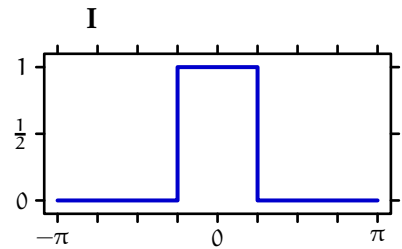
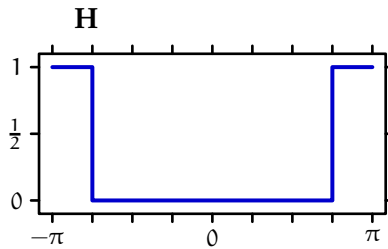
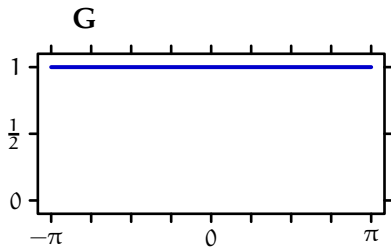
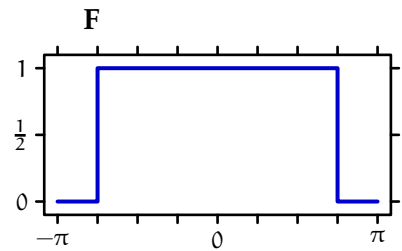
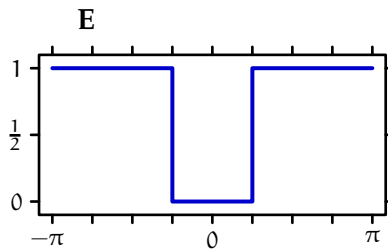
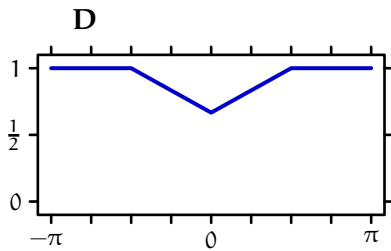
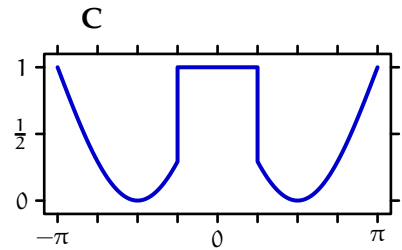
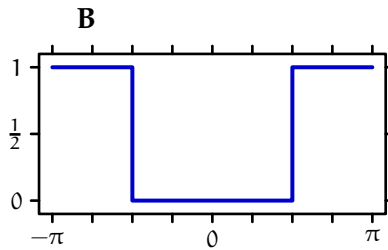
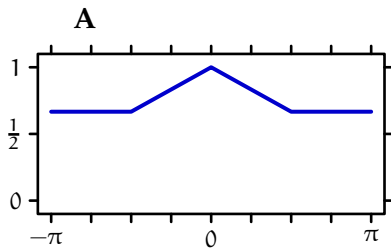


The crosshatched region of the angle plot indicates a region of Ω for which the angle is undefined (because $|F(\Omega)|$ is zero at those frequencies). The dashed line in the crosshatched region is shown as an aid to visualization of trends outside the crosshatched region.

The left column in the chart below shows seven signals that are derived from $f[n]$. Determine which (if any) of the magnitude plots on the following page and the angle plots on the subsequent page match your results for each of the **seven derived signals**.

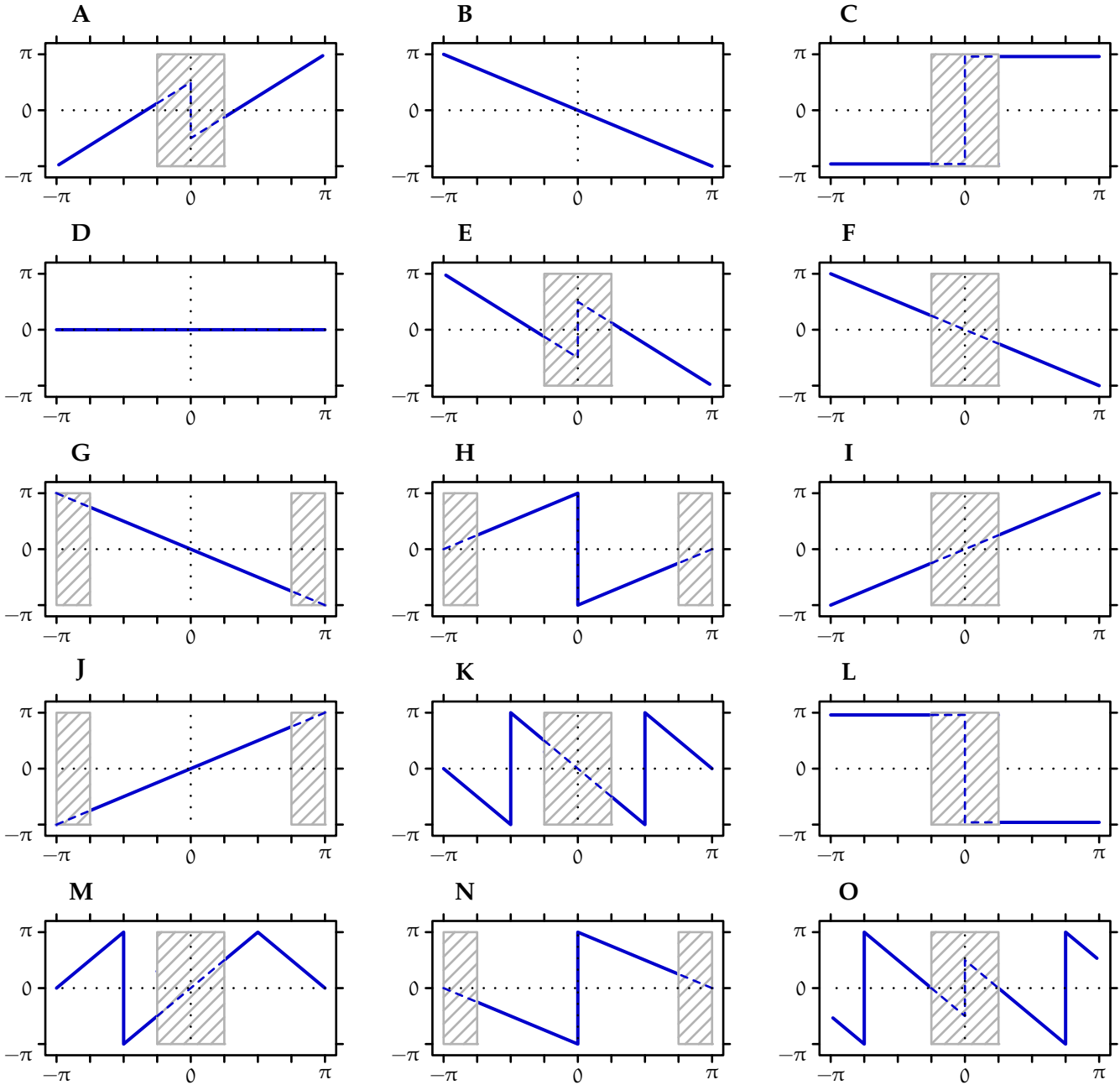
	magnitude (A-O) or X if none	angle (A-O) or X if none
$f[n-1]$:		
$f[n] \times (-1)^n$:		
$f[n] \times (-1)^{(n-1)}$:		
$(f[n] - f[n-1])/2$:		
$(f[n] - f[n-2])/2$:		
$f^2[n]$:		
$(f * f)[n]$:		

Magnitude plots: Each of the following panels shows the magnitude of a DTFT as a function of radian frequency Ω . The magnitudes in each panel are normalized so that the maximum value in each panel is 1. Determine which (if any) of the following plots matches the magnitude of each of the **seven derived signals** shown on the previous page. Some of these plots may match more than one of the seven derived magnitudes. If none of these plots are a match, put X in the corresponding box(es).



X = none of the above.

Angle plots: Each of the following panels shows the angle of a DTFT as a function of radian frequency Ω . Determine which (if any) of the following plots matches the angle of each of the **seven derived signals**. The cross-hatched regions indicate values of Ω for which the angle is undefined (because the corresponding magnitude function is zero at those frequencies). Angle plots should only be considered a match if the crosshatched regions match **AND** the plots match outside the crosshatched regions. The dashed lines in the crosshatched regions are provided to help visualize the trends outside the crosshatched region.

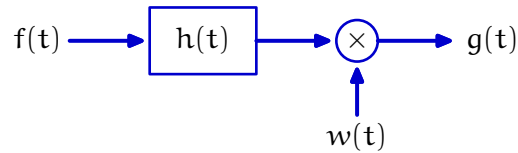


X = none of the above.

Worksheet (intentionally blank)

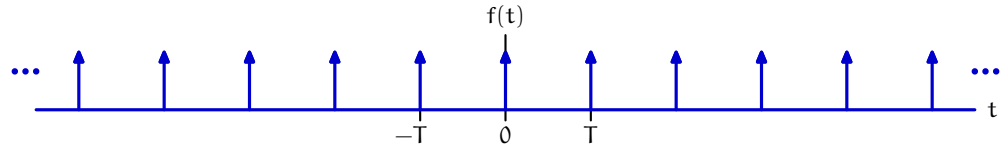
4 Continuous Processing (20 points)

In the following block diagram, $h(t)$ represents the impulse response of a linear, time-invariant subsystem whose input is $f(t)$, and whose output is multiplied by a window function $w(t)$ to generate the final output $g(t)$.



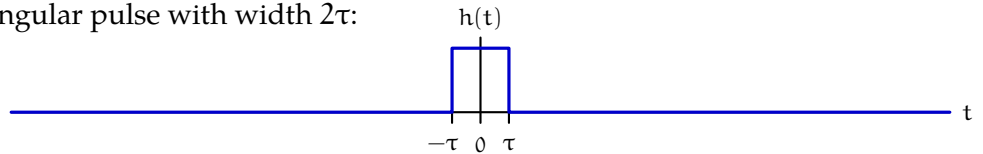
The input $f(t)$ is a periodic sequence of impulses with period T , as follows:

$$f(t) = \sum_{m=-\infty}^{\infty} \delta(t - mT)$$



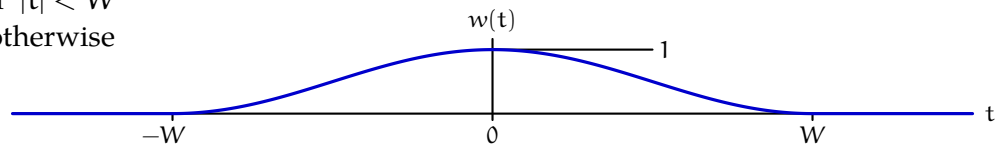
The impulse response $h(t)$ is a rectangular pulse with width 2τ :

$$h(t) = \begin{cases} 1 & \text{if } |t| < \tau \\ 0 & \text{otherwise} \end{cases}$$



The window function $w(t)$ is a Hann window with width $2W$:

$$w(t) = \begin{cases} (1 + \cos(\pi t/W))/2 & \text{if } |t| < W \\ 0 & \text{otherwise} \end{cases}$$

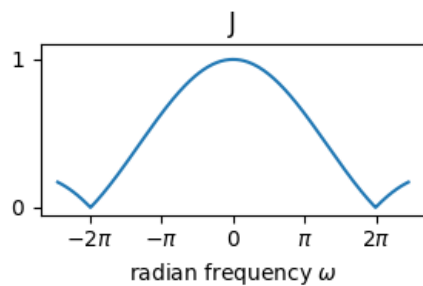
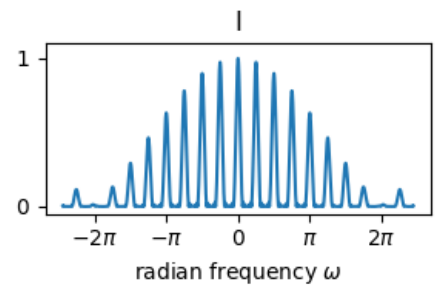
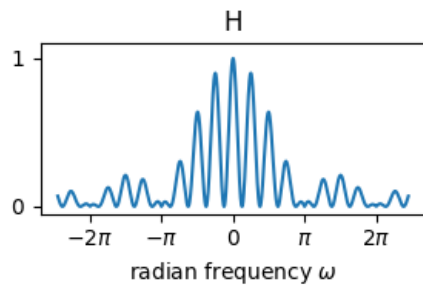
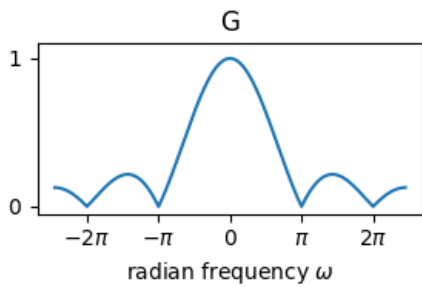
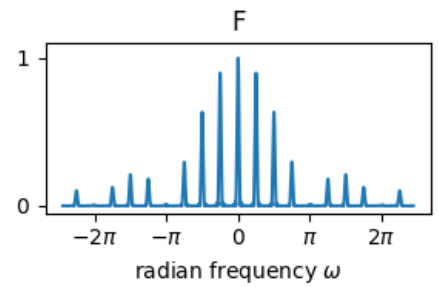
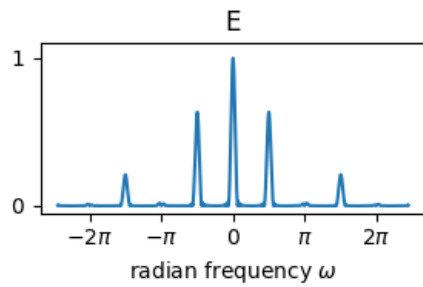
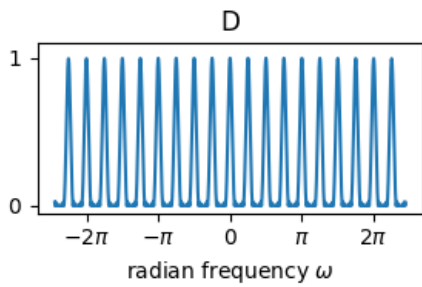
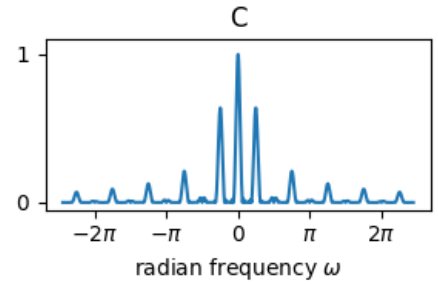
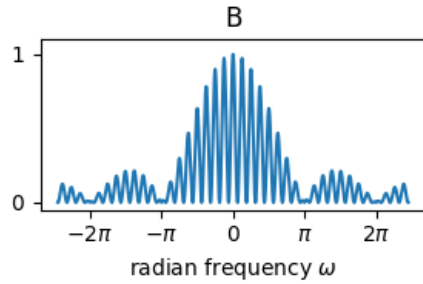
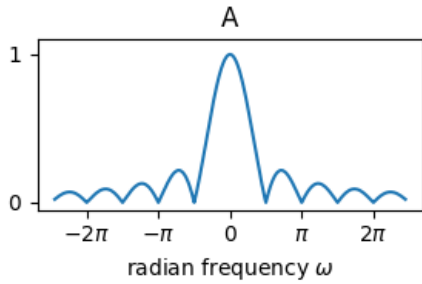
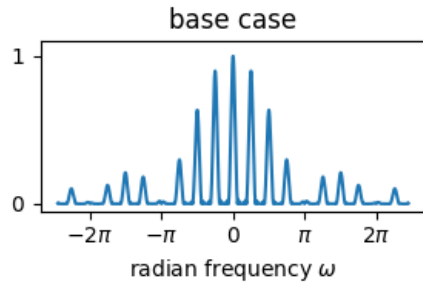


The top-left plot on the following page shows the magnitude of the Continuous-Time Fourier Transform (CTFT) of $g(t)$ for a "base case" in which $T = 8$ seconds, $\tau = 1$ second, and $W = 32$ seconds.

Determine which of plots (A-N) on the following page shows the CTFT of the output that results when a single change is made to the base case. Enter your answers in the boxes provided.

- double W :
- double T :
- halve τ :
- divide τ by 64:
- multiply T by 4:

Each of the following panels shows the magnitude of a CTFT as a function of radian frequency ω . The magnitudes in each panel are normalized so that the maximum value in each panel is 1.



Worksheet (intentionally blank)

Worksheet (intentionally blank)

Worksheet (intentionally blank)

Worksheet (intentionally blank)

Worksheet (intentionally blank)