6.3000 Quiz 1 Spring 2026

Name: Solutions

Kerberos (Athena) username:

Please WAIT until we tell you to begin.

This quiz is closed book, but you may use one 8.5×11 sheet of notes (both sides).

You may NOT use any electronic devices (such as calculators and phones).

If you have questions, please **come to us** at the front of the room to ask.

Please enter all solutions in the boxes provided.

Work on other pages with QR codes will be considered for partial credit.
Please provide a note if you continue work on worksheets at the end of the exam.

Please do not write on the QR codes at the bottom of each page.

We use those codes to identify which pages belong to each student.

Trigonometric Identities Reference

$\cos(a+b) = \cos(a)\cos(b) - \sin(a)\sin(b)$

$\cos(a-b) = \cos(a)\cos(b) + \sin(a)\sin(b)$

$\sin(a+b) = \sin(a)\cos(b) + \cos(a)\sin(b)$

$\sin(a-b) = \sin(a)\cos(b) - \cos(a)\sin(b)$

$$\cos(a) + \cos(b) = 2\cos\left(\frac{a+b}{2}\right)\cos\left(\frac{a-b}{2}\right)$$

$$\cos(a) - \cos(b) = -2\sin\left(\frac{a+b}{2}\right)\sin\left(\frac{a-b}{2}\right)$$

$$\sin(a) + \sin(b) = 2\sin\left(\frac{a+b}{2}\right)\cos\left(\frac{a-b}{2}\right)$$

$$\sin(a) - \sin(b) = 2\cos\left(\frac{a+b}{2}\right)\sin\left(\frac{a-b}{2}\right)$$

$\cos(a+b)+\cos(a-b) = 2\cos(a)\cos(b)$

$\cos(a+b)-\cos(a-b) = -2\sin(a)\sin(b)$

$\sin(a+b)+\sin(a-b) = 2\sin(a)\cos(b)$

$\sin(a+b)-\sin(a-b) = 2\cos(a)\sin(b)$

$2\cos(a)\cos(b) = \cos(a-b)+\cos(a+b)$

$2\sin(a)\sin(b) = \cos(a-b)-\cos(a+b)$

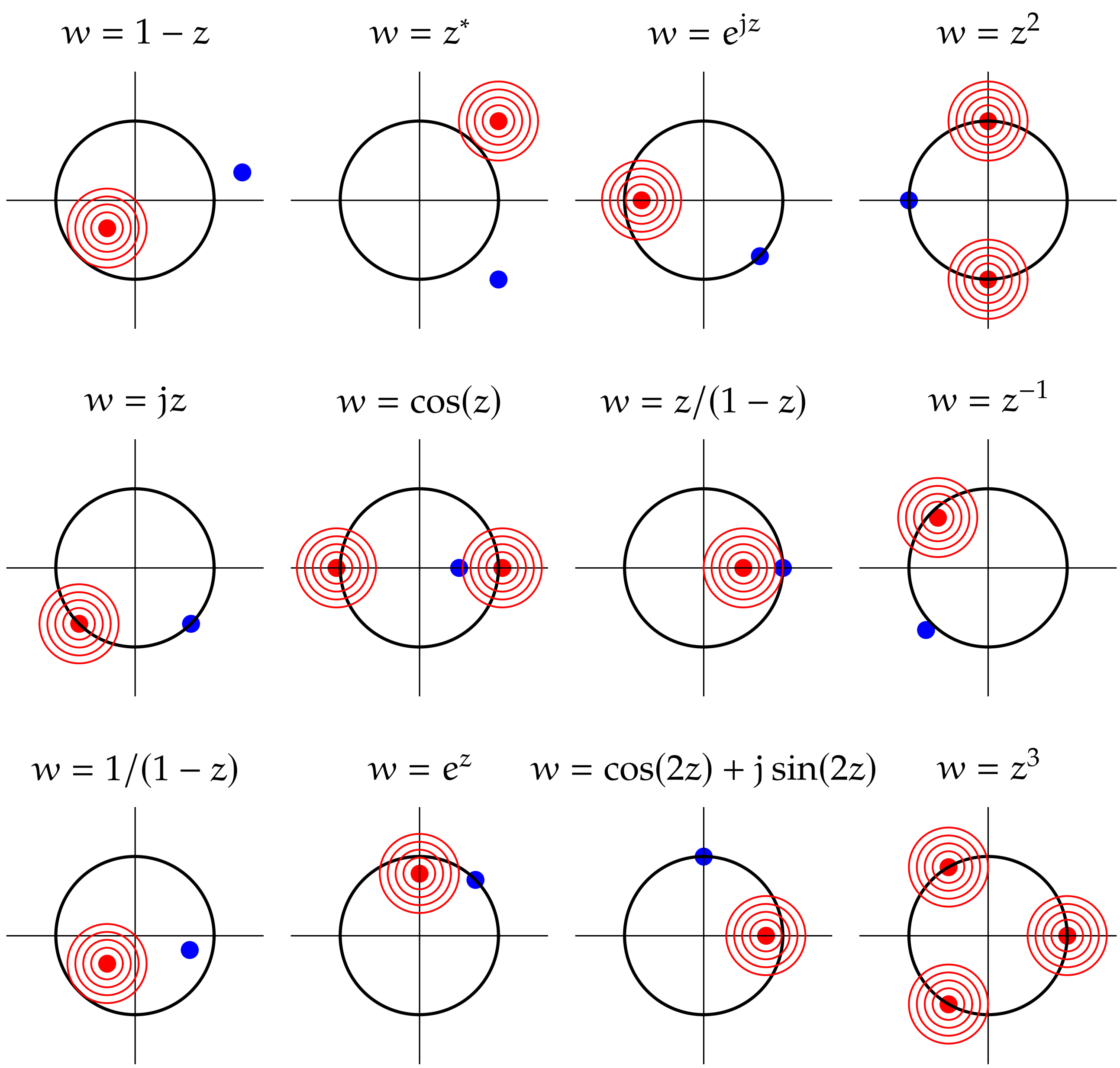
$2\sin(a)\cos(b) = \sin(a+b)+\sin(a-b)$

$2\cos(a)\sin(b) = \sin(a+b)-\sin(a-b)$

1 Dot, Dot, Dot (25 points)

Twelve plots are shown below. In each plot, a dot (•) indicates where the complex number $w = f(z)$ lies in the complex plane. Your task is to mark with a dot (•) a value of z such that $w = f(z)$. If there are multiple values of z such that $w = f(z)$, you need mark only one such value of z to receive full credit.

[√] I have read the instructions carefully: In each plot, I will mark a value of z such that $w = f(z)$.



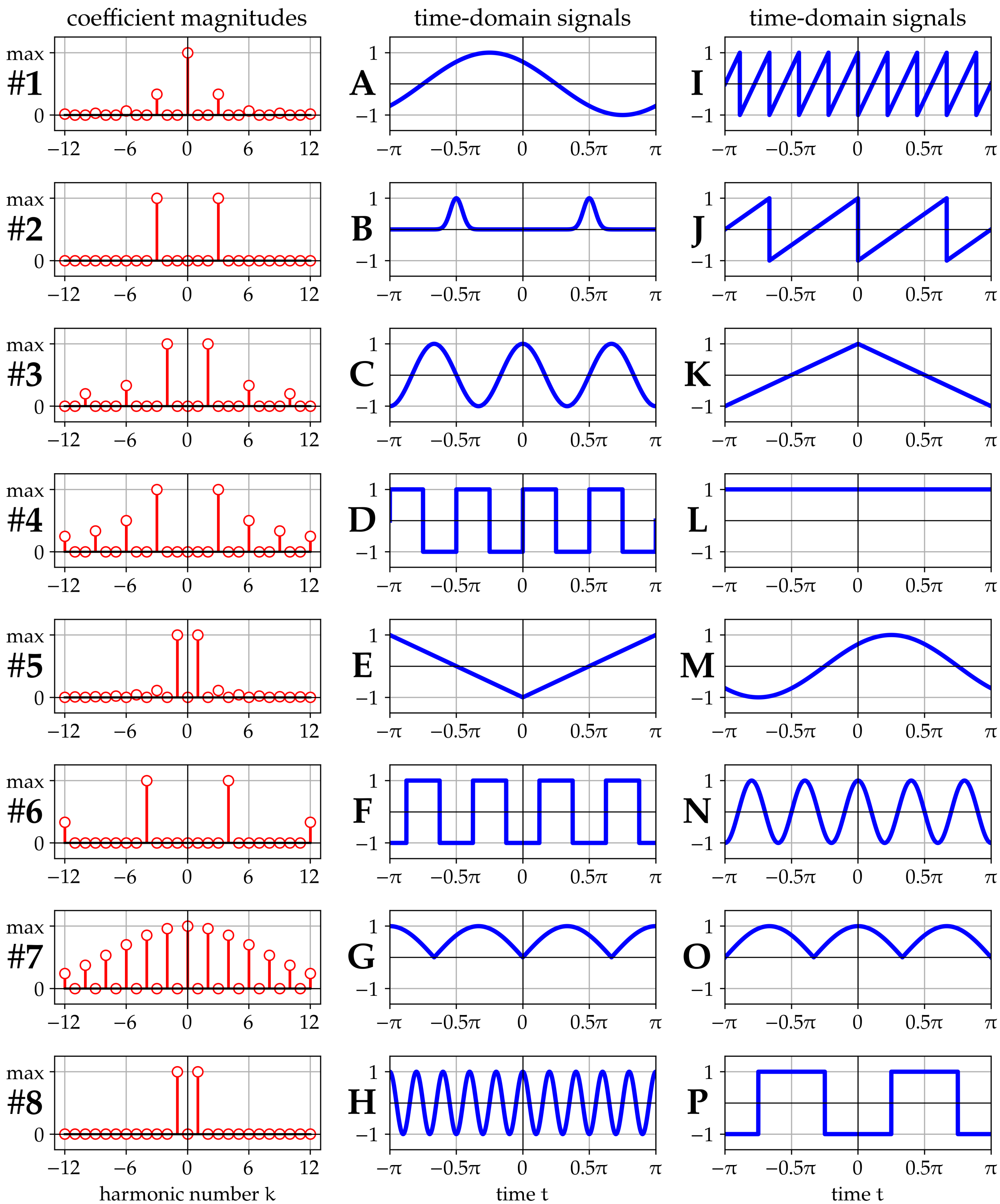
Note: Any $z = 2\pi n \pm \frac{1}{3}\pi$ (where n is an integer) yields $w = \cos(z) = \frac{1}{2}$.

2 Matching Magnitudes (25 points)

Twenty-four plots are shown on the next page. Each plot along the left column (#1, #2, #3, . . . , #8) shows the magnitudes of the Fourier series coefficients for a periodic signal, computed with period $T = 2\pi$. The sixteen plots along the next two columns (A, B, C, . . . , P) show periodic continuous-time signals.

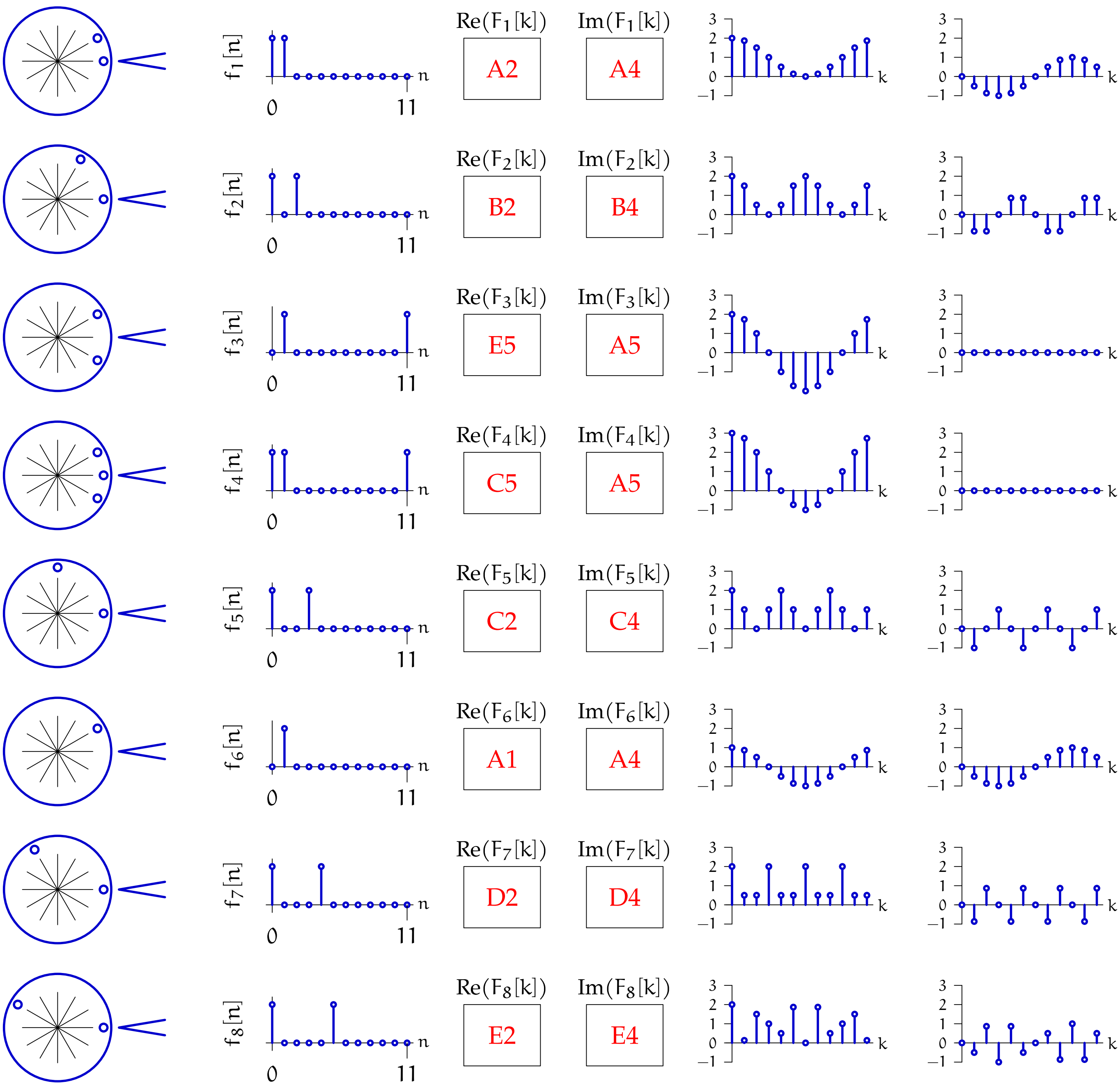
For each set of Fourier series coefficients (#1, #2, #3, . . . , #8), determine **all** time-domain signals (A, B, C, . . . , P) with the same set of Fourier series coefficient magnitudes. If there is no match, write an **X**.

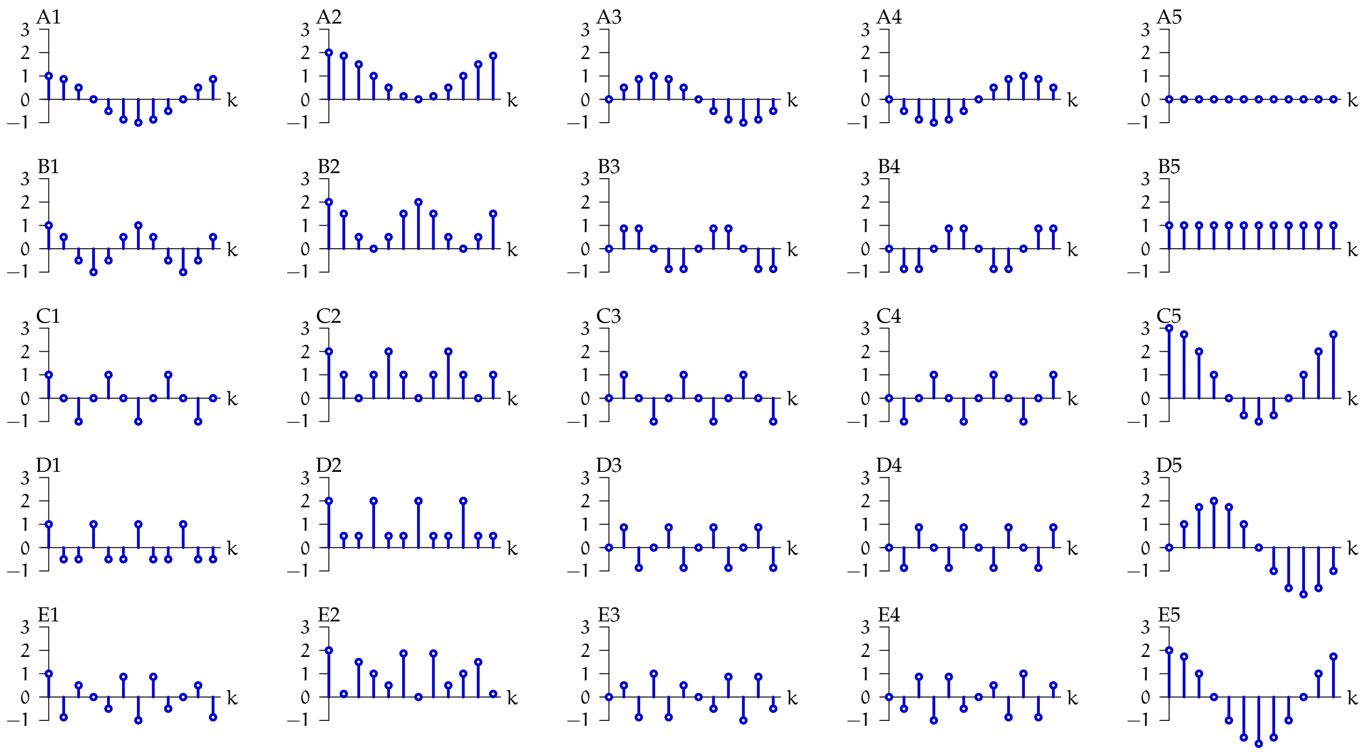
#1:	G, O
#2:	C
#3:	P
#4:	J
#5:	E, K
#6:	D, F
#7:	B
#8:	A,M


coefficient magnitudes
time-domain signals
time-domain signals
#1
#2
#3
#4
#5
#6
#7
#8
max
0
−12 −6 0 6 12
harmonic number k
A
B
C
D
E
F
G
H
I
J
K
L
M
N
O
P
1
−1
−π −0.5π 0 0.5π π
time t
time t

3 Siren Sounds (25 points)

A siren is a device that generates sounds by passing a jet of compressed air through holes in a spinning disk. The pattern of holes in the disk determines the pattern of pulses in each period of disk rotation, as illustrated in the following figure, where each plot shows one period ($N = 12$) of a periodic, discrete-time signal ($f_i[n]$) that can be represented by its Fourier series coefficients $F_i[k]$. Find the plots on the next page that show the real and imaginary parts of $F_i[k]$ and enter their identifiers in the boxes below. If none of the plots match, enter **X** in that box. Some plots on the next page may be used more than once.




A1
A2
A3
A4
A5
B1
B2
B3
B4
B5
C1
C2
C3
C4
C5
D1
D2
D3
D4
D5
E1
E2
E3
E4
E5
k

4 Aliasing Harmonics (25 points)

Let $f_c(t)$ represent a continuous-time sinusoid with (radian) frequency ω:

$$f_c(t) = \cos(\omega t)$$

Sampling $f_c(t)$ at integer multiples of the sampling period Δ produces a discrete-time signal:

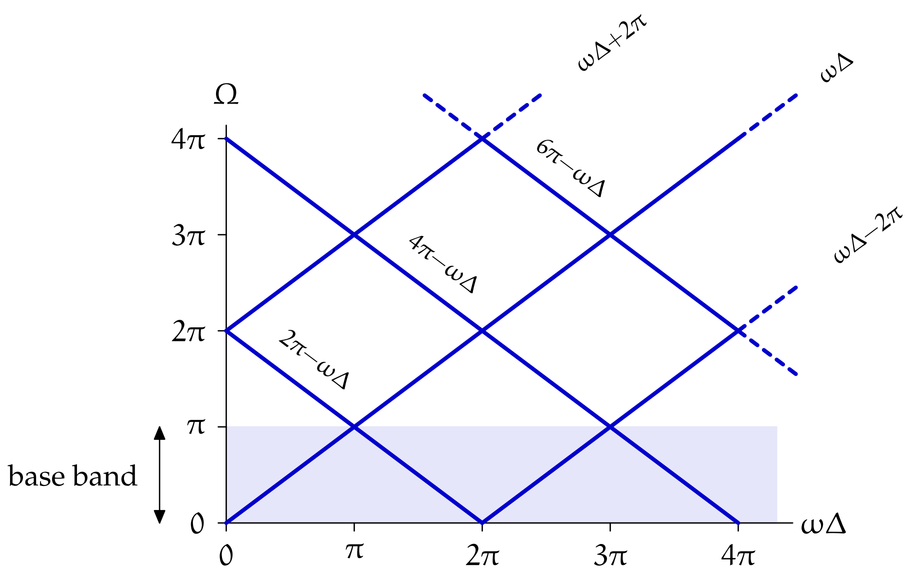
$$f_d[n] = f_c(n\Delta) = \cos(\omega\Delta n)$$

whose samples are given by $f_d[n] = \cos(\Omega n)$ where $\Omega = \omega\Delta$. However, the cosine function is periodic:

$$\cos(\Omega n) = \cos(\Omega n + 2\pi n)$$

so the samples of $f_d[\cdot]$ could equivalently be described as samples of a discrete-time sinusoid at frequency $\Omega+2\pi$. The cosine function is also symmetric about π, so the samples $f_d[\cdot]$ could equivalently be described as samples of a discrete-time sinusoid at frequency $2\pi-\Omega$. We refer to these alternative (equivalent) expressions for the discrete-time frequency as "aliases."

While there are infinitely many such aliases for each value of $\omega\Delta$, there is a unique value in the range $0<\Omega<\pi$. We call that range of frequencies the "base band," and we refer to the unique value of Ω that is in the base band as the "base-band frequency." These relations are summarized graphically in the following figure.



In this problem, we consider three continuous-time signals:

$$f_{c1}(t) = \cos(300t)$$

$$f_{c2}(t) = \cos(400t)$$

$$f_{c3}(t) = \cos(500t)$$

that are sampled at integer multiples of a sampling period Δ to generate three discrete-time sinusoids:

$$f_{d1}[n] = f_{c1}(n\Delta) = \cos(300\Delta n) = \cos(\Omega_1 n)$$

$$f_{d2}[n] = f_{c2}(n\Delta) = \cos(400\Delta n) = \cos(\Omega_2 n)$$

$$f_{d3}[n] = f_{c3}(n\Delta) = \cos(500\Delta n) = \cos(\Omega_3 n)$$

with base-band (discrete-time) frequencies Ω_1, Ω_2, and Ω_3, respectively.

Part a. Determine closed-form numerical expressions for the base-band frequencies Ω_1, Ω_2, and Ω_3 that result when $\Delta = \pi/600$. Enter your expressions in the boxes below.

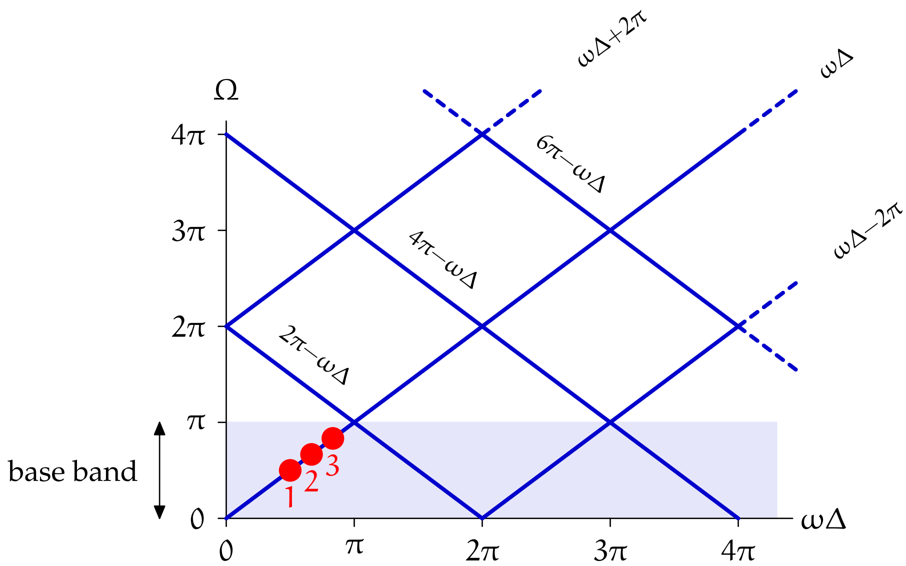
$\Omega_1 =$ $3\pi/6$

$\Omega_2 =$ $4\pi/6$

$\Omega_3 =$ $5\pi/6$

Show these results as points on the following plot:

- Draw a dot at horizontal location $\omega_1\Delta$ and vertical location Ω_1. Label this dot as number 1.
- Draw a dot at horizontal location $\omega_2\Delta$ and vertical location Ω_2. Label this dot as number 2.
- Draw a dot at horizontal location $\omega_3\Delta$ and vertical location Ω_3. Label this dot as number 3.



Start by finding $\omega_1\Delta = 300 \times \pi/600 = \pi/2$, $\omega_2\Delta = 400 \times \pi/600 = 2\pi/3$, and $\omega_3\Delta = 500 \times \pi/600 = 5\pi/6$. All of these frequencies are less than π, so the base-band discrete frequencies Ω_i are equal to the corresponding $\omega_i\Delta$ for $i = 1, 2$, and 3.

Part b. Determine closed-form numerical expressions for the base-band frequencies Ω_1, Ω_2, and Ω_3 that result when $\Delta = 3\pi/800$. Enter your expressions in the boxes below.

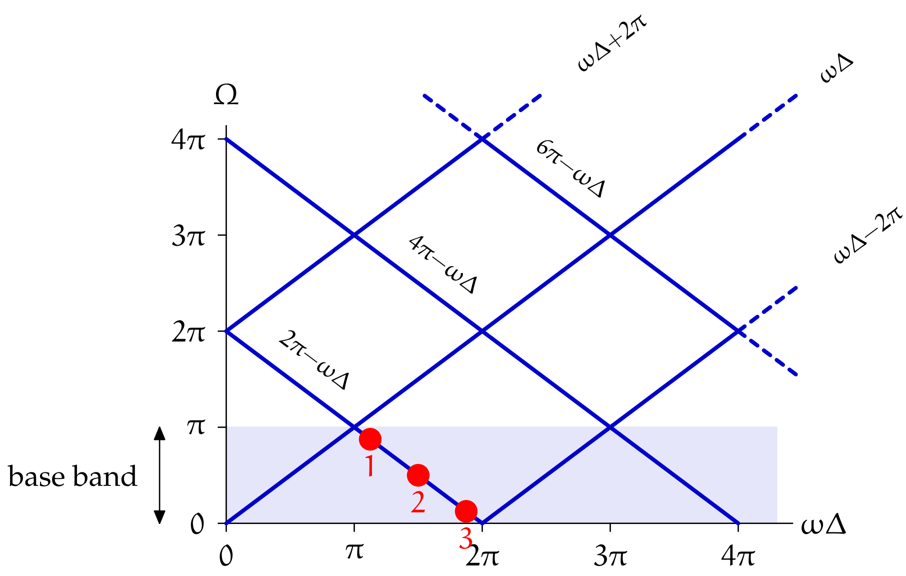
$\Omega_1 = \frac{7\pi}{8}$

$\Omega_2 = \frac{4\pi}{8}$

$\Omega_3 = \frac{\pi}{8}$

Show these results as points on the following plot:

- Draw a dot at horizontal location $\omega_1\Delta$ and vertical location Ω_1. Label this dot as number 1.
- Draw a dot at horizontal location $\omega_2\Delta$ and vertical location Ω_2. Label this dot as number 2.
- Draw a dot at horizontal location $\omega_3\Delta$ and vertical location Ω_3. Label this dot as number 3.



Start by finding $\omega_1\Delta=300\frac{3\pi}{800}=\frac{9\pi}{8}$, $\omega_2\Delta=400\frac{3\pi}{800}=\frac{12\pi}{8}$, and $\omega_3\Delta=500\frac{3\pi}{800}=\frac{15\pi}{8}$. All of these values of $\omega\Delta$ are between π and 2π, so

$$\Omega_1 = 2\pi - \omega_1\Delta = \frac{16\pi}{8} - \frac{9\pi}{8} = \frac{7\pi}{8},$$

$$\Omega_2 = 2\pi - \omega_2\Delta = \frac{16\pi}{8} - \frac{12\pi}{8} = \frac{4\pi}{8} \text{ and}$$

$$\Omega_3 = 2\pi - \omega_3\Delta = \frac{16\pi}{8} - \frac{15\pi}{8} = \frac{1\pi}{8}.$$

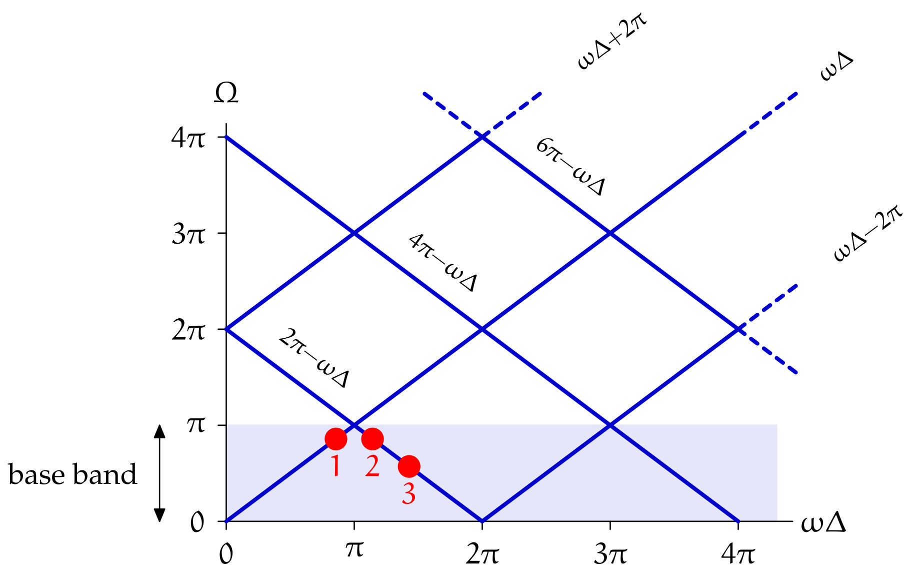
Part c. Is there a non-zero sampling period Δ for which the base-band discrete-time frequencies $\Omega_1 = \Omega_2$? If yes, enter the value of Δ in the box below. If there are no such values of Δ write **none** in the box below.

Δ or **none**: $\frac{2\pi}{700}$

$\omega_1\Delta =$	$\frac{6}{7}\pi$	$\Omega_1 =$	$\frac{6}{7}\pi$
$\omega_2\Delta =$	$\frac{8}{7}\pi$	$\Omega_2 =$	$2\pi - \frac{8}{7}\pi = \frac{6}{7}\pi$
$\omega_3\Delta =$	$\frac{10}{7}\pi$	$\Omega_3 =$	$2\pi - \frac{10}{7}\pi = \frac{4}{7}\pi$

Show these results as points on the following plot:

- Draw a dot at horizontal location $\omega_1\Delta$ and vertical location Ω_1. Label this dot as number 1.
- Draw a dot at horizontal location $\omega_2\Delta$ and vertical location Ω_2. Label this dot as number 2.
- Draw a dot at horizontal location $\omega_3\Delta$ and vertical location Ω_3. Label this dot as number 3.



While the general relation between $\omega\Delta$ and Ω is nonlinear, it is linear within subregions where $\omega\Delta$ is in the range $(n\pi\,,\ n\pi{+}\pi)$ and Ω is in the range $(m\pi\,,\ m\pi{+}\pi)$, for integers n and m.

First, consider the possibility that both $\omega_1\Delta$ and $\omega_2\Delta$ are in $(0, \pi)$. Since $\omega_1{=}300$ and $\omega_2{=}400$, ω_1 cannot equal ω_2 and therefore Ω_1 cannot equal Ω_2. It follows that both $\omega_1\Delta$ and $\omega_2\Delta$ cannot be in $(0, \pi)$.

Next, consider the possibility that $\omega_1\Delta$ is in $(0, \pi)$ but $\omega_2\Delta$ is in $(\pi, 2\pi)$. For this case, $\Omega_1 = \omega_1\Delta$, and $\Omega_2 = 2\pi - \omega_2\Delta$. Setting $\Omega_1 = \Omega_2$ and solving yields $\Delta = \frac{2\pi}{700}$. It follows that $\Omega_1 = 300\frac{2\pi}{700} = \frac{6}{7}\pi$, $\Omega_2 = 2\pi - 400\frac{2\pi}{700} = \frac{6}{7}\pi$, and $\Omega_3 = 2\pi - 500\frac{2\pi}{700} = \frac{4}{7}\pi$. This is an aceptable answer, and there could also be other acceptable answers.

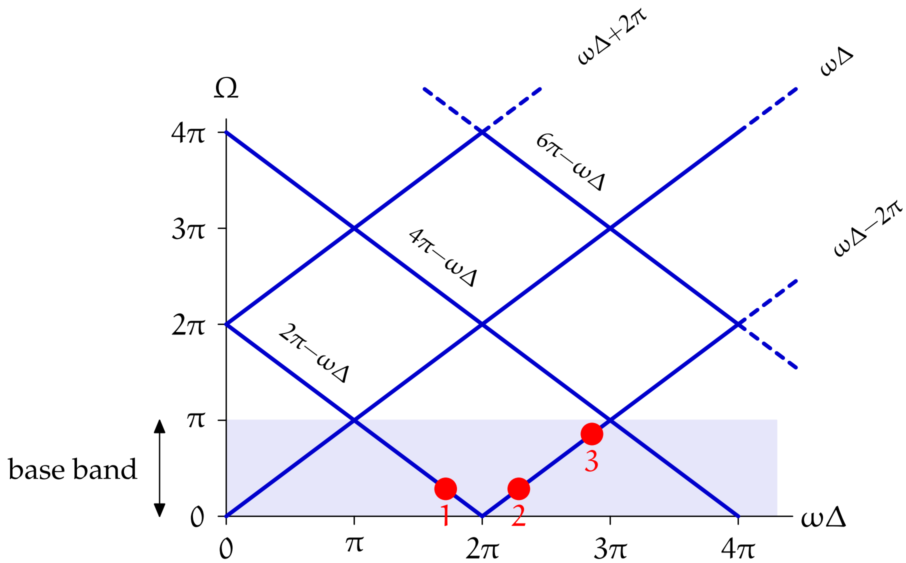
Is there a non-zero sampling period Δ for which the base-band discrete-time frequencies $\Omega_1 = \Omega_2$? If yes, describe all such values of Δ in the box below. If there are no such values of Δ write **none** in the box below.

Δ or **none**: $\frac{4\pi}{700}$

$\omega_1\Delta =$	$\frac{12}{7}\pi$	$\Omega_1 =$	$2\pi - \frac{12}{7}\pi = \frac{2}{7}\pi$
$\omega_2\Delta =$	$\frac{16}{7}\pi$	$\Omega_2 =$	$\frac{16}{7}\pi - 2\pi = \frac{2}{7}\pi$
$\omega_3\Delta =$	$\frac{20}{7}\pi$	$\Omega_3 =$	$\frac{20}{7}\pi - 2\pi = \frac{6}{7}\pi$

Show these results as points on the following plot:

- Draw a dot at horizontal location $\omega_1\Delta$ and vertical location Ω_1. Label this dot as number 1.
- Draw a dot at horizontal location $\omega_2\Delta$ and vertical location Ω_2. Label this dot as number 2.
- Draw a dot at horizontal location $\omega_3\Delta$ and vertical location Ω_3. Label this dot as number 3.



Next, consider the possibility that $\omega_1\Delta$ is in $(0,\pi)$ but $\omega_2\Delta$ is in $(2\pi, 3\pi)$. For this case, $\Omega_1 = \omega_1\Delta$, and $\Omega_2 = \omega_2\Delta - 2\pi$. Setting $\Omega_1 = \Omega_2$ and solving yields $\Delta = \frac{2\pi}{100}$. It would follow that $\omega_1 = 300\frac{2\pi}{100} = 6\pi$, which contradicts our original assumption that ω_1 is in $(2\pi, 3\pi)$. Thus, there is no solution with $\omega_1\Delta$ in $(0,\pi)$ and $\omega_2\Delta$ in $(2\pi, 3\pi)$.

Next, consider the possibility that $\omega_1\Delta$ is in $(\pi, 2\pi)$ and $\omega_2\Delta$ is in $(2\pi, 3\pi)$. For this case, $\Omega_1 = 2\pi - \omega_1\Delta$ and $\Omega_2 = \omega\Delta_2 - 2\pi$. Setting $\Omega_1 = \Omega_2$ and solving yields $\Delta = \frac{4\pi}{700}$. Thus $\omega_1\Delta = \frac{12\pi}{7}$ and $\Omega_1 = 2\pi - \frac{12\pi}{7} = \frac{2\pi}{7}$. Similarly, $\omega_2\Delta = \frac{16\pi}{7} - 2\pi$ and $\Omega_2 = \frac{16\pi}{7} - 2\pi = \frac{2\pi}{7}$. Lastly, $\omega_3\Delta = \frac{20\pi}{7}$ and $\Omega_3 = \frac{20\pi}{7} - 2\pi = \frac{6\pi}{7}$.

So far, we have found two solutions to the question posed in part c. There could also be other acceptable answers.

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