6.003 Final Exam Spring 2022

Name:

Kerberos (Athena) username:

Please WAIT until we tell you to begin.

This quiz is closed book, but you may use three 8.5 × 11 sheets of notes (six sides).

You may NOT use any electronic devices (such as calculators and phones).

If you have questions, please **come to us** at the front of the room to ask.

Please enter all solutions in the boxes provided.

Work on other pages with QR codes will be considered for partial credit.
Please provide a note if you continue work on worksheets at the end of the exam.

Please do not write on the QR codes at the bottom of each page.

We use those codes to identify which pages belong to each student.

Trigonometric Identities Reference

$$\cos(a+b) = \cos(a)\cos(b) - \sin(a)\sin(b)$$
$$\cos(a-b) = \cos(a)\cos(b) + \sin(a)\sin(b)$$
$$\sin(a+b) = \sin(a)\cos(b) + \cos(a)\sin(b)$$
$$\sin(a-b) = \sin(a)\cos(b) - \cos(a)\sin(b)$$
$$\cos(a) + \cos(b) = 2\cos\left(\frac{a+b}{2}\right)\cos\left(\frac{a-b}{2}\right)$$
$$\cos(a) - \cos(b) = -2\sin\left(\frac{a+b}{2}\right)\sin\left(\frac{a-b}{2}\right)$$
$$\sin(a) + \sin(b) = 2\sin\left(\frac{a+b}{2}\right)\cos\left(\frac{a-b}{2}\right)$$
$$\sin(a) - \sin(b) = 2\cos\left(\frac{a+b}{2}\right)\sin\left(\frac{a-b}{2}\right)$$
$$\cos(a+b)+\cos(a-b) = 2\cos(a)\cos(b)$$
$$\cos(a+b)-\cos(a-b) = -2\sin(a)\sin(b)$$
$$\sin(a+b)+\sin(a-b) = 2\sin(a)\cos(b)$$
$$\sin(a+b)-\sin(a-b) = 2\cos(a)\sin(b)$$
$$2\cos(a)\cos(b) = \cos(a-b)+\cos(a+b)$$
$$2\sin(a)\sin(b) = \cos(a-b)-\cos(a+b)$$
$$2\sin(a)\cos(b) = \sin(a+b)+\sin(a-b)$$
$$2\cos(a)\sin(b) = \sin(a+b)-\sin(a-b)$$

1 Frequency Responses (20 points)

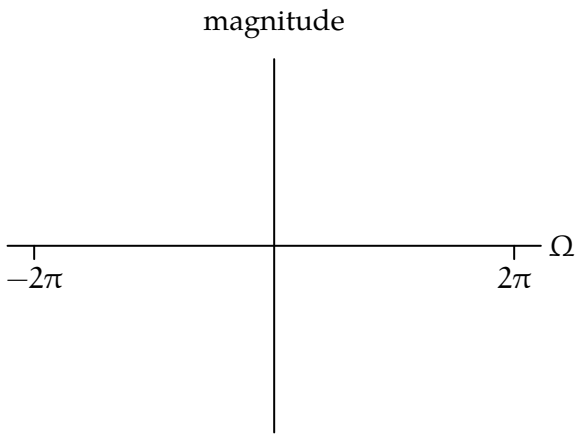
Part a. Determine an expression for the frequency response of a discrete-time system with unit sample response

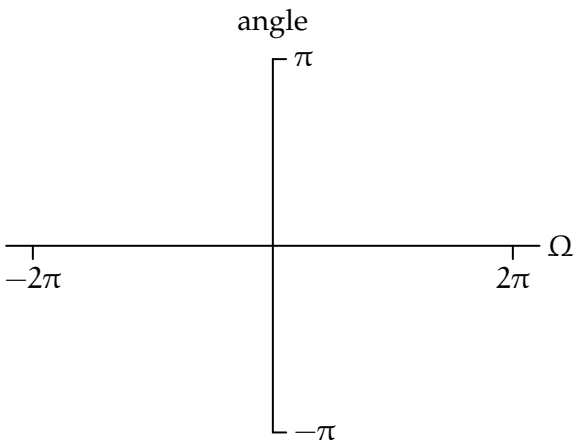
$$h[n] = \delta[n] - \delta[n-1]$$

Enter your expression in the box below.

$H(\Omega) =$ ____

Sketch the magnitude and angle of the frequency response on the axes below. Label the key points.





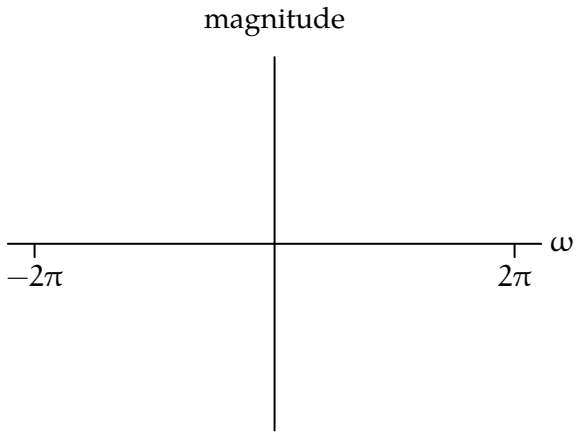
Part b. Determine an expression for the frequency response of a continuous-time system with impulse response

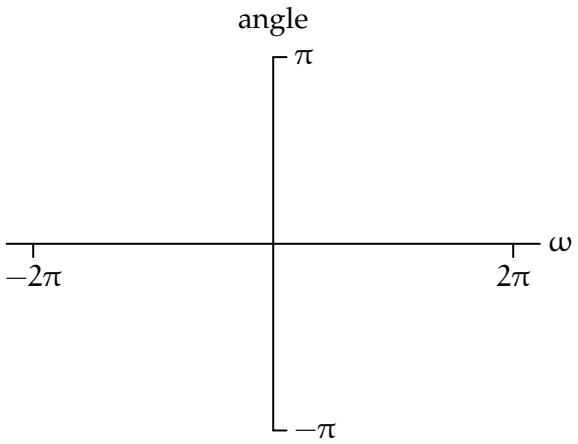
$$h(t) = \begin{cases} 1 & \text{if } 0 \leqslant t < 1 \\ -1 & \text{if } 1 \leqslant t < 2 \\ 0 & \text{otherwise} \end{cases}$$

Enter your expression in the box below.

$H(\omega) =$ [ ]

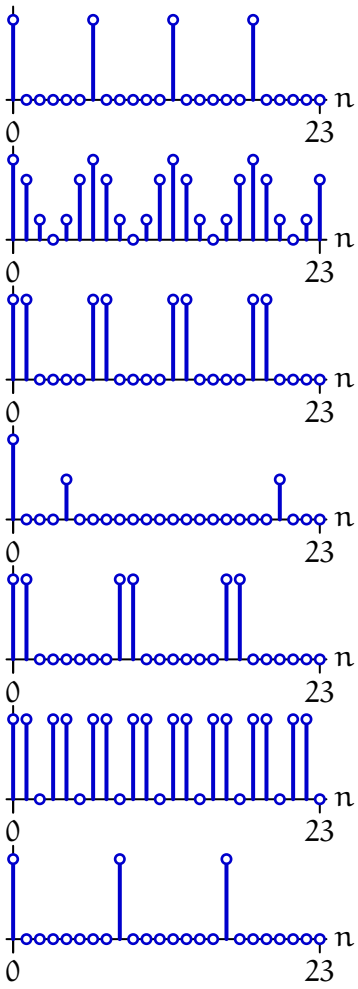
Sketch the magnitude and angle of the frequency response on the axes below. Label the key points.





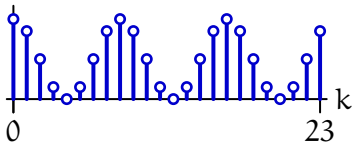
2 Discrete Fourier Transform Matching (21 points)

Each of the following plots shows the first 24 samples of a discrete-time signal. Find the plot on the following page that corresponds to the 24-point Discrete Fourier Transform (DFT) for each of these signals. Enter the letter of the plot (A-N) in the box provided.

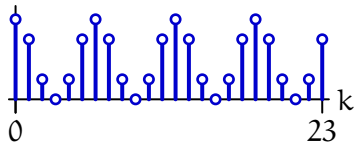


Each of the following plots shows the magnitude of a DFT computed with an analysis window N = 24. The vertical scale for each plot is different: it has been normalized so that the peak value in each plot is 1.

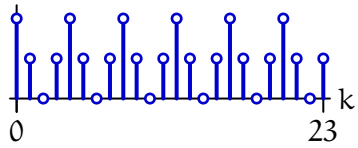
A



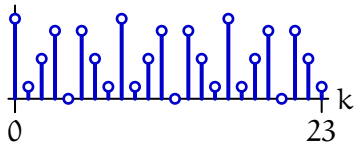
B



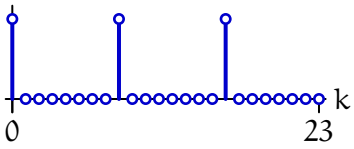
C



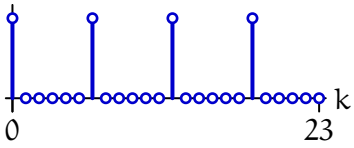
D



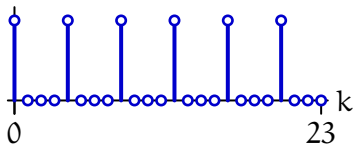
E



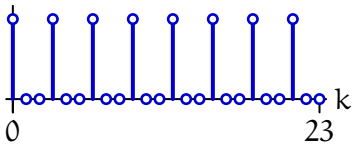
F



G

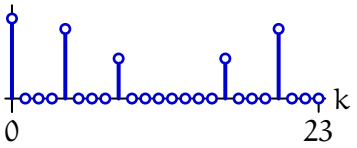


H

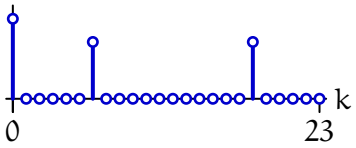


I

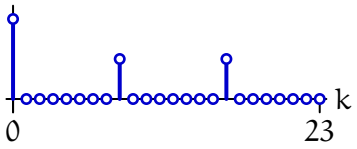
J



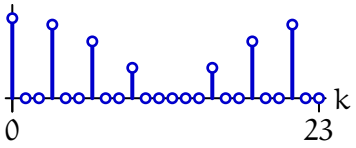
K



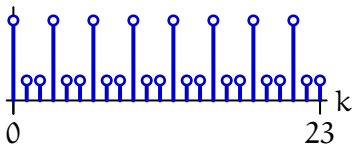
L



M



N

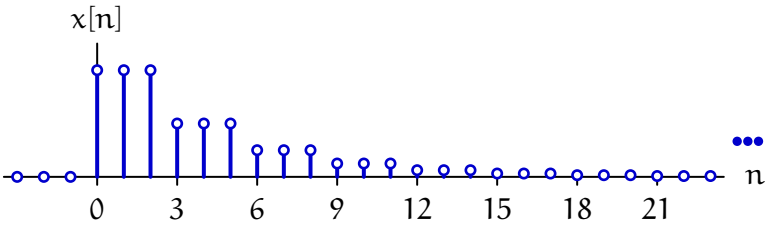


3 Steps (23 points)

Part 1. Let $x[n]$ represent the following discrete-time signal

$$x[n] = \begin{cases} 0 & \text{for } n < 0 \\ a^0 & \text{for } n = 0, 1, 2 \\ a^1 & \text{for } n = 3, 4, 5 \\ a^2 & \text{for } n = 6, 7, 8 \\ \dots \end{cases}$$

where a is a real number between 0 and 1, as shown in the plot below.



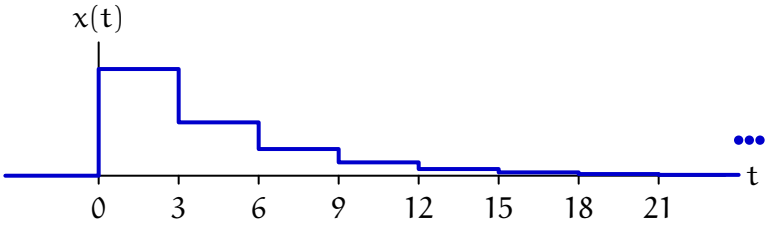
Determine a closed form expression for $X(\Omega)$, which is the discrete-time Fourier transform of $x[n]$.

$X(\Omega) =$

Part 2. Let $x(t)$ represent the following continuous-time signal

$$x(t) = \begin{cases} 0 & \text{for } t < 0 \\ a^0 & \text{for } 0 \leqslant t < 3 \\ a^1 & \text{for } 3 \leqslant t < 6 \\ a^2 & \text{for } 6 \leqslant t < 9 \\ \ldots & \end{cases}$$

where a is a real number between 0 and 1, as shown in the plot below.

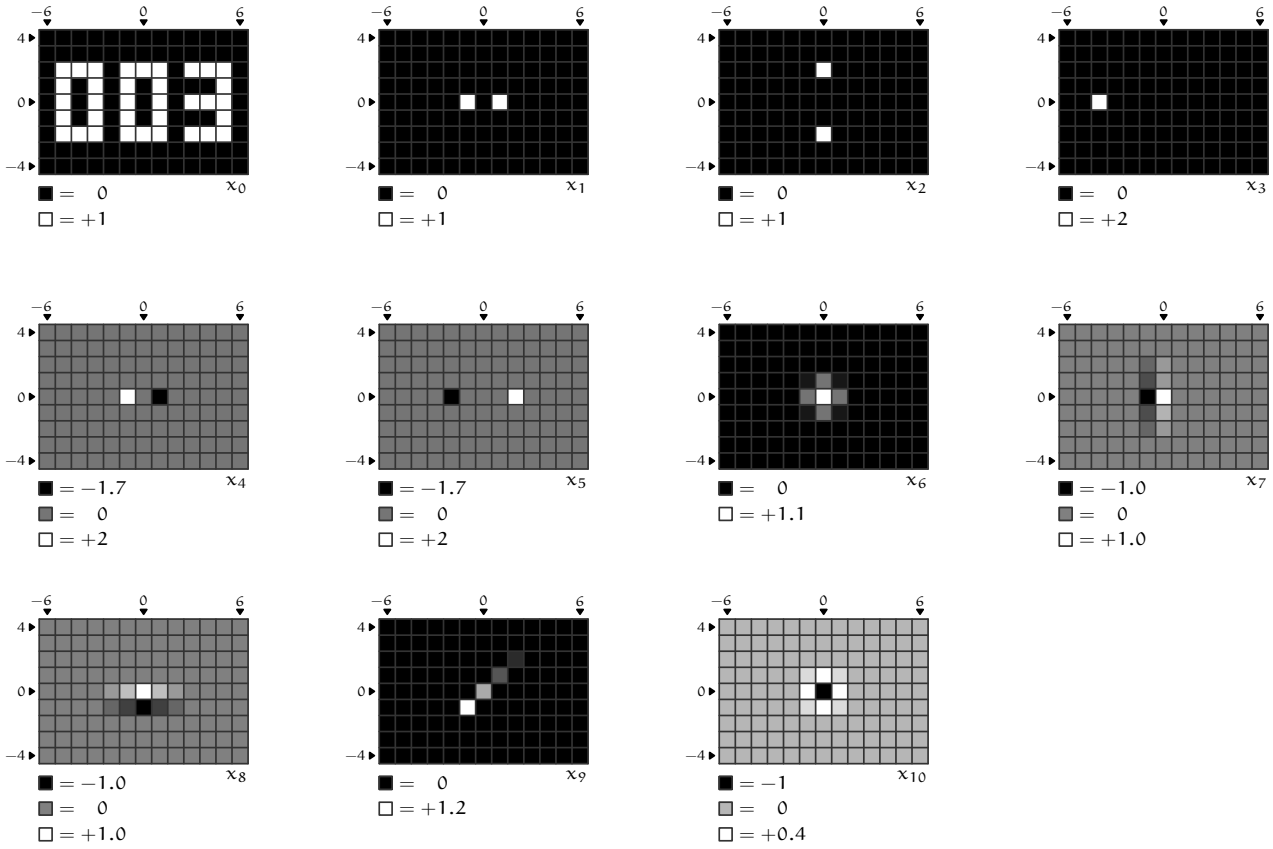


Determine a closed-form expression for $X(\omega)$, which is the continuous-time Fourier transform of $x(t)$.

$X(\omega) =$ [ ]

4 2D Convolution (20 Points)

For this problem, we will consider the following 2D signals, labeled x_0 through x_{10}, each of which is 9 rows $\times$ 13 columns. Note that the color scale is different between some of the signals.

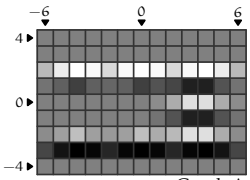


For each circular convolution below, indicate which of the graphs on the facing page most closely matches the result by entering a single letter in each box. Note that, for each graph on the facing page, black corresponds to the lowest value in the signal (not necessarily 0), and white corresponds to the highest value in the signal (not necessarily 1).

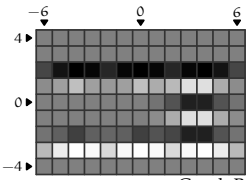
$x_1 \circledast x_0$ ☐ $x_2 \circledast x_0$ ☐ $x_3 \circledast x_0$ ☐

$x_4 \circledast x_0$ ☐ $x_5 \circledast x_0$ ☐ $x_6 \circledast x_0$ ☐ $x_7 \circledast x_0$ ☐

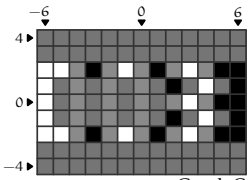
$x_8 \circledast x_0$ ☐ $x_9 \circledast x_0$ ☐ $x_{10} \circledast x_0$ ☐



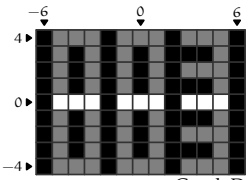
Graph A



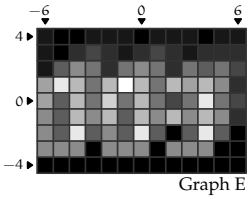
Graph B



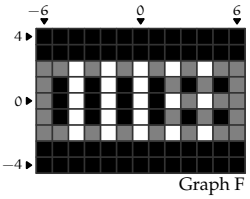
Graph C



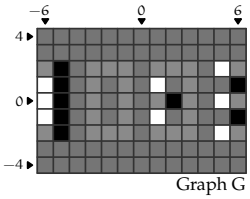
Graph D



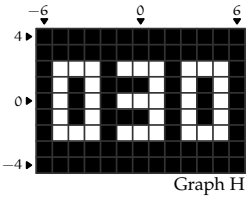
Graph E



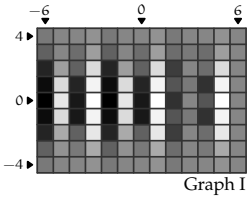
Graph F



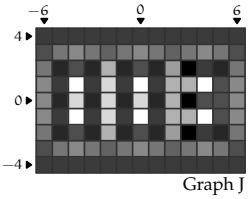
Graph G



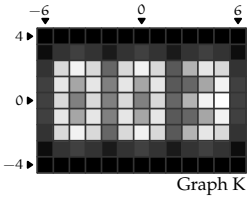
Graph H



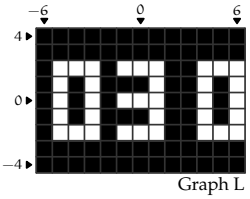
Graph I



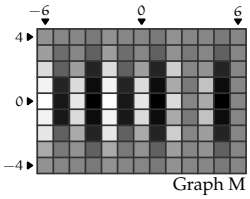
Graph J



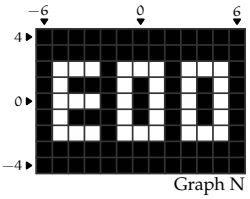
Graph K



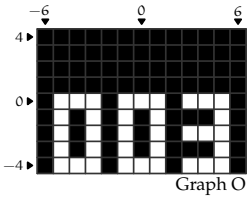
Graph L



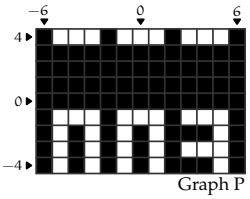
Graph M



Graph N



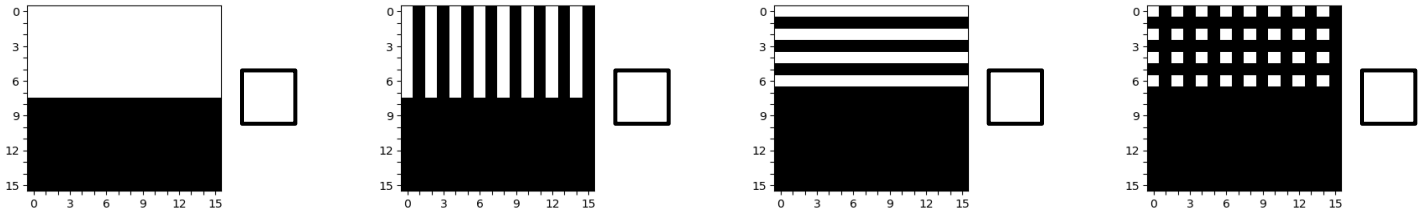
Graph O



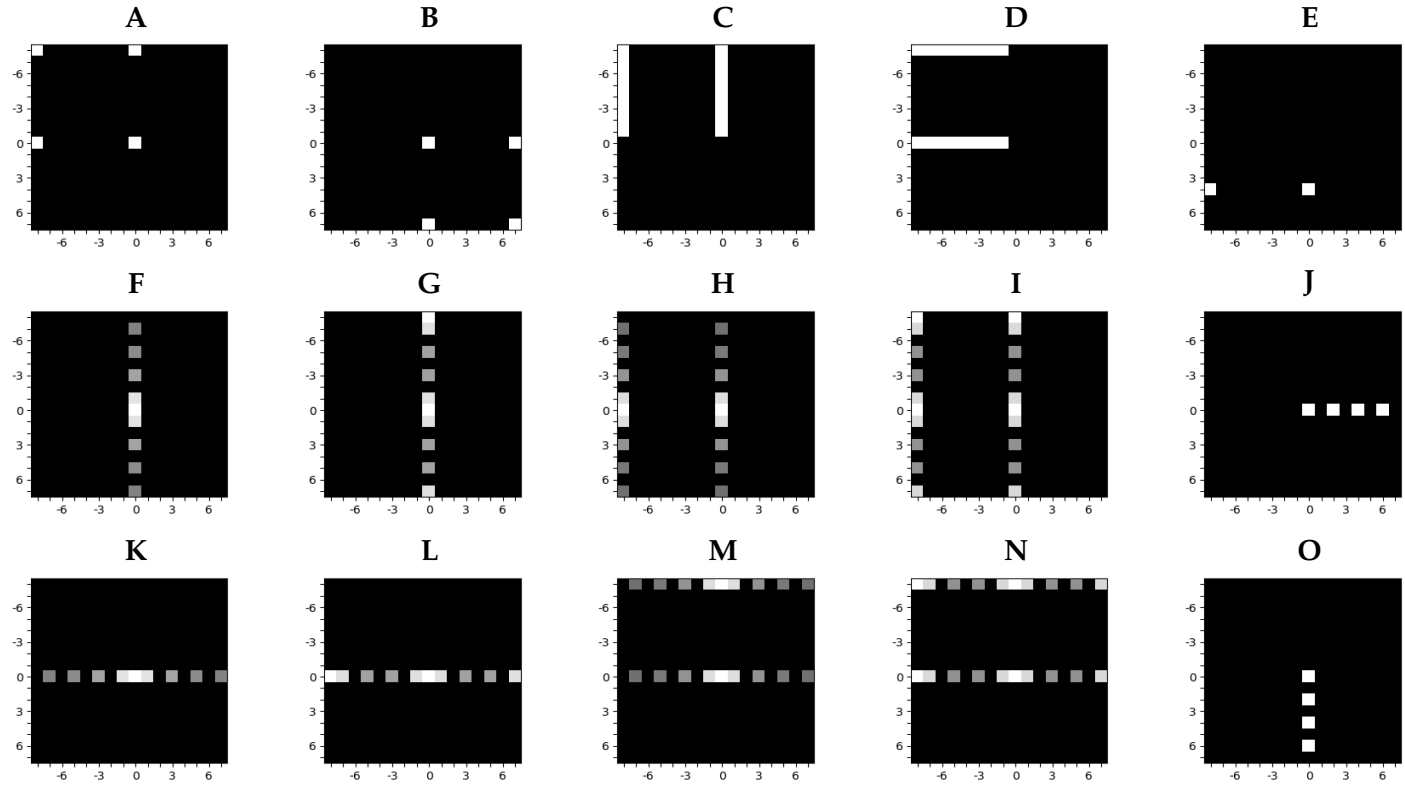
Graph P

5 2D Discrete Fourier Transforms (16 points)

Each of the following images contain 16 × 16 pixels that are either black (representing a value of 0) or white (representing a value of 1).



Determine which of the following images (**A-O**) shows the magnitude of the 2D DFT of each of the preceding images, and enter that letter in the corresponding box above.



In images A-O, white pixels represent the most positive magnitude in that image and black pixels a magnitude of 0. Notice that the zero-location inf images A-O is near the center of the image.

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Worksheet (*intentionally blank*)

Worksheet (*intentionally blank*)

Worksheet (*intentionally blank*)