6.3000 Quiz 2 Spring 2025

Name:

Kerberos (Athena) username:

Please WAIT until we tell you to begin.

This quiz is closed book, but you may use two 8.5×11 sheets of notes (four sides).

You may NOT use any electronic devices (such as calculators and phones).

If you have questions, please **come to us** at the front of the room to ask.

Please enter all solutions in the boxes provided.

Work on other pages with QR codes will be considered for partial credit.
Please provide a note if you continue work on worksheets at the end of the exam.

Please do not write on the QR codes at the bottom of each page.

We use those codes to identify which pages belong to each student.

Trigonometric Identities Reference

$$\cos(a+b) = \cos(a)\cos(b) - \sin(a)\sin(b)$$
$$\cos(a-b) = \cos(a)\cos(b) + \sin(a)\sin(b)$$
$$\sin(a+b) = \sin(a)\cos(b) + \cos(a)\sin(b)$$
$$\sin(a-b) = \sin(a)\cos(b) - \cos(a)\sin(b)$$
$$\cos(a) + \cos(b) = 2\cos\left(\frac{a+b}{2}\right)\cos\left(\frac{a-b}{2}\right)$$
$$\cos(a) - \cos(b) = -2\sin\left(\frac{a+b}{2}\right)\sin\left(\frac{a-b}{2}\right)$$
$$\sin(a) + \sin(b) = 2\sin\left(\frac{a+b}{2}\right)\cos\left(\frac{a-b}{2}\right)$$
$$\sin(a) - \sin(b) = 2\cos\left(\frac{a+b}{2}\right)\sin\left(\frac{a-b}{2}\right)$$
$$\cos(a+b)+\cos(a-b) = 2\cos(a)\cos(b)$$
$$\cos(a+b)-\cos(a-b) = -2\sin(a)\sin(b)$$
$$\sin(a+b)+\sin(a-b) = 2\sin(a)\cos(b)$$
$$\sin(a+b)-\sin(a-b) = 2\cos(a)\sin(b)$$
$$2\cos(a)\cos(b) = \cos(a-b)+\cos(a+b)$$
$$2\sin(a)\sin(b) = \cos(a-b)-\cos(a+b)$$
$$2\sin(a)\cos(b) = \sin(a+b)+\sin(a-b)$$
$$2\cos(a)\sin(b) = \sin(a+b)-\sin(a-b)$$

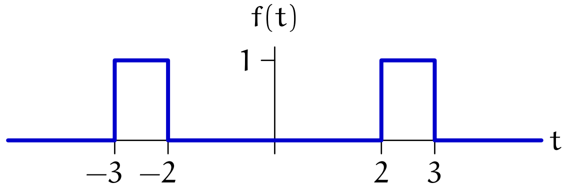
1 Short-Answer Questions (24 points)

Part a. The left column of the following table shows the input/output relations for four different systems, where x represents the input and y represents the output of the system. For each system, determine if that system is additive and/or homogeneous and/or time-invariant, and enter **Yes** or **No** in the boxes to the right.

	additive? (**Yes** or **No**)	homogeneous? (**Yes** or **No**)	time-invariant? (**Yes** or **No**)
$y[n] = \left(-\frac{1}{2}\right)^n \left(x[n]+1\right)$			
$y[n] = \sin\left(x[n]\right)$			
$y(t) = \int_{-\infty}^{t} x(\tau)d\tau$			
$y(t) = tx(t)$			

Part b. Determine the frequency response $F(\omega)$ of a linear, time-invariant system with the following impulse response:

$$f(t) = \begin{cases} 1 & \text{if } 2 < |t| < 3 \\ 0 & \text{otherwise} \end{cases}$$



Enter a closed form expression[1] for $F(\omega)$ in the box below.

$F(\omega) =$

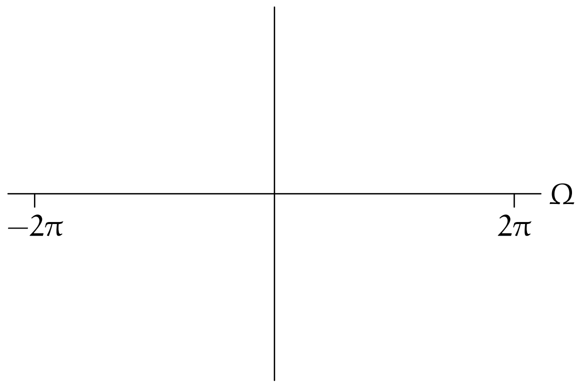
[1] Your expression should not include integrals or derivatives.

Part c. Determine the frequency response of a linear, time-invariant system with the following unit-sample response:

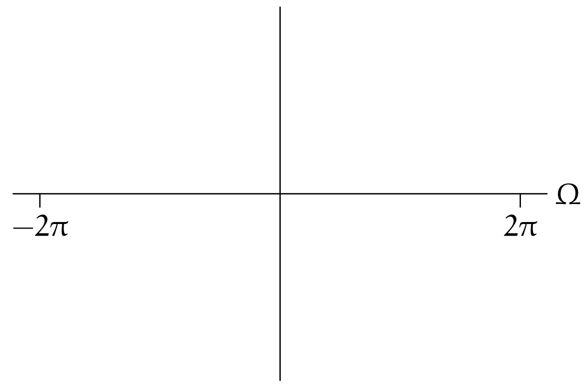
$$g[n] = \delta[n] - \delta[n-2]$$

Sketch the magnitude and angle of the frequency response on the axes below. Label the key points.

magnitude



angle



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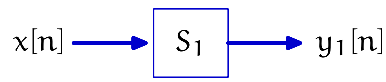
2 Cascaded Systems (22 points)

Part a.

Let S_1 represent a linear, time-invariant (LTI) system whose unit-sample response $h_1[n]$ is a unit step:



$$h_1[n] = \begin{cases} 1 & \text{if } n \geq 0 \\ 0 & \text{otherwise} \end{cases}$$

Determine the response $y_1[n]$ of this system when the input $x[n]$ is the following geometric sequence:

$$x[n] = \begin{cases} (0.9)^n & \text{if } n \geq 0 \\ 0 & \text{otherwise} \end{cases}$$

Enter the first 5 values of $y_1[n]$ in the boxes below.

$y_1[0] =$

$y_1[1] =$

$y_1[2] =$

$y_1[3] =$

$y_1[4] =$

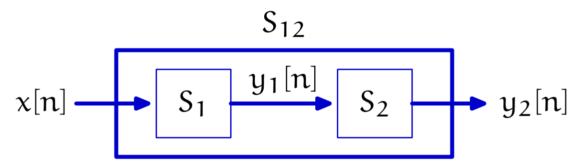
Enter a closed-form expression[2] for $y_1[n]$ in the box below.

$y_1[n] =$

[2] Your expression can contain additions, subtractions, multiplications, divisions, and exponentiations, but no other operators. Also, the number of operations required for each $y_1[n]$ should be bounded by a constant as $n \to \infty$.

Part b.

Let S_{12} represent the linear, time-invariant (LTI) system that results when two LTI systems (S_1 and S_2) are connected in cascade (so that the output of S_1 is the input to S_2) as shown in the figure to the right.



Determine the unit-sample response $h_{12}[n]$ of S_{12} when the unit-sample response $h_1[n]$ of system S_1 is a unit-step function $u[n]$ and the unit-sample response $h_2[n]$ of system S_2 is also a unit-step function $u[n]$:

$$h_1[n] = u[n]$$
$$h_2[n] = u[n]$$

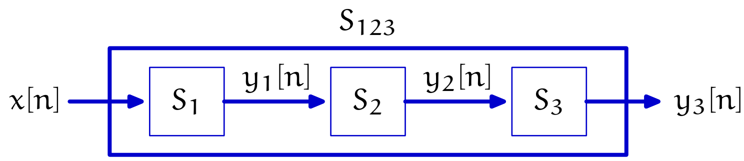
Enter a closed-form expression[3] for $h_{12}[n]$ in the box below.

$h_{12}[n] =$

[3] Your expression can contain additions, subtractions, multiplications, divisions, and exponentiations, but no other operators. Also, the number of operations required for each $h_{12}[n]$ should be bounded by a constant as $n \to \infty$.

Part c.

Let S_{123} represent the LTI system that results when three LTI systems (S_1, S_2, and S_3) are connected in cascade, as shown in the figure to the right.



Determine the unit-sample response $h_{123}[n]$ of S_{123} when the unit-sample response $h_1[n]$ of system S_1 is a unit-step function $u[n]$, the unit-sample response $h_2[n]$ of system S_2 is a unit-step function $u[n]$, and the unit-sample response $h_3[n]$ of system S_3 is also a unit-step function $u[n]$:

$h_1[n] = u[n]$
$h_2[n] = u[n]$
$h_3[n] = u[n]$

Enter the first 5 values of $h_{123}[n]$ in the boxes below.

$h_{123}[0] =$

$h_{123}[1] =$

$h_{123}[2] =$

$h_{123}[3] =$

$h_{123}[4] =$

Worksheet (intentionally blank)

3 Pulsed Relations (27 points)

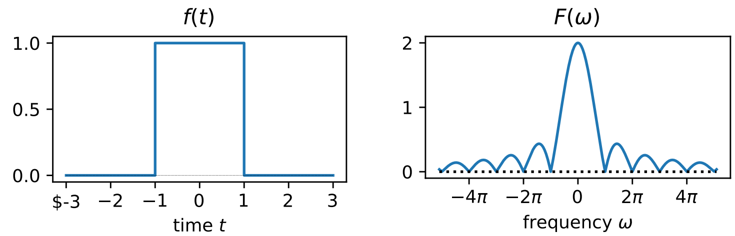
Let $f(t)$ represent the following signal

$$f(t) = \begin{cases} 1 & \text{if } |t| < 1 \\ 0 & \text{otherwise} \end{cases}$$

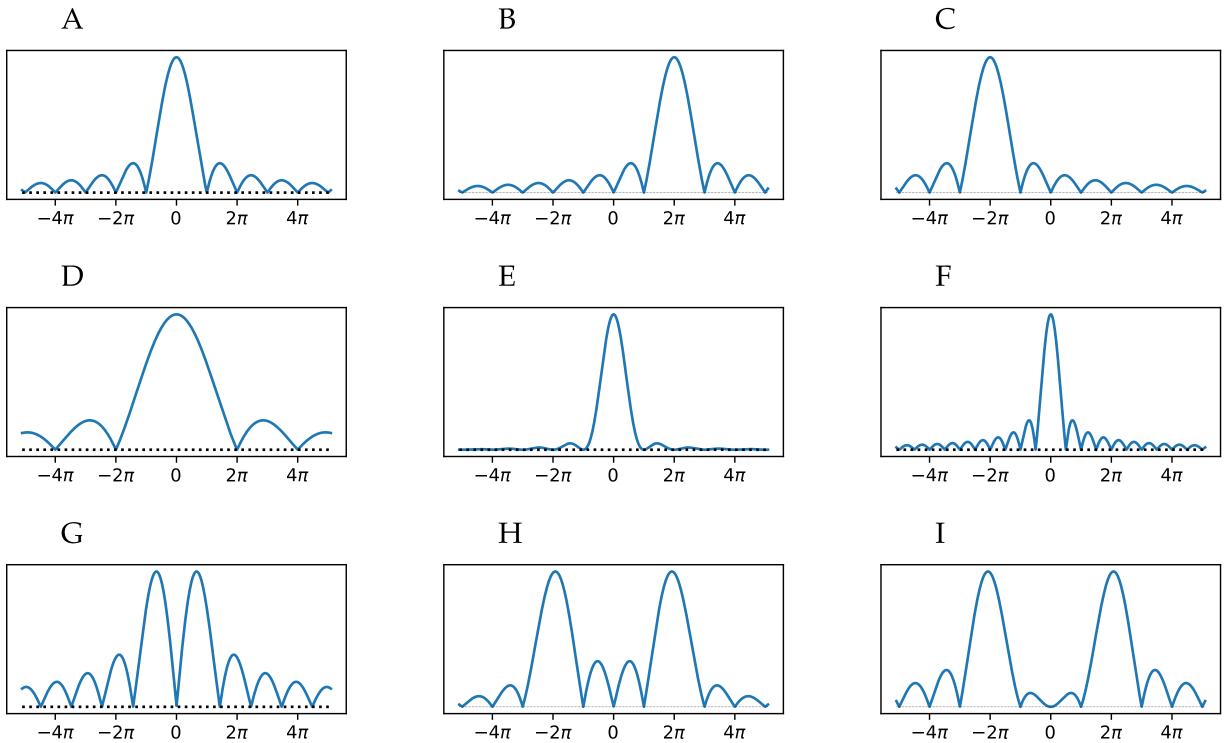
and let $F(\omega)$ represent its Fourier transform

$$F(\omega) = 2\,\frac{\sin(\omega)}{\omega}$$

as shown on the right.



Each of the following plots shows the magnitude of the Fourier transform of a signal derived from $f(t)$. Note that the magnitude scales for these plots are not specified and may differ from one another.



Identify which plot (A-I) shows the magnitude of each of the following derived signals.

$f(t/2)$:

$(f * f)(t)$:

$f(2t)$:

$f(t-1)$:

$tf(t)$:

$f(t)\sin(2\pi t)$:

$f(t)\cos(2\pi t)$:

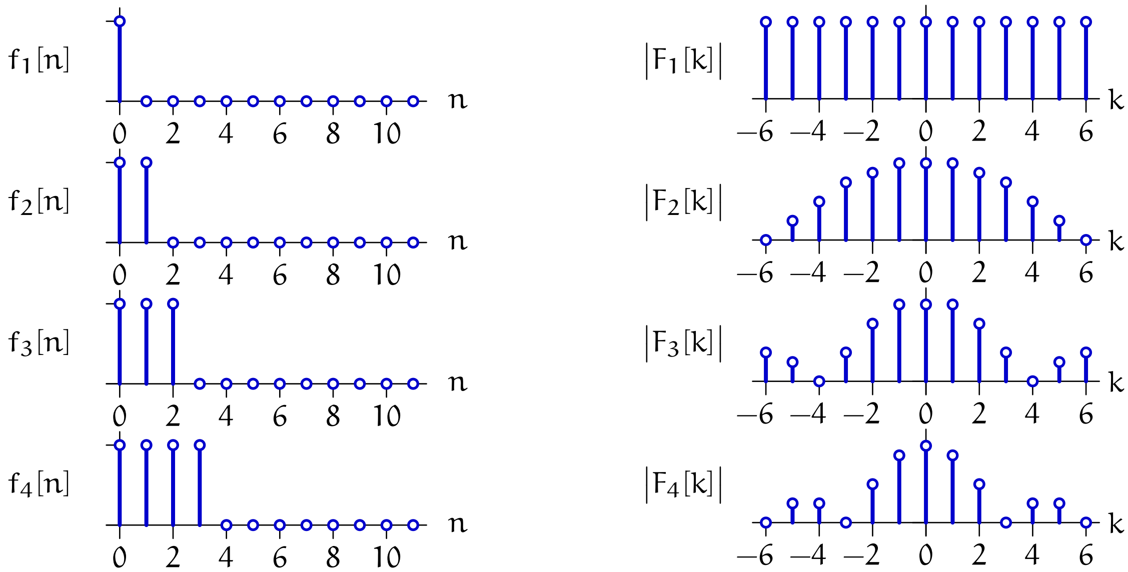
$f(t)e^{j2\pi t}$:

$f(t)e^{-j2\pi t}$:

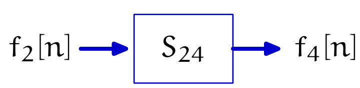
Worksheet (*intentionally blank*)

4 Pulses In Pulses Out (27 points)

The first 12 samples of four **periodic** signals that are each periodic in N=12 are shown in the left column below, and the magnitudes of their Fourier series coefficients are shown in the right column.



Part a. Consider a system S_{24} that produces $f_4[n]$ as output when $f_2[n]$ is its input.

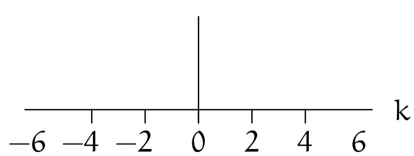


Determine the unit-sample response $h_{24}[n]$ and frequency response $H_{24}[k]$ of system S_{24} and enter expressions for $h_{24}[n]$ and $H_{24}[k]$ in the boxes below.

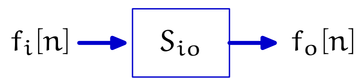
$h_{24}[n] =$

$H_{24}[k] =$

Sketch the magnitude of the frequency response $H_{24}[k]$ on the axes below.



Part b. Consider 16 possible systems that are each defined by its input signal $f_i[n]$ and output signal $f_o[n]$:



Some of these systems could be linear and time invariant (LTI); others cannot.

Determine which of the 16 systems cannot possibly be LTI, and enter **X** in the corresponding box below.

i \ o	1	2	3	4
1				
2				
3				
4				

Worksheet (*intentionally blank*)

Worksheet (*intentionally blank*)

Worksheet (*intentionally blank*)