6.003 Quiz 2 Spring 2022

Name:

Kerberos (Athena) username:

Please WAIT until we tell you to begin.

This quiz is closed book, but you may use two 8.5 × 11 sheets of notes (four sides).

You may NOT use any electronic devices (such as calculators and phones).

If you have questions, please **come to us** at the front of the room to ask.

Please enter all solutions in the boxes provided.

Work on other pages with QR codes will be considered for partial credit.
Please provide a note if you continue work on worksheets at the end of the exam.

Please do not write on the QR codes at the bottom of each page.

We use those codes to identify which pages belong to each student.

Trigonometric Identities Reference

$$\cos(a+b) = \cos(a)\cos(b) - \sin(a)\sin(b)$$

$$\cos(a-b) = \cos(a)\cos(b) + \sin(a)\sin(b)$$

$$\sin(a+b) = \sin(a)\cos(b) + \cos(a)\sin(b)$$

$$\sin(a-b) = \sin(a)\cos(b) - \cos(a)\sin(b)$$

$$\cos(a) + \cos(b) = 2\cos\left(\frac{a+b}{2}\right)\cos\left(\frac{a-b}{2}\right)$$

$$\cos(a) - \cos(b) = -2\sin\left(\frac{a+b}{2}\right)\sin\left(\frac{a-b}{2}\right)$$

$$\sin(a) + \sin(b) = 2\sin\left(\frac{a+b}{2}\right)\cos\left(\frac{a-b}{2}\right)$$

$$\sin(a) - \sin(b) = 2\cos\left(\frac{a+b}{2}\right)\sin\left(\frac{a-b}{2}\right)$$

$$\cos(a+b)+\cos(a-b) = 2\cos(a)\cos(b)$$

$$\cos(a+b)-\cos(a-b) = -2\sin(a)\sin(b)$$

$$\sin(a+b)+\sin(a-b) = 2\sin(a)\cos(b)$$

$$\sin(a+b)-\sin(a-b) = 2\cos(a)\sin(b)$$

$$2\cos(a)\cos(b) = \cos(a-b)+\cos(a+b)$$

$$2\sin(a)\sin(b) = \cos(a-b)-\cos(a+b)$$

$$2\sin(a)\cos(b) = \sin(a+b)+\sin(a-b)$$

$$2\cos(a)\sin(b) = \sin(a+b)-\sin(a-b)$$

1 Closing Sums (12 points)

In each of the following parts, k and n represent integers, and N is an integer multiple of 2 that is greater than 0. Simplify each expression by closing the sum and removing indeterminate forms (such as divisions by 0). Enter your answers in the boxes provided. Your answers should be valid for $0 \leqslant k < N$, and need not apply outside that range.

$$f_0[k] = \sum_{n=0}^{N-1} e^{-j\frac{2\pi kn}{N}} \quad =$$

$$f_1[k] = \sum_{n=0}^{N-1} e^{j\frac{2\pi k_0 n}{N}} e^{-j\frac{2\pi kn}{N}} \quad =$$

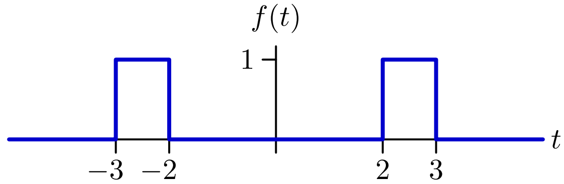
$$f_2[k] = \sum_{n=0}^{N-1} e^{-j\frac{4\pi kn}{N}} \quad =$$

Worksheet (*intentionally blank*)

2 Frequency Responses (20 points)

Part a. Determine the frequency response $F(\omega)$ of a linear, time-invariant system with the following impulse response:

$$f(t) = \begin{cases} 1 & \text{if } 2 < |t| < 3 \\ 0 & \text{otherwise} \end{cases}$$



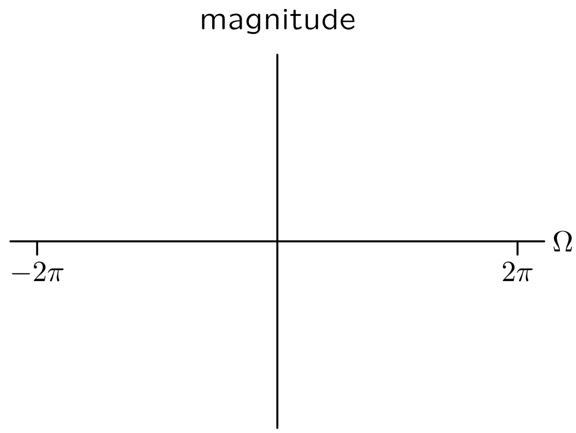
Enter a closed form expression for $F(\omega)$ in the box below.

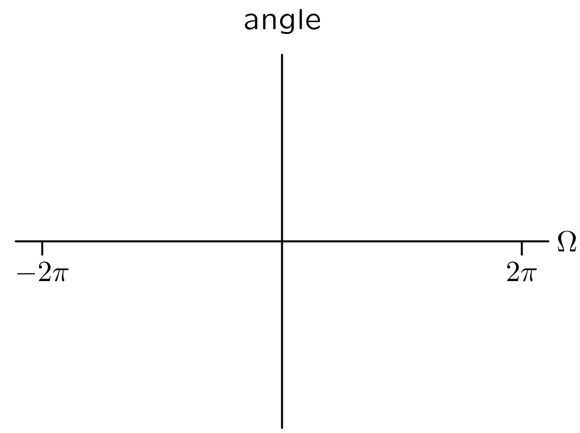
$F(\omega) =$

Part b. Determine the frequency response of a linear, time-invariant system with the following unit-sample response.

$$g[n] = \delta[n] - \delta[n-2]$$

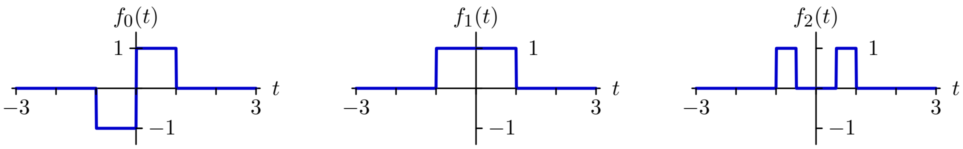
Sketch the magnitude and angle of the frequency response on the axes below. Label the key points.



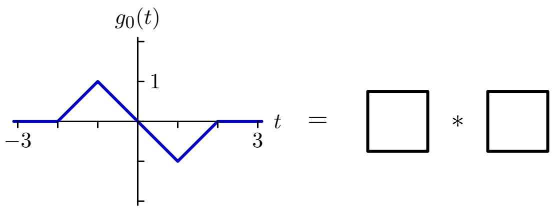


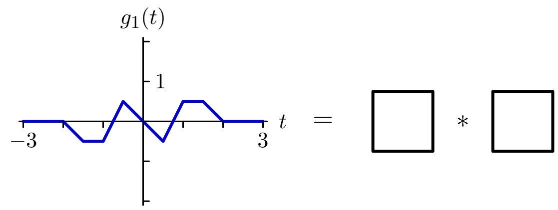
3 Convolving Steps (24 points)

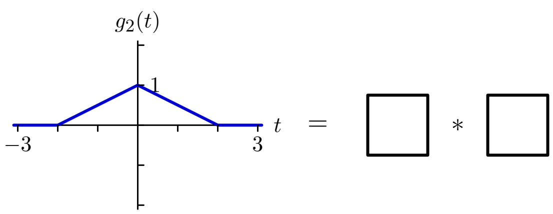
Let $f_0(t)$, $f_1(t)$, and $f_2(t)$ represent the following signals.

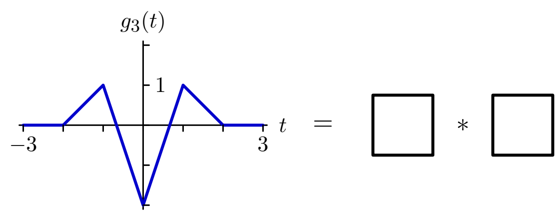


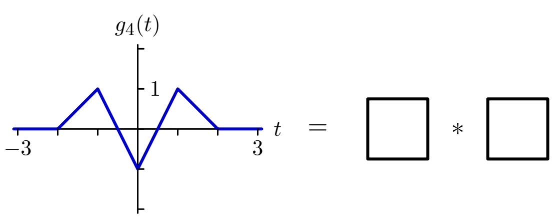
Determine if each of the signals below can be constructed by convolving (f_0 or f_1 or f_2) with (f_0 or f_1 or f_2). If it can, then enter the signals to be convolved in the corresponding boxes. If it cannot, then put an X in both boxes.

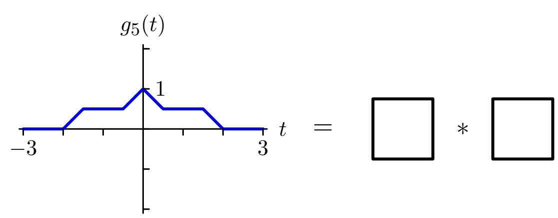


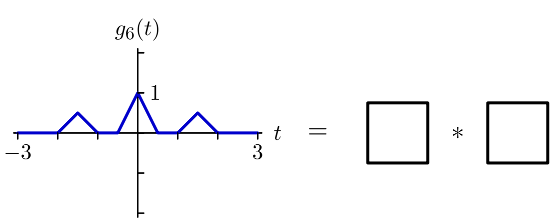


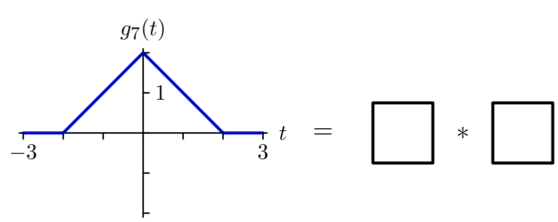












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4 Systems (20 points)

Part a. Let $\mathcal{S}$ represent a linear, time-invariant system with unit-sample response

$$h[n] = \begin{cases} \left(\frac{1}{2}\right)^n & \text{if } n \text{ is both even and greater than or equal to } 0 \\ 0 & \text{otherwise} \end{cases}$$

If the input to $\mathcal{S}$ is

$$x[n] = \cos(\pi n/4)$$

then the output can be written in the following form:

$$y[n] = A\cos(\pi n/4) + B\sin(\pi n/4)$$

Determine A and B and enter these numbers in the following boxes.

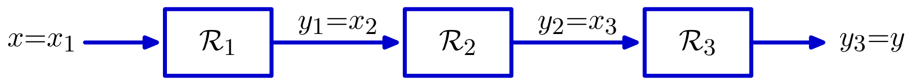
$A =$ [ ]

$B =$ [ ]

Part b. A system is constructed from three identical subsystems: $\mathcal{R}_1$, $\mathcal{R}_2$, and $\mathcal{R}_3$. The input signal x_i and output signal y_i of each subsystem are related by the following differential equation:

$$y_i(t) + \frac{d}{dt}y_i(t) = x_i(t)$$

where i is 1, 2, and 3 for $\mathcal{R}_1$, $\mathcal{R}_2$, and $\mathcal{R}_3$, respectively. The three subsystems are connected so that the output of $\mathcal{R}_1$ becomes the input to $\mathcal{R}_2$, and the output of $\mathcal{R}_2$ becomes the input to $\mathcal{R}_3$ as shown in the following diagram.



If the input to the combined system is

$$x(t) = x_1(t) = \cos(t)$$

then the output can be written in the following form:

$$y(t) = y_3(t) = C\cos(t - \phi)$$

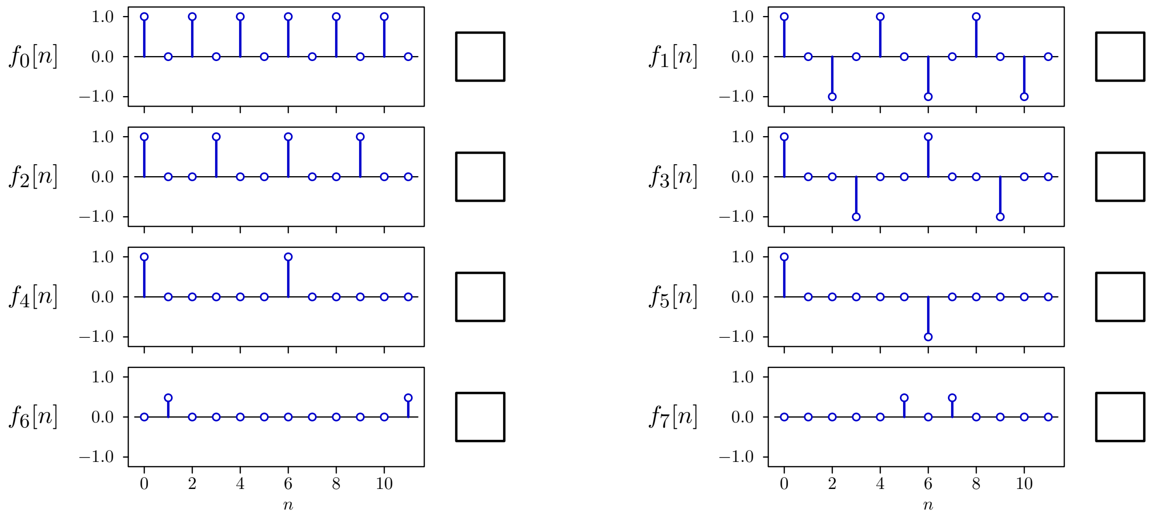
Determine C and ϕ, and enter these numbers in the following boxes.

$C =$

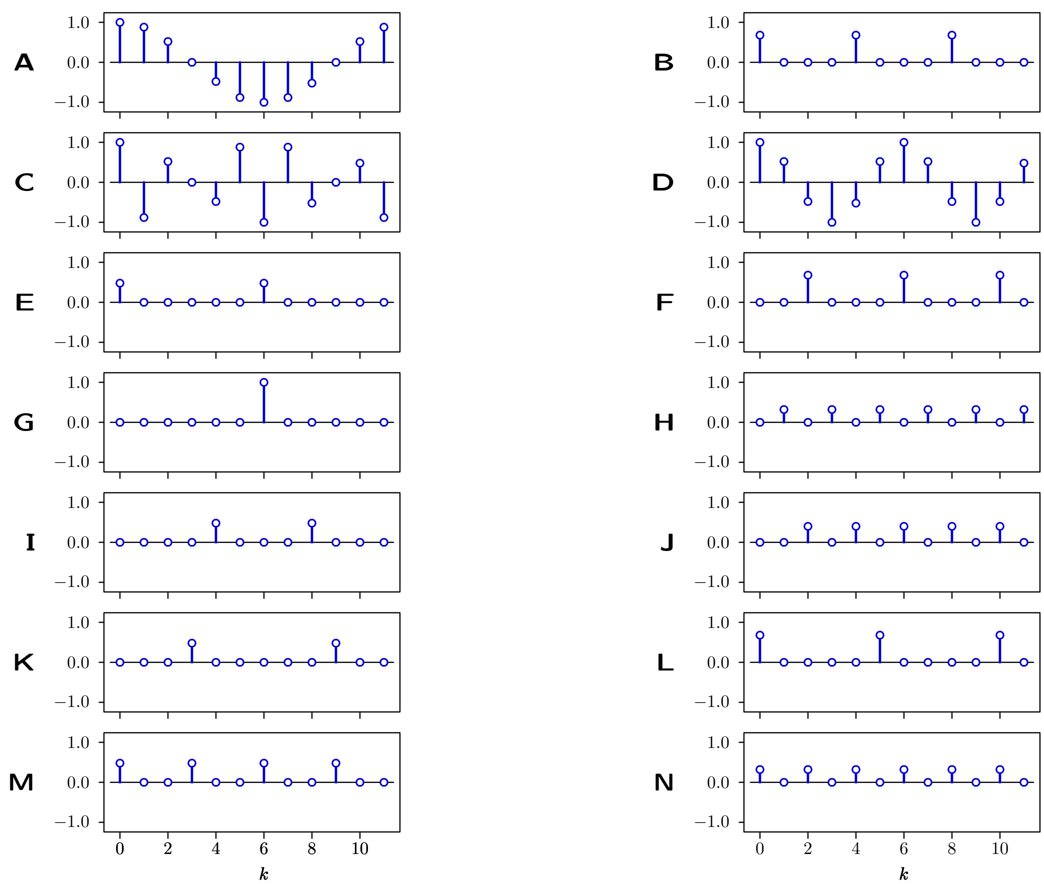
$\phi =$

5 DFT Matching (24 points)

Each of the following plots shows samples $n=0$ through $n=11$ of a discrete-time signal $f_i[n]$.



Determine which of the following plots (**A-N**) shows the real part of the 12-point DFT of each of the preceding signals ($f_0[n]$–$f_7[n]$), and enter that letter in the corresponding box above.



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