6.3000: Signal Processing

Communication Systems

- Matching Signals to Communications Media
- Amplitude Modulation
- Frequency-Division Multiplexing
- Radio Receivers

March 19, 2026

Communications Systems

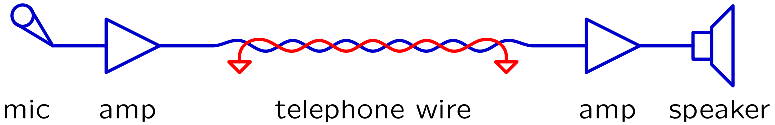
Beginning with commercial radio (1900s), communications technologies continue to be among the **fastest growing** applications of signal processing.

Examples:

- cellular communications
- wifi
- bluetooth
- GPS (Global Positioning System)
- broadband
- IOT (Internet of Things)
  - smart house / smart appliances
  - smart car
  - medical devices
- cable
- private networks: fire departments, police
- radar and navigation systems

Telephone

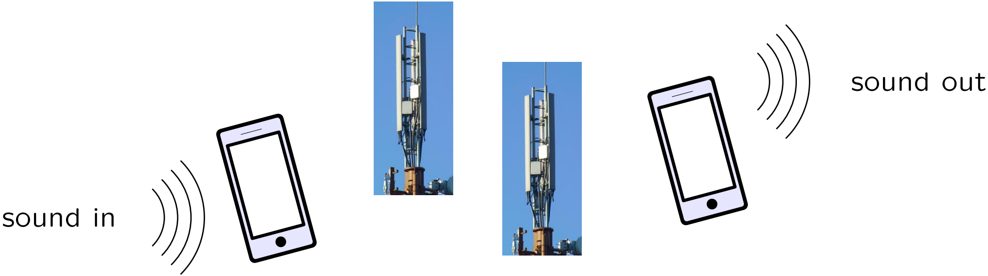
Popular thirst for communications has been evident since the early days of telephony.

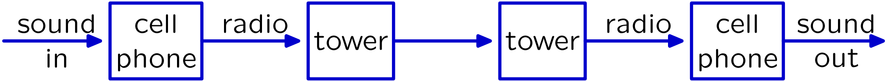


Patented by Alexander Graham Bell (1876), this technology flourished first as a network of **copper wires** and later as **optical fibers** (“long-distance” network) connecting virtually every household in the US by the 1980s.

Cellular Communication

First demonstrated by Motorola (1973), cellular communications quickly revolutionized the field. There are now more cell phones than people in the world.





Much of the popularity and convenience of cellular communications is that the communication is **wireless** (at least to the local tower).

Communications Systems

Beginning with commercial radio (1900s), communications technologies continue to be among the **fastest growing** applications of signal processing. Notice the proliferation of wireless technologies.

Examples:

- cellular communications $\surd$
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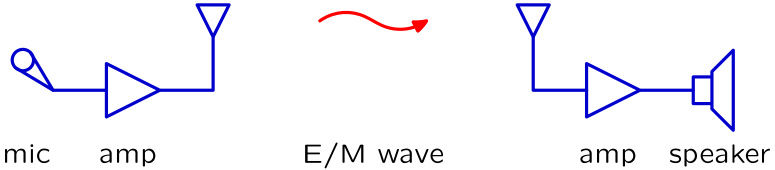
Wireless Communication of Speech

Challenges and solutions.

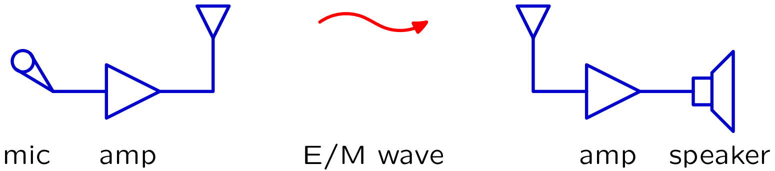


What is possible? (physics)

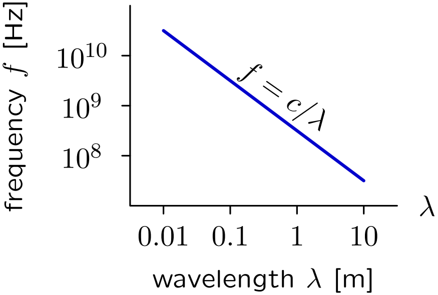
What is desireable? (FCC)

Wireless Communication

Wireless signals are transmitted via electromagnetic (E/M) waves.



For energy-efficient transmission and reception, the dimensions of the antenna should be on the order of the wavelength. But the wavelength of an electromagnetic wave depends on frequency.



Check Yourself

A key problem in the design of any communications system is matching characteristics of the signal to those of the media.

Telephone-quality speech contains frequencies from 200 Hz to 3000 Hz.

How long must the antenna be for efficient transmission and reception of E/M waves at audio frequencies?

1. < 1 mm
2. $\sim$ cm
3. $\sim$ m
4. $\sim$ km
5. > 100 km

Matching Signals to Communications Media

A key problem in the design of any communications system is matching characteristics of the signal to those of the media.

Telephone-quality speech contains frequencies from 200 Hz to 3000 Hz.

How large must the antenna be for efficient transmission and reception of E/M waves at audio frequencies?

The lowest frequencies (200 Hz) produce the longest wavelengths.

$$\lambda = \frac{c}{f} = \frac{3 \times 10^8 \, \text{m/s}}{200 \, \text{Hz}} = 1.5 \times 10^6 \, \text{m} = 1500 \, \text{km} \, .$$

The size of the antenna should be on the order of 900 miles!

Check Yourself

For efficient transmission and reception, the antenna length should be on the order of the wavelength.

Telephone-quality speech contains frequencies between 200 Hz and 3000 Hz.

How long should the antenna be? 5

1. < 1 mm
2. $\sim$ cm
3. $\sim$ m
4. $\sim$ km
5. > 100 km

Clearly untenable.

Check Yourself

What frequency E/M wave is well matched to an antenna with a length of 10 cm (about 4 inches)?

1. < 100 kHz
2. 1 MHz
3. 10 MHz
4. 100 MHz
5. > 1 GHz

Matching Signals to Communications Media

What frequency E/M wave is well matched to an antenna with a length of 10 cm (about 4 inches)?

A wavelength of $10\,\mathrm{cm}$ corresponds to a frequency of

$$f = \frac{c}{\lambda} \sim \frac{3 \times 10^8\,\mathrm{m/s}}{10\,\mathrm{cm}} \approx 3\,\mathrm{GHz}\,.$$

4G phones use frequencies near 2.5 GHz.

5G phones use a mix of frequencies ... up to 40 GHz.

Check Yourself

What frequency E/M wave is well matched to an antenna with a length of 10 cm (about 4 inches)? 5

1. < 100 kHz
2. 1 MHz
3. 10 MHz
4. 100 MHz
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Wireless Communication

Speech is not well matched to the wireless medium.

Many applications require the use of signals that are not well matched to the required media.

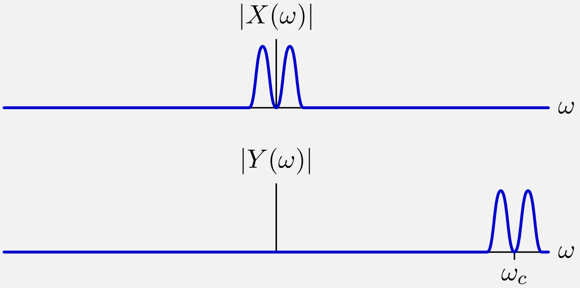
signal	applications
audio	radio, CD, cellular, optical fiber (TOSLINK), flash memory, MP3
video	DVD, flash memory, MP4
internet	twisted pair (Cat 5), optical fiber (backbone), wifi, bluetooth

Much of current research in communications focuses on modifying the intended signals to better accomodate constraints imposed by the media.

Today we will introduce simple matching strategies based on **modulation**.

Check Yourself

Let $X(\omega)$ represent audio frequency information to be transmitted.
Let $Y(\omega)$ represent that same information – shifted to center on $\omega{=}\omega_c$.



Which if any of the following statements is true?

1. $y(t) = x(t)\, e^{j\omega_c t}$
2. $y(t) = x(t) * e^{j\omega_c t}$
3. $y(t) = x(t) \cos(\omega_c t)$
4. $y(t) = x(t) * \cos(\omega_c t)$
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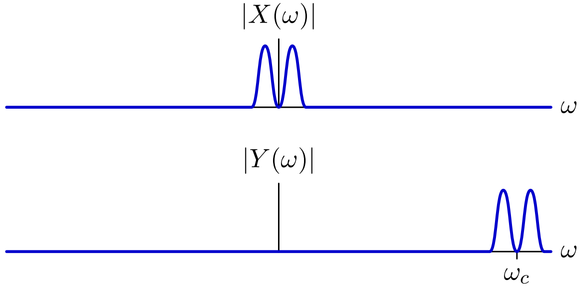

$Y(\omega)$ is a shifted version of $X(\omega)$:

$$Y(\omega) = X(\omega - \omega_c)$$

Find $y(t)$ from its Fourier transform:

$$y(t) = \frac{1}{2\pi}\int_{-\infty}^{\infty} Y(\omega)e^{j\omega t}d\omega = \frac{1}{2\pi}\int_{-\infty}^{\infty} X(\omega{-}\omega_c)e^{j\omega t}d\omega$$

$$= \frac{1}{2\pi}\int_{-\infty}^{\infty} X(\lambda)e^{j(\lambda+\omega_c)t}d\omega = e^{j\omega_c t}\frac{1}{2\pi}\int_{-\infty}^{\infty} X(\lambda)e^{j\lambda t}d\lambda = e^{j\omega_c t}x(t)$$

Check Yourself

Let $X(\omega)$ represent audio frequency information to be transmitted.
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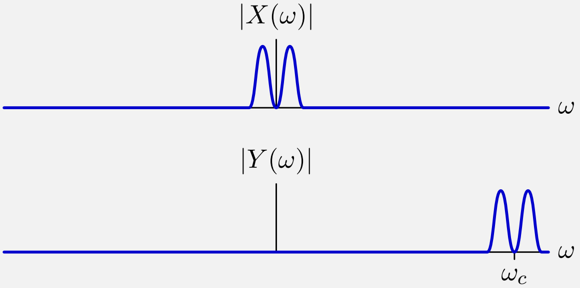

Which if any of the following statements is true? 1

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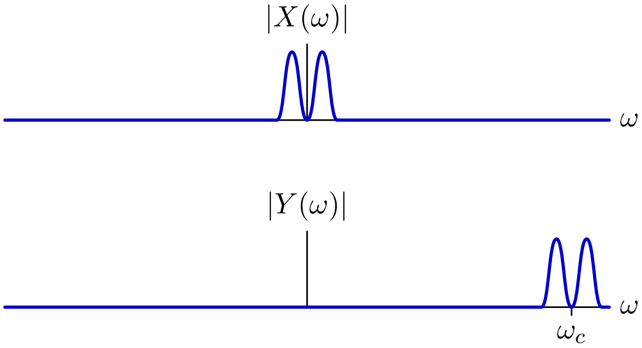
Matching Signals to Communications Media

This scheme cannot be implemented **physically**. Why?



Matching Signals to Communications Media

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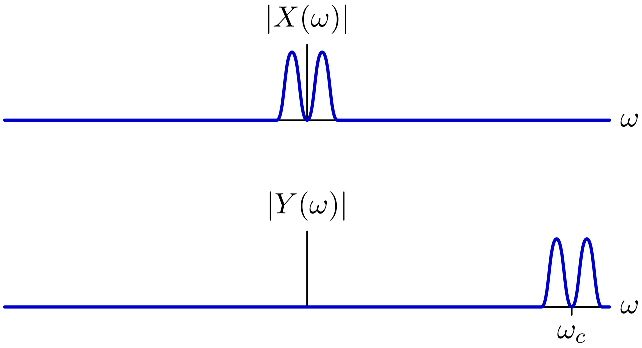


The problem with this scheme

$$y(t) = x(t)e^{j\omega_c t}$$

is that $y(t)$ has both real and imaginary parts, even if $x(t)$ is purely real. Signals that are transmitted through physical media (such as wires and optical fibers) are purely real.

Matching Signals to Communications Media

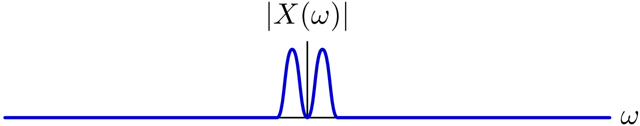
A purely real-valued signal results if we shift a copy of $X(\omega)$ upward in frequency by ω_c and a second copy downward by the same amount.

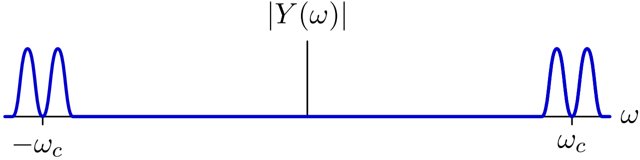
If

$$x(t) \overset{\text{CTFT}}{\Longrightarrow} X(\omega)$$

then

$$e^{j\omega_c t}x(t) + e^{-j\omega_c t}x(t) = 2\cos(\omega_c t)x(t) \overset{\text{CTFT}}{\Longrightarrow} X(\omega-\omega_c) + X(\omega+\omega_c)$$



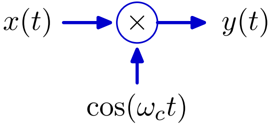


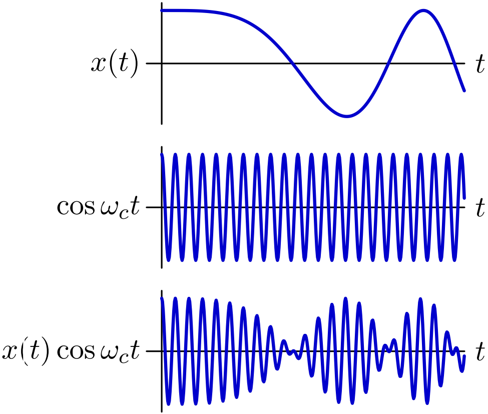
We refer to this scheme as **amplitude modulation.**

Amplitude Modulation (Time Domain)

Multiplying by a sinusoidal "carrier" is called **amplitude modulation.**

The signal "modulates" the amplitude of the carrier.





Multiplication Property of Fourier Transform

Multiplication in time corresponds to convolution in frequency.

Let

$$z(t) = x(t)y(t)$$

then

$$Z(\omega) = \int x(t)y(t)e^{-j\omega t}\,dt$$

Substitute $y(t) = \frac{1}{2\pi}\int Y(\omega)e^{j\omega t}d\omega = \frac{1}{2\pi}\int Y(\lambda)e^{j\lambda t}d\lambda$.

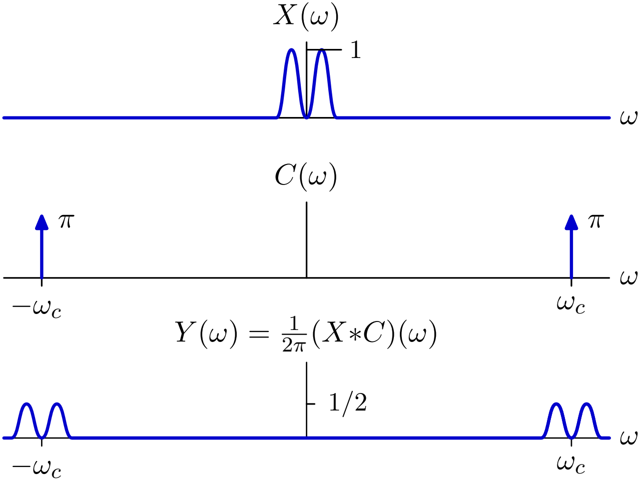
$$\begin{aligned} Z(\omega) &= \int x(t)\left(\frac{1}{2\pi}\int Y(\lambda)e^{j\lambda t}\,d\lambda\right)e^{-j\omega t}\,dt \\ &= \frac{1}{2\pi}\int Y(\lambda)\left(\int x(t)e^{-j(\omega-\lambda)t}dt\right)d\lambda \\ &= \frac{1}{2\pi}\int Y(\lambda)X(\omega-\lambda)\,d\lambda = \frac{1}{2\pi}(X*Y)(\omega) \end{aligned}$$

This result is the **dual** of filtering, where convolution in time corresponds to multiplication in frequency.

Amplitude Modulation (Frequency Domain)

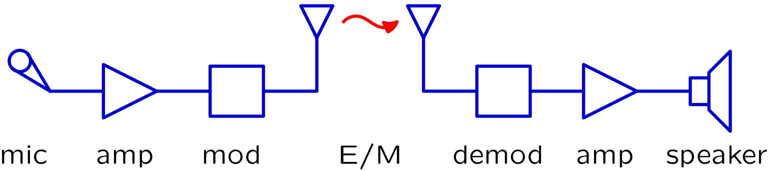
Multiplication in time corresponds to convolution in frequency.

Let $X(\omega)$ represent the Fourier transform of the signal to be transmitted.
Let $C(\omega)$ represent the Fourier transform of $\cos(\omega_c t) = \frac{1}{2}\left(e^{j\omega_c t} + e^{-j\omega_c t}\right)$.
Then $Y(\omega)$ is the result of convolving $X(\omega)$ with $C(\omega)$.



Demodulating the Received Signal

We can match the signal to the medium with a modulator as shown by the **mod** box below. But then we must demodulate the received signal to get back our original message (the **demod** box below).



How can we recover the original message from the modulated signal?

Synchronous Demodulation

The original message can be recovered from an amplitude modulated signal by multiplying by the carrier and then low-pass filtering.

Assume that

$$y(t) = x(t)\cos(\omega_c t)$$

Then

$$z(t) = y(t)\cos(\omega_c t) = x(t) \times \cos(\omega_c t) \times \cos(\omega_c t) = x(t)\left(\frac{1}{2} + \frac{1}{2}\cos(2\omega_c t)\right)$$

This process is called **synchronous demodulation**.

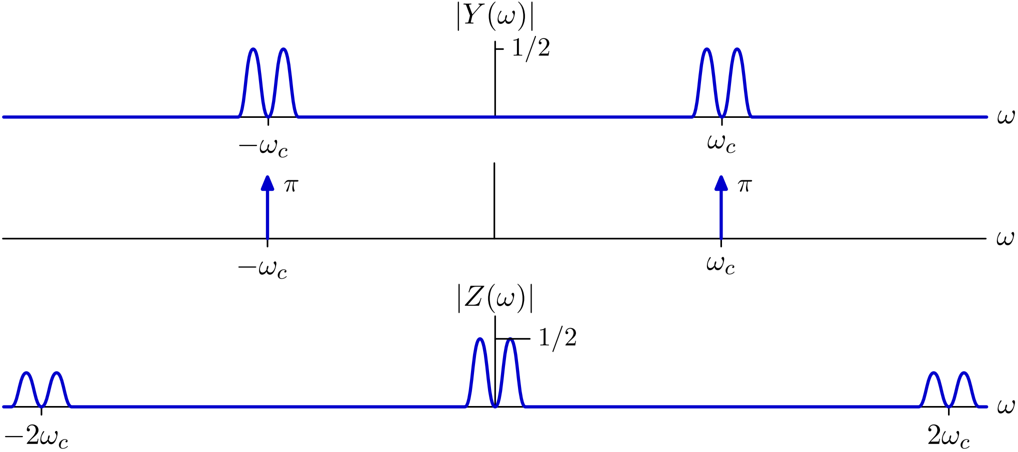
Synchronous Demodulation

Synchronous demodulation is equivalent to convolution in frequency.

$$y(t) = x(t)\cos(\omega_c t)$$

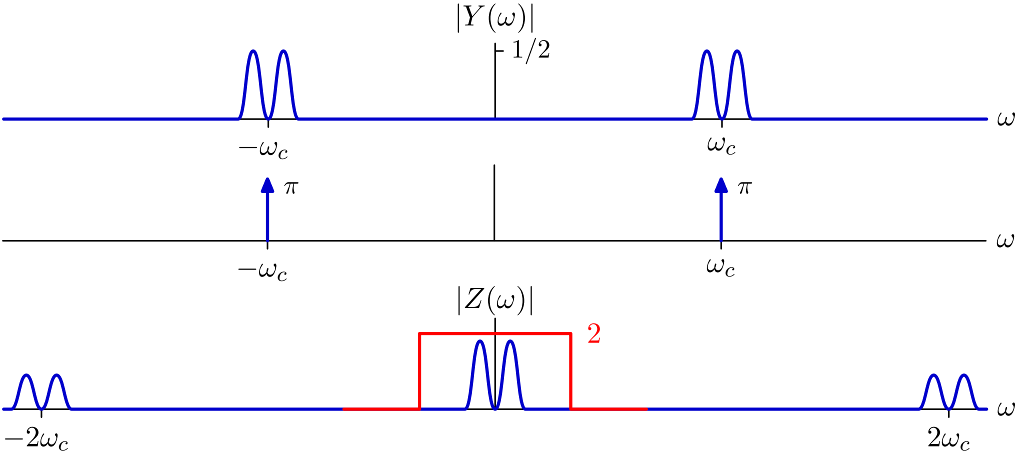
$$z(t) = y(t)\cos(\omega_c t)$$

$$Z(\omega) = \frac{1}{2\pi}Y(\omega) * \Big(\pi\delta(\omega{-}\omega_c) + \pi\delta(\omega{+}\omega_c)\Big)$$



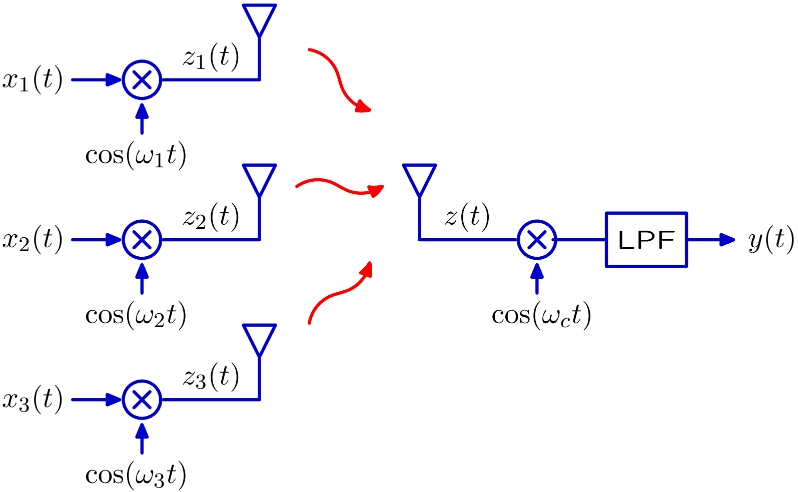
Synchronous Demodulation

Recover the original message by low-pass filtering (and multiplying by 2).



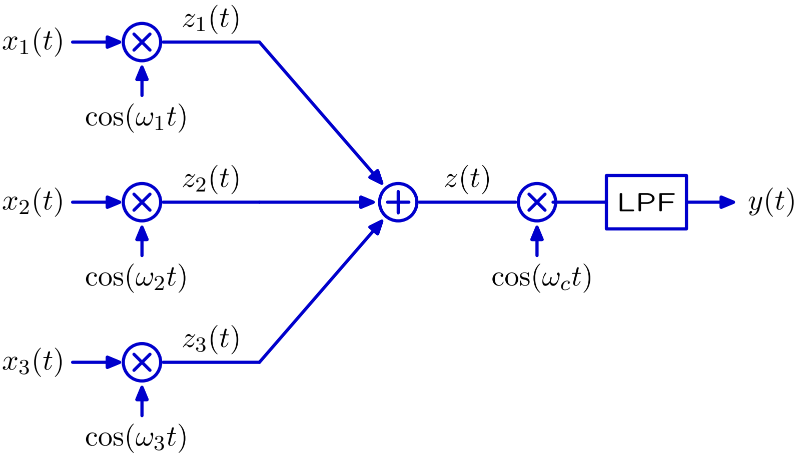
Frequency-Division Multiplexing

Multiple transmitters can co-exist, as long as the frequencies that they transmit do not overlap.



Frequency-Division Multiplexing

To first order, multiple transmitters simply sum.



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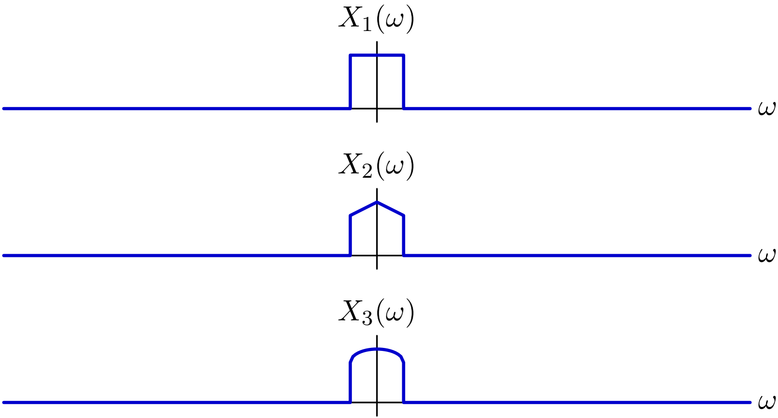

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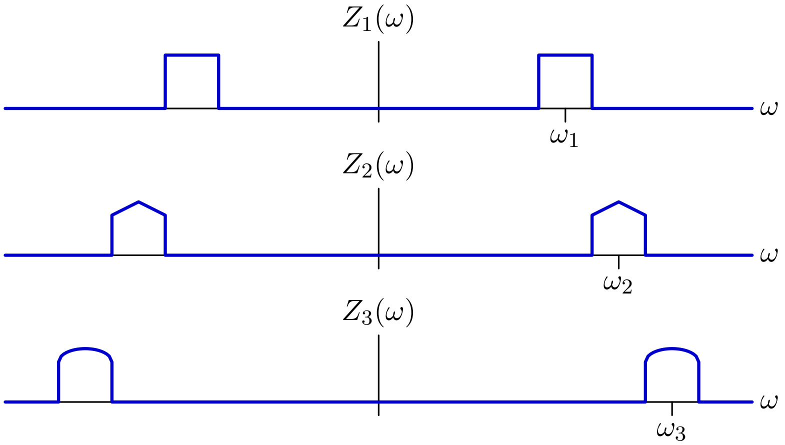

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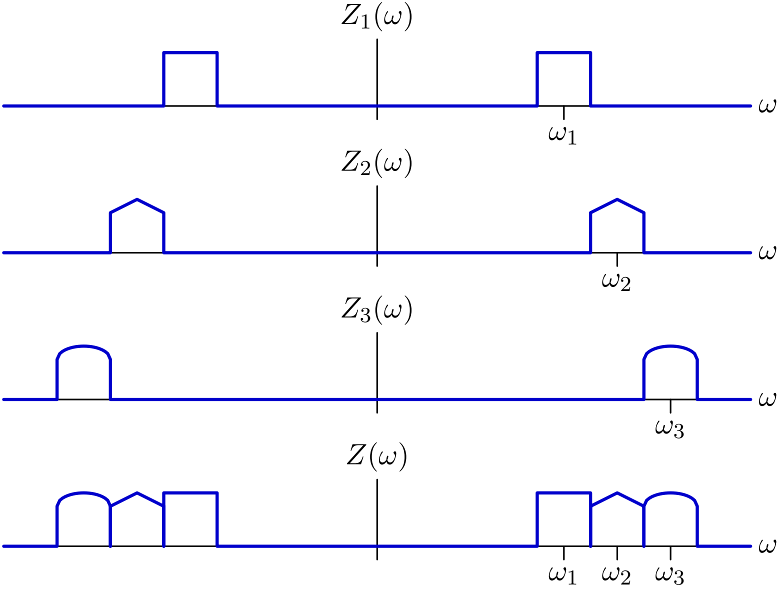
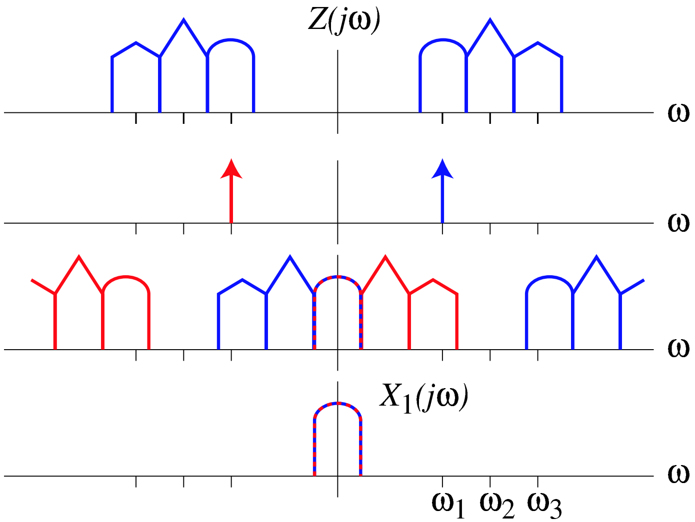


Frequency-Division Multiplexing

The receiver can select the transmitter of interest by choosing the corresponding demodulation frequency.



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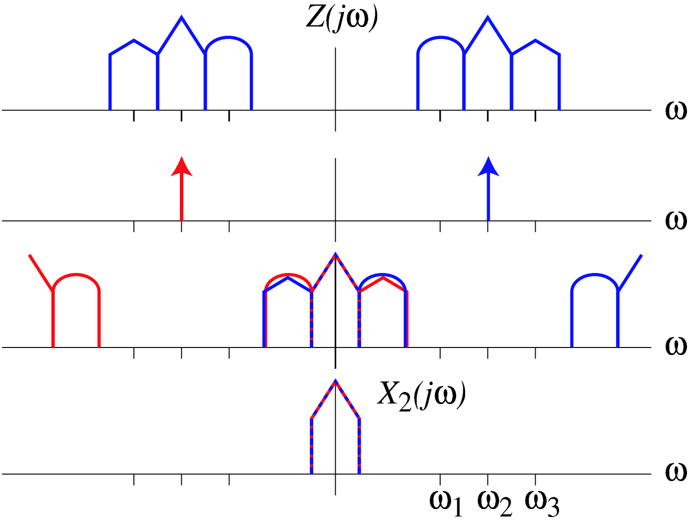

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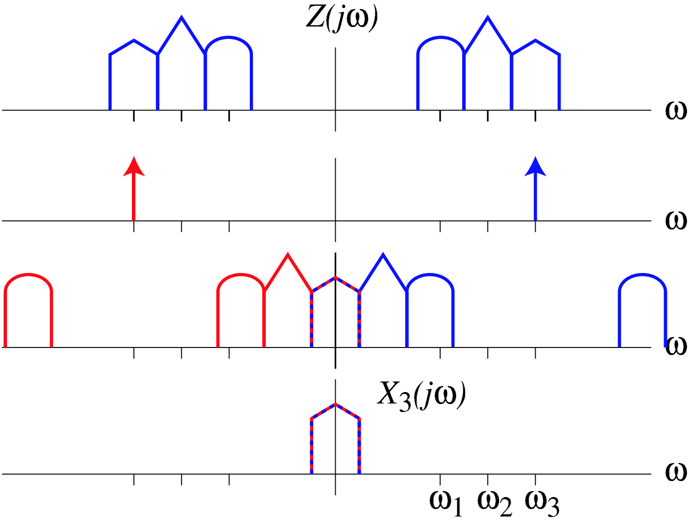
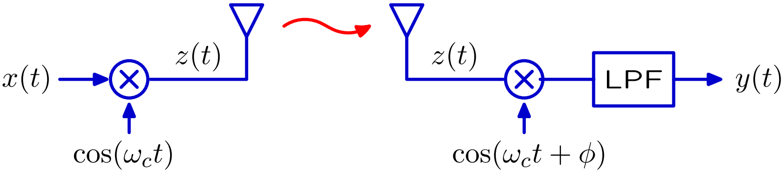


Radio Receiver

Synchronous demodulation requires knowing the carrier signal exactly.



What happens if there is a phase shift ϕ between the signal used to modulate and the one used to demodulate?

Radio Receiver

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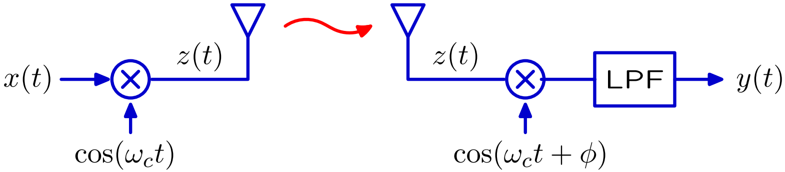


What happens if there is a phase shift ϕ between the signal used to modulate and the one used to demodulate?

$$
\begin{aligned}
y(t) &= x(t)\cos(\omega_c t)\cos(\omega_c t + \phi) \\
&= \frac{1}{2}x(t)\Big(\cos\phi + \cos(2\omega_c t + \phi)\Big) \\
&= \frac{1}{2}x(t)\cos\phi + \frac{1}{2}x(t)\cos(2\omega_c t + \phi)
\end{aligned}
$$

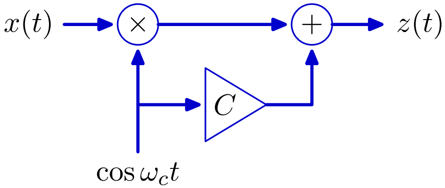
The second term is at a high frequency, so we can filter it out.

But multiplying by $\cos\phi$ in the first term is a problem: the signal "fades."

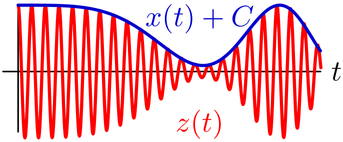
For example, if $\phi = \frac{\pi}{2}$, there is no output at all!

AM with Carrier

One way to know the carrier exactly is to send it along with the message.



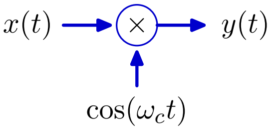
$$z(t) = x(t)\cos\omega_c t + C\cos\omega_c t = (x(t) + C)\cos\omega_c t$$

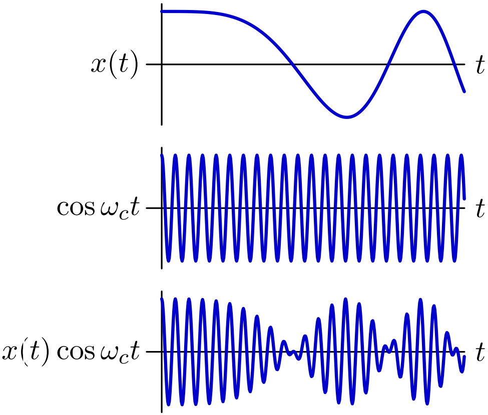


Adding carrier is equivalent to shifting the DC value of $x(t)$.
If we shift the DC value sufficiently, the message is easy to decode: it is just the envelope (minus the DC shift).

Amplitude Modulation (Time Domain)

Multiplying by a sinusoidal "carrier" is called **amplitude modulation.**
The signal "modulates" the amplitude of the carrier.





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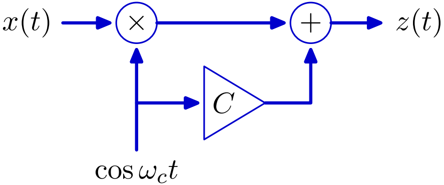

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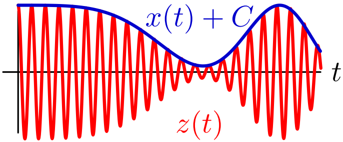

Adding carrier is equivalent to shifting the DC value of $x(t)$.

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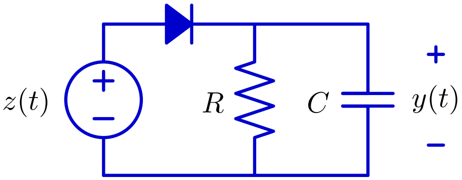
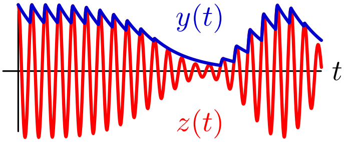
Radio Receiver

If the carrier frequency is much greater than the highest frequency in the message, AM with carrier can be demodulated with a peak detector.





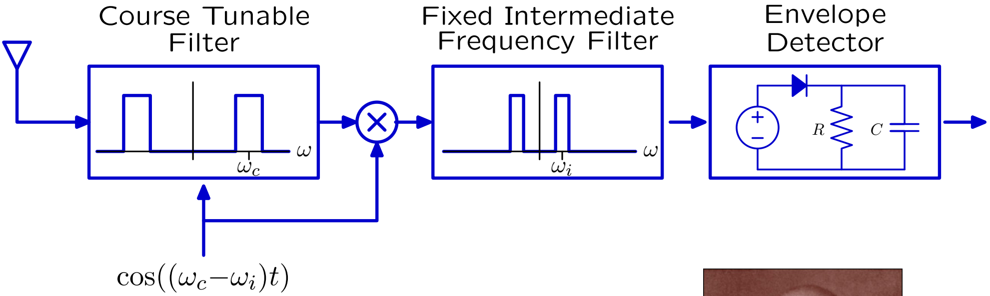
In AM radio, the highest frequency in the message is 5 kHz and the carrier frequency is between 500 kHz and 1500 kHz.

This circuit is simple and inexpensive.

But there is a problem.
Envelope detection cannot separate multiple senders.

Superheterodyne Receiver

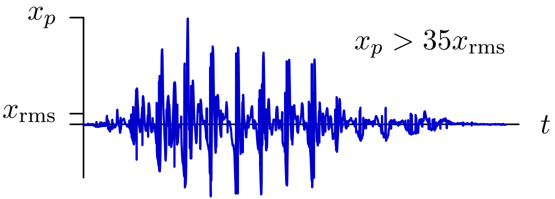
Edwin Howard Armstrong invented the superheterodyne receiver, which made broadcast AM practical.



Edwin Howard Armstrong also invented and patented the "regenerative" (positive feedback) circuit for amplifying radio signals (while he was a junior at Columbia University). He also invented wideband FM.

Radio Transmission

AM with carrier requires more power to transmit the carrier than to transmit the message!



Speech sounds have high crest factors (peak value divided by rms value). Envelope detection will only work if the DC offset C is larger than x_p.

The power needed to transmit the carrier can be $35^2 \approx 1000\times$ that needed to transmit the message.

Okay for broadcast radio (WBZ: 50 kwatts).
Not for point-to-point (cell phone batteries wouldn't last long!).

Broadcast Radio

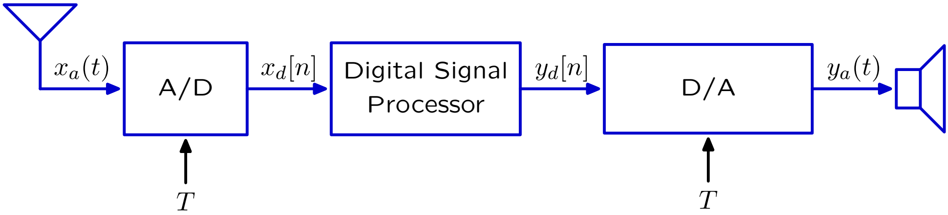
"Broadcast" radio was championed by David Sarnoff, who previously worked at Marconi Wireless Telegraphy Company (point-to-point).

- envisioned "radio music boxes"
- analogous to newspaper, but at speed of light
- receiver must be cheap (as with newsprint)
- transmitter can be expensive (as with printing press)

Sarnoff (left) and Marconi (right)

Digital Radio

Today's radios are very different from those that launched broadcast radio.



Some issues remain the same:

- power utilization
- bandwidth limitations

Other issues are newer:

- more users
- more messages per user
- more different kinds of messages (audio, video, data)
- privacy and security

Signal processing plays an important role in all of these areas.

Questions of the Day

Let $\mathcal{S}$ represent a system whose output $y(t) = e^{-j\omega_o t}x(t)$ when its input is $x(t)$.

$$x(t) \rightarrow \boxed{\mathcal{S}} \rightarrow y(t) = e^{-j\omega_o t}x(t)$$

Is the system $\mathcal{S}$ additive? homogeneous? time-invariant? LTI?

Please enter your responses at the following url:

http://bit.ly/4qehFmF