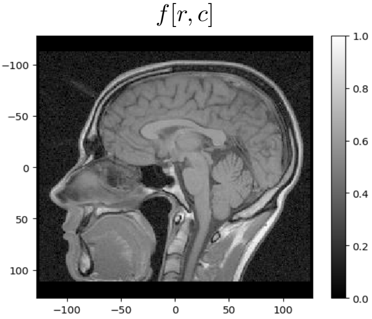
6.3000: Signal Processing

MRI

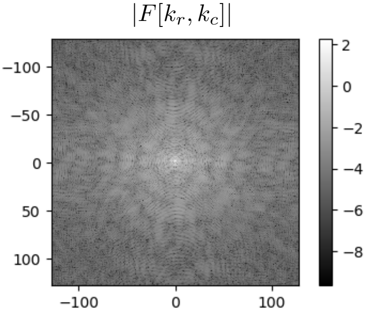
May 06, 2025

Magnetic Resonance Imaging

Magnetic resonance images are constructed from measurements of Fourier (k-space) data.



2D DFT $\Longleftrightarrow$



optical imaging
starts here

magnetic resonance imaging
starts here

This different has profound effects on processing MR images.

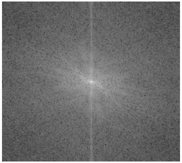
Example: MR imaging is slow – often tens of minutes.

Today: brief introduction to accelerating MR imaging.

Downsampling

Measure $F[k_r, k_c]$ and inverse transform to get $f[r, c]$.

$F[k_r, k_c]$

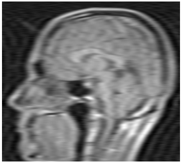


$\overset{\text{IDFT}}{\Longrightarrow}$

$f[r, c]$

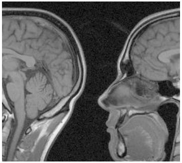
Measure just even-numbered columns of F. Set others to 0. This would half the data collected for each image. Which image would result?

A

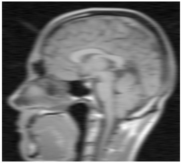


B

C

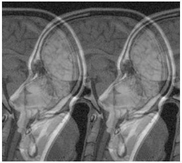


D

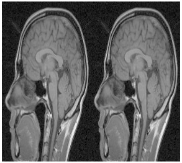


E

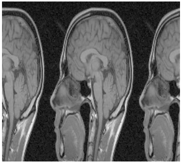
F



G



H



Downsampling

Which image is reconstructed from just even-numbered columns?

Setting the odd-numbered columns of the transform to zero is equivalent to multiplying the DFT by

$$H[k_r,k_c] = \frac{1}{2}\left(1+(-1)^{k_c}\right) = \frac{1}{2}\left(1+e^{-j\pi k_c}\right)$$

Multiplying in frequency is equivalent to convolving in space by

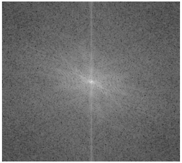
$$\begin{aligned} h[r,c] &= \sum_{k_r,k_c} H[k_r,k_c]\, e^{j2\pi\left(\frac{k_r r}{R}+\frac{k_c c}{C}\right)} \\ &= \sum_{k_r,k_c} \frac{1}{2}\left(1+e^{-j\pi k_c}\right)\, e^{j2\pi\left(\frac{k_r r}{R}+\frac{k_c c}{C}\right)} \\ &= \frac{1}{2}\sum_{k_r,k_c} e^{j2\pi\left(\frac{k_r r}{R}+\frac{k_c c}{C}\right)} + \frac{1}{2}\sum_{k_r,k_c} e^{j2\pi\left(\frac{k_r r}{R}+\frac{k_c (c-C/2)}{C}\right)} \\ &= \frac{1}{2}\delta[r,c] + \frac{1}{2}\delta\left[r,c-\frac{C}{2}\right] \end{aligned}$$

Convolving by $h[r,c]$ adds a half-frame horizaontal circular shift of the image to the original image → panel F.

Downsampling

Measure $F[k_r, k_c]$ and inverse transform to get $f[r, c]$.

$F[k_r, k_c]$

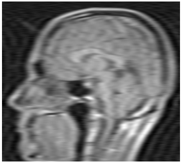


$\stackrel{\text{IDFT}}{\Longrightarrow}$

$f[r, c]$

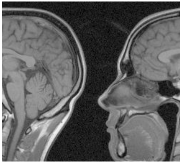
Measure just even-numbered columns of F. Set others to 0. This would half the data collected for each image. Which image would result? **F**

A

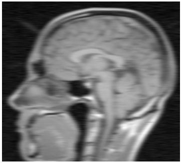


B

C

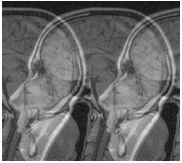


D

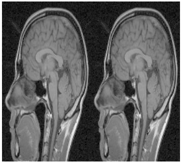


E

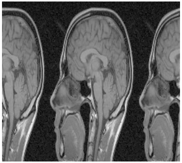
F



G



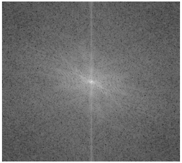
H



Downsampling with Linear Interpolation

Measure $F[k_r, k_c]$ and inverse transform to get $f[r, c]$.

$F[k_r, k_c]$

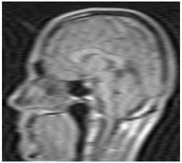


$\overset{\text{IDFT}}{\Longrightarrow}$

$f[r, c]$

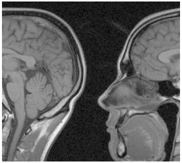
Measure even-numbered columns of $F[k_r, k_c]$. Set pixels in odd numbered columns to the average of adjacent columns. Result?

A

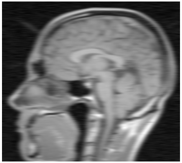


B

C

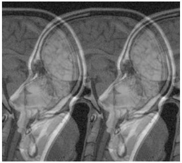


D

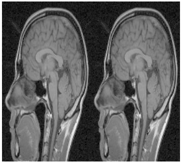


E

F



G



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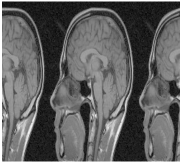


Downsampling with Linear Interpolation

Setting the odd numbered columns to the average of the adjacent columns is equivalent to zeroing the odd numbered columns and then convolving by a three-point triangular averager:

$$H[k_r,k_c]=\frac{1}{4}\delta[k_r,k_c-1]+\frac{1}{2}\delta[k_r,k_c]+\frac{1}{4}\delta[k_r,k_c+1]$$

original data:	F_0	F_1	F_2	F_3	F_4	F_5	F_6	F_7
just evens:	F_0	0	F_2	0	F_4	0	F_6	0
3-point averaged:	$\frac{1}{2}F_0$	$\frac{F_0+F_2}{4}$	$\frac{1}{2}F_2$	$\frac{F_2+F_4}{4}$	$\frac{1}{2}F_4$	$\frac{F_4+F_6}{4}$	$\frac{1}{2}F_6$	$\frac{F_6+F_8}{4}$

Convolving with a three-point triangular averager in frequency is equivalent to multiplying in time by

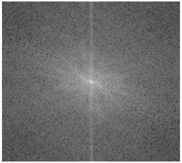
$$h[r,c]=\frac{1}{4}e^{-j2\pi c/C}+\frac{1}{2}+\frac{1}{4}e^{j2\pi c/C}=\frac{1}{2}+\frac{1}{2}\cos(2\pi c/C)$$

$\rightarrow$ panel E

Downsampling with Linear Interpolation

Measure $F[k_r, k_c]$ and inverse transform to get $f[r, c]$.

$F[k_r, k_c]$

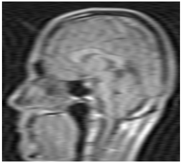


$\overset{\text{IDFT}}{\Longrightarrow}$

$f[r, c]$

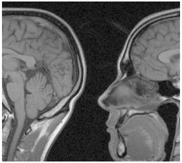
Measure even-numbered columns of $F[k_r, k_c]$. Set pixels in odd numbered columns to the average of adjacent columns. Result? E

A



B

C



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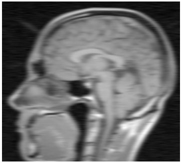
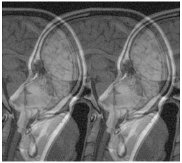
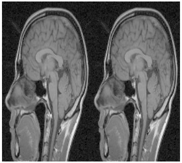


E

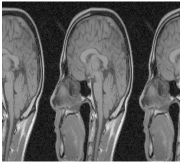
F



G

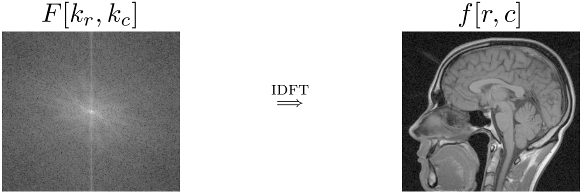


H

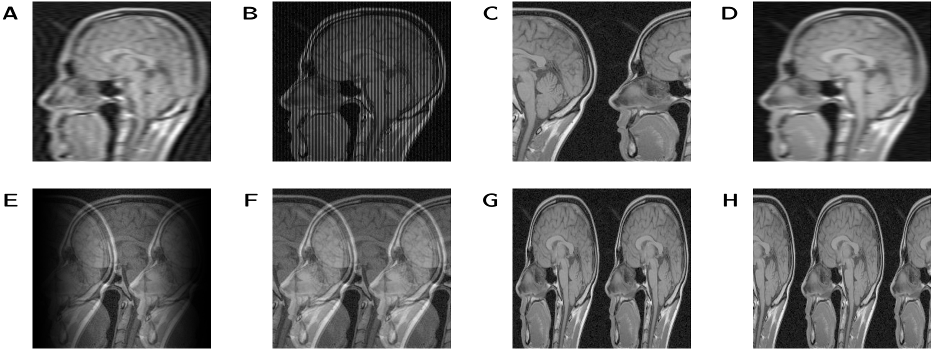


Swapping Places

Measure $F[k_r, k_c]$ and inverse transform to get $f[r, c]$.



What operation would result in panel C?



Swapping Places

What operation would result in panel C?

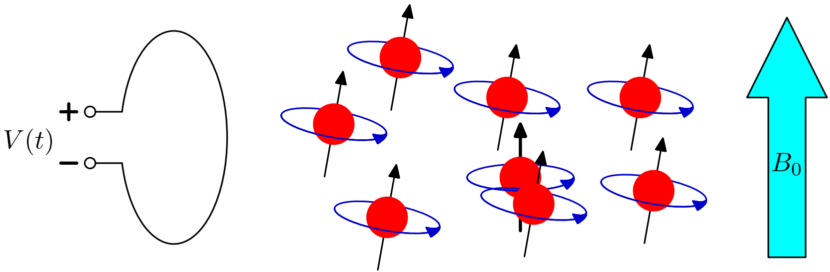
Shift the whole image $f[r, c]$ to the right (or left) by half a frame.

This is equivalent to convolving $f[r, c]$ with $\delta[k_c \pm C/2]$ or by multiplying the original transform data $F[k_r, k_c]$ by $e^{-j\pi k_C} = (-1)^{k_C}$.

This operation is part of the operation that generated panel F (and E). However, this operation requires ALL of the pixels (both even and odd numbered entries). It is not useful by itself for data compression.

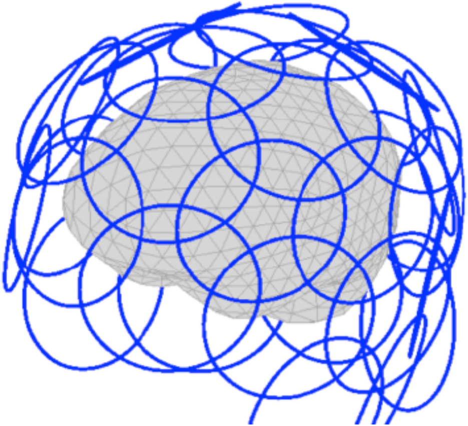
Multi-Coil Imaging

Multiple readout coils can be read in parallel, and thereby provide additional data without increasing imaging time.



Multi-Coil Imaging

“Helmets” with as many as 16 to 32 readout coils have been used to increase the resolution of brain images.



This can be a very effective means of decreasing the time to get an MR image.

Reconstructing Images from Multi-Coil Data

Consider two coils, one on each side of the head. The left coil will be more sensitive to the left portions of the image, and vice versa.

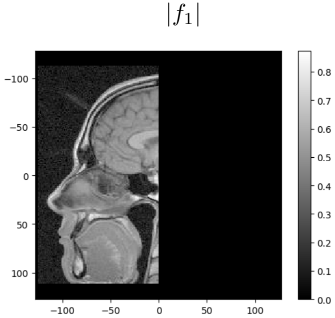
Characterize the **sensitivity** of each coil by specifying a number between 0 (insensitive) and 1 (sensitive) for each pixel in the image.

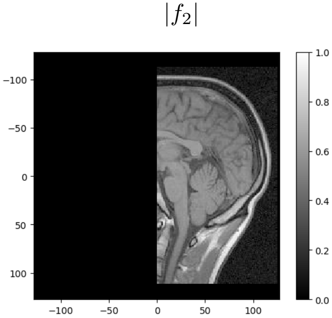
Here is an idealized 1D version with two coils $c_1[c]$ and $c_2[c]$.

What would be the effect of these coils on the resulting image(s)?

Images From Coils 1 and 2

Since coil 1 is only sensitive to half of the head, the image produced with data from coil 1 shows just that half. Similarly, the image produced with data from coil 2 shows just that half.

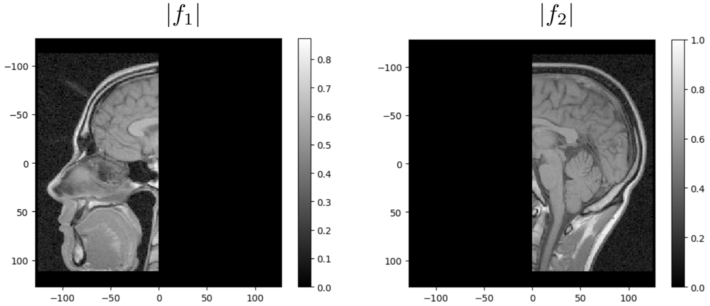




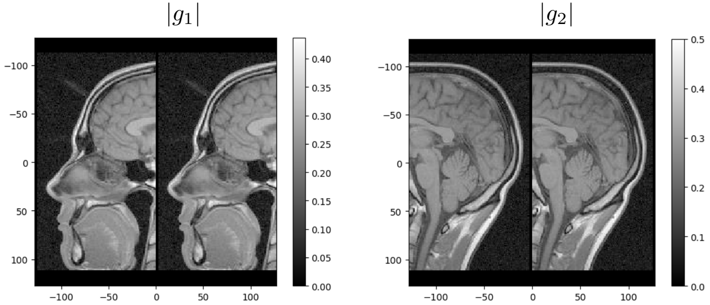
How would these images change if we only measured $F_1[k_r, k_c]$ at even-numbered k_c?

Images From Coils 1 and 2

Images f_1 and f_2 are derived from full-resolution data F_1 and F_2.

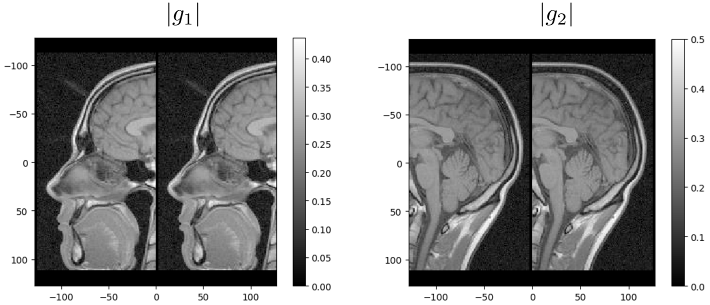


Images g_1 and g_2 are derived from just the even-numbered k_c.



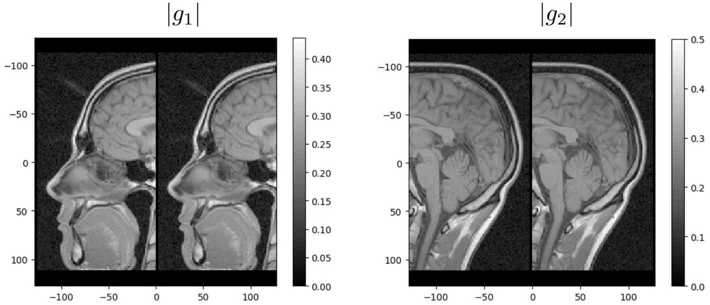
Images From Coils 1 and 2

Could we construct a full-frame, full-resolution image from g_1 and g_2?



Images From Coils 1 and 2

Could we construct a full-frame, full-resolution image from g_1 and g_2?

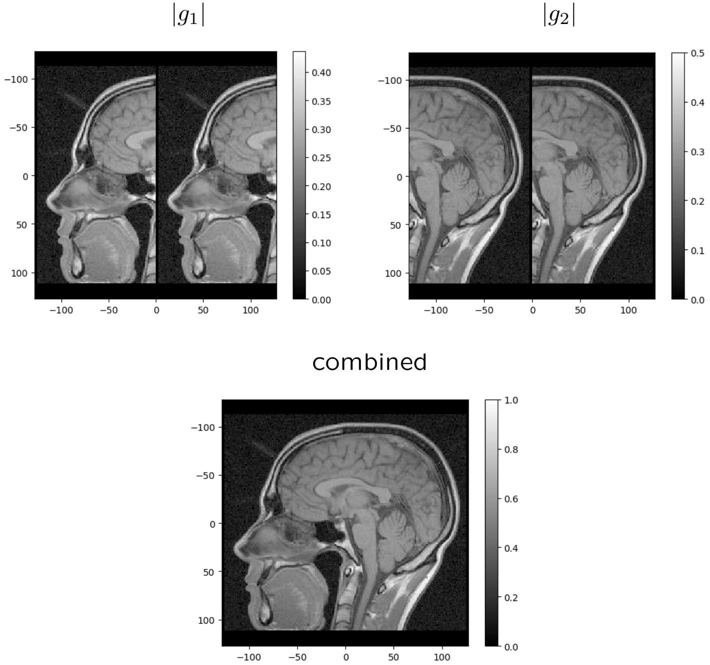


Yes. Combine the left half of the image from coil 1 with the right half of the image from coil 2.

Advantage:
$|g_1|$ was acquired in half the time required for a full-frame full-resolution image. Similar with $|g_2|$.

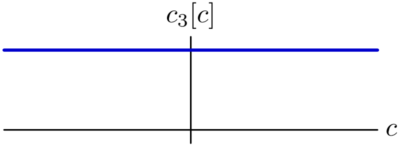
But $|g_1|$ and $|g_2|$ data were acquired simultaneously!

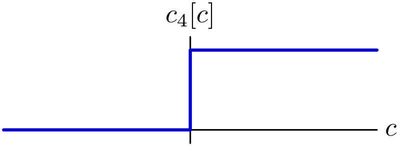
Constructing Full-Frame Image From Coil 1 and 2 Data



Multi-Coil MRI

What if the coils had the following sensitivities?

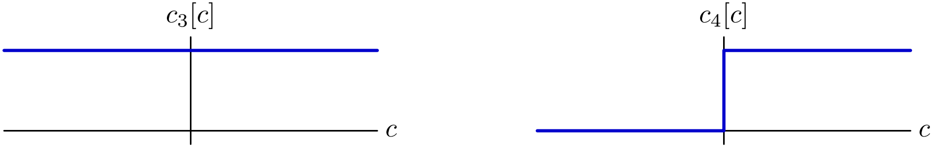




What would be the effect of each of these coils on the image?

Multi-Coil MRI

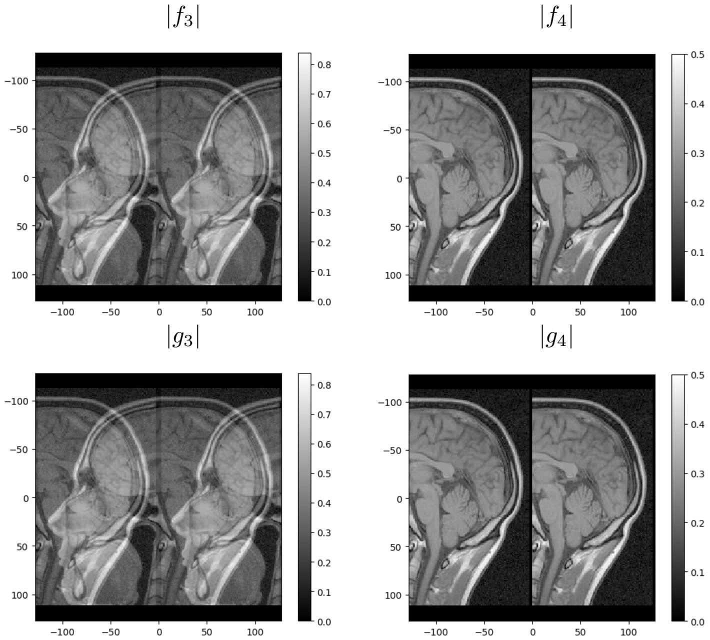
What if the coils had the following sensitivities?



What would be the effect of each of these coils on the image?

c_3 is a full-frame image. Omitting the odd number columns from G_3 will produce the aliased image we started with.
c_4 is the same as the previous c_2, so $|g_4|$ is the same as $|g_2|$.

Images From Coils 3 and 4



Can we create a full-frame full-resolution image from this data?

Images From Coils 3 and 4

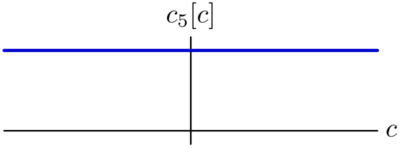
The $|f_3|$ image can be viewed as the sum of results for the left and right sides of the image (as in the c_1 and c_2 example).

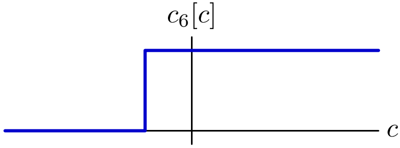
Subtracting $|g_4|$ from $|g_3|$ would generate the previous $|g_1|$ image.

Algorithm:
Combine the left part of $|g_3| - |g_4|$ with the right part of $|g_4|$.

Multi-Coil MRI

What if the coils had the following sensitivities?

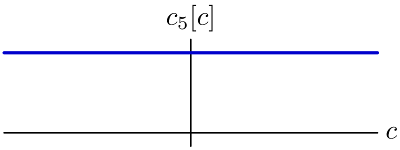


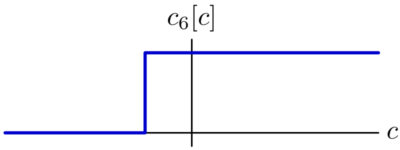


What would be the effect of each of these coils on the image?

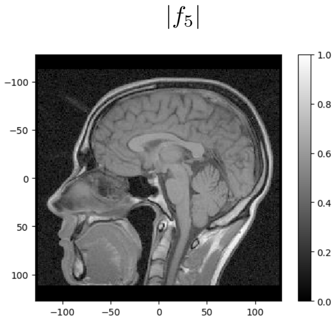
Multi-Coil MRI

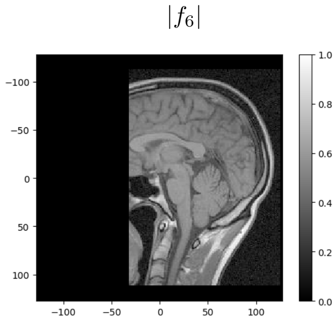
What if the coils had the following sensitivities?





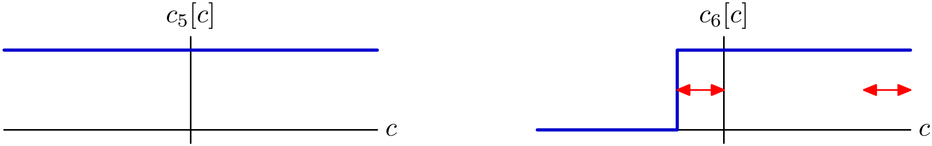
What would be the effect of each of these coils on the image?





Multi-Coil MRI

What if the coils had the following sensitivities?



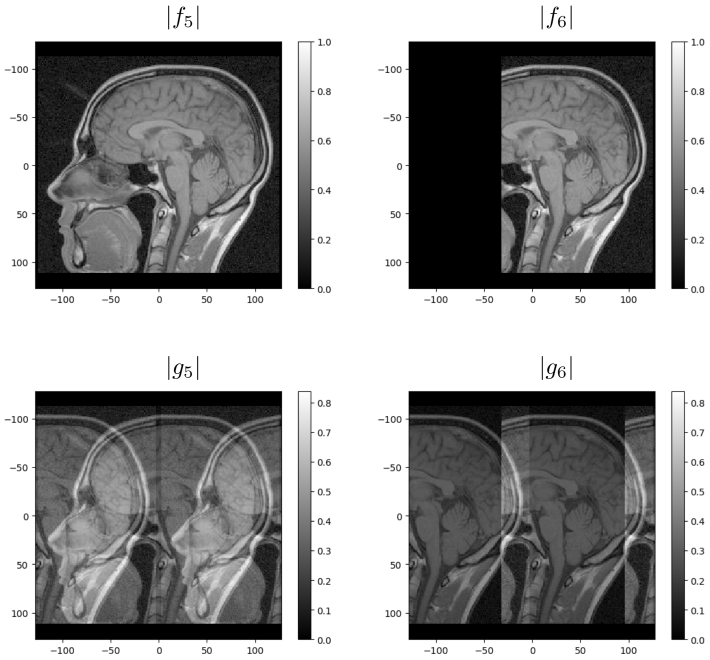
Notice that c_6 weights contributions from pixels in the range $-32 \leq c < 0$ exactly the same as those in $96 \leq c < 128$.

Therefore the c_6 image contains no information that is useful for separating these two bands of pixels.

Similar statements apply for c_5.

Images From Coils 5 and 6

g_5, g_6 are after omitting odd numbered columns from $|f_5|$, $|f_6|$.



Images From Coils 5 and 6

Highlighted regions are identical: both represent sum $f[r,c]+f[r,c+128]$.