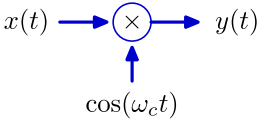
6.3000: Signal Processing

Communications Systems

April 08, 2025

Amplitude Modulation

Multiplication by a sinusoid shifts the frequencies in an input signal.



Compare the frequency representation of the output signal to that of the input signal.

Notice that each frequency in the input is represented twice in the output. (For that reason, this modulation scheme is sometimes called "double side-band."

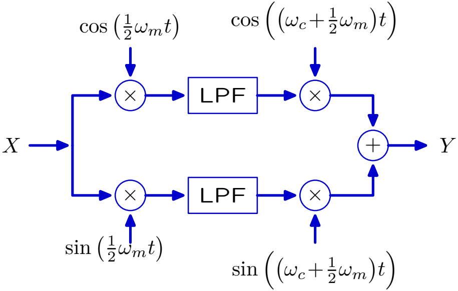
This reduces the number of independent messages that can be transmitted over a particular medium.

Despite this limitation, this scheme is used in commercial AM radio.

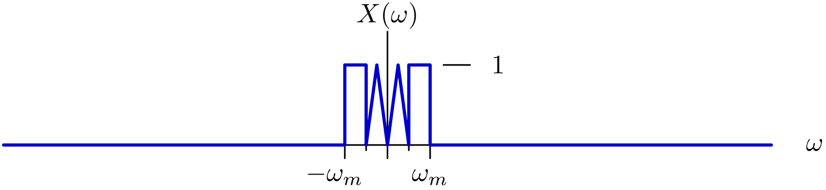
It was originally popular because it was easy to decode.

Bandwidth Conservation

Consider the following modulation scheme, where $\omega_c >> \omega_m$.



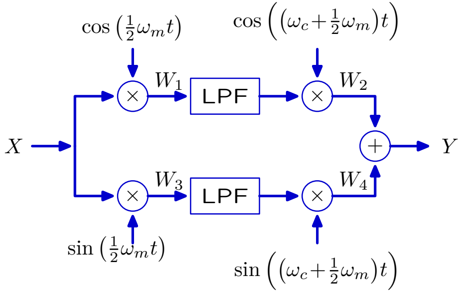
Assume each LPF is ideal, with cutoff frequency $\omega_m/2$, and DC gain of 2. Also assume that the input signal has the following Fourier transform.

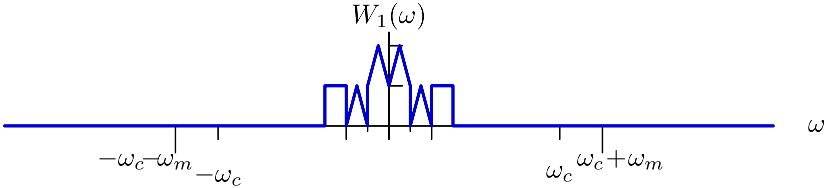


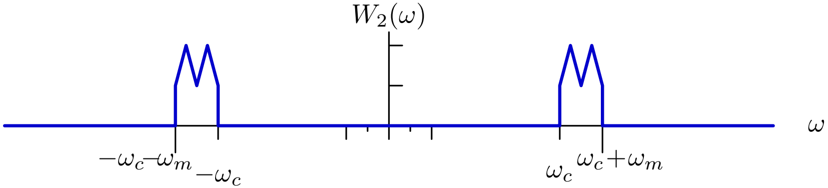
Determine $Y(\omega)$.

Bandwidth Conservation

Consider the following modulation scheme, where $\omega_c >> \omega_m$.







Bandwidth Conservation

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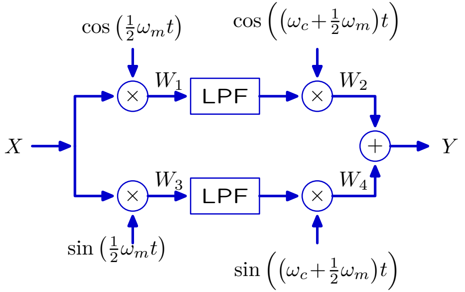
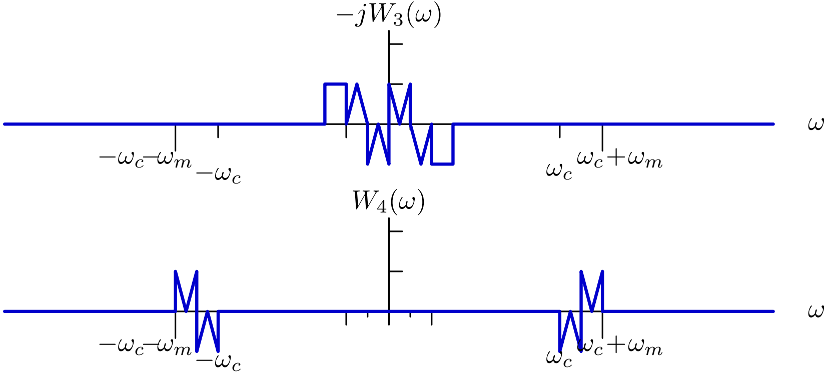
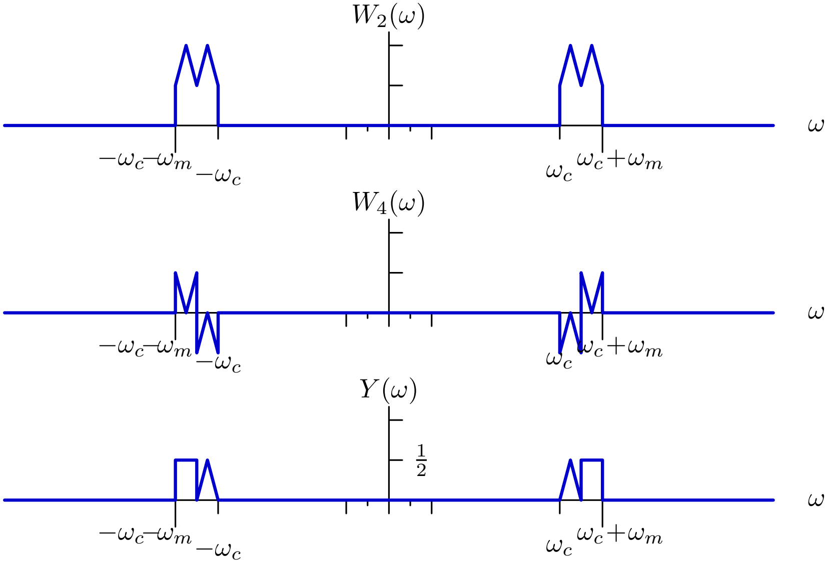




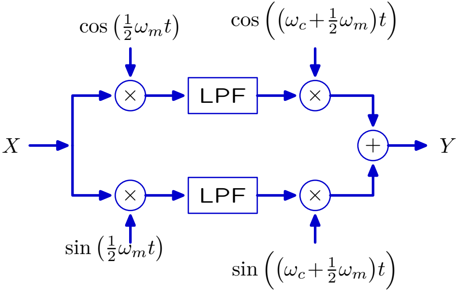
Bandwidth Conservation

Add $W_2(\omega)$ to $W_4(\omega)$ to get $Y(\omega)$.



Bandwidth Conservation

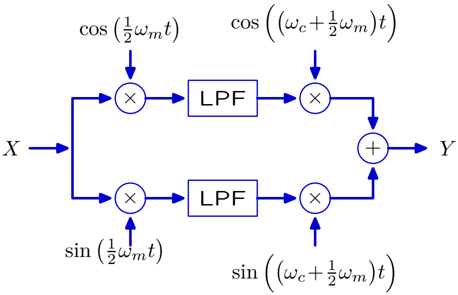
Notice that the frequency components of $Y(\omega)$ are near ω_c (not 0). Thus this scheme could be used to modulate a low frequency signal so that it can be transmitted via a high-frequency medium.



What (if any) advantages would this scheme offer?

Bandwidth Conservation

Notice that the frequency components of $Y(\omega)$ are near ω_c (not 0). Thus this scheme could be used to modulate a low frequency signal so that it can be transmitted via a high-frequency medium.



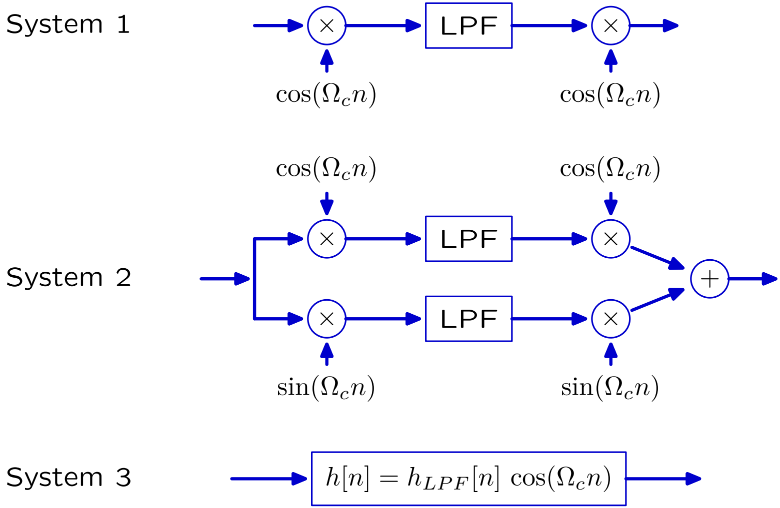
What (if any) advantages would this scheme offer?

This schemes uses just half the high-frequency bandwidth as the simple scheme that multiples by the cosine of the carrier frequency. This would allow twice as many signals to be conveyed simultaneously through a given medium.

Implementing a Bandpass Filter

The critical components of many modulation systems include bandpass filters, which are filters that pass only frequencies in a given range.

Which of following systems implement a bandpass filter?



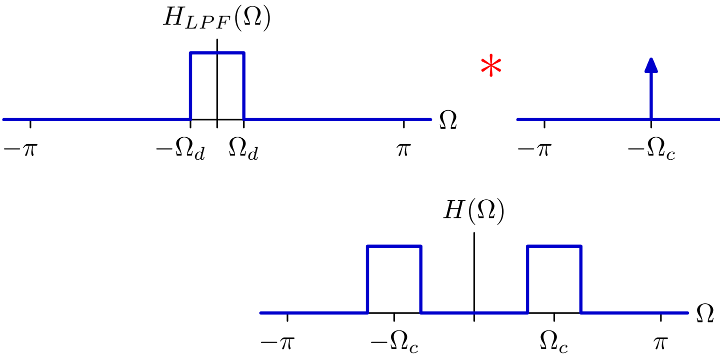
Implementing a Bandpass Filter

Consider system 3.

Since the unit sample response of system 3 is the product of two signals,

$$h[n] = h_{LPF}[n]\cos(\Omega_c n)$$

the Fourier transform of $h[n]$ can be expressed as the convolution of the Fourier transform of h_{LPF} with the Fourier transform of $\cos(\Omega_c n)$.



The result is a bandpass filter.

(Note that all of these DTFT's are periodic in $\Omega = 2\pi$, as are all DTFTs.)

Implementing a Bandpass Filter

We get a different representation of the system by implementing system 3

$$h[n] = h_{LPF}[n]\cos(\Omega_c n)$$

as a convolution:

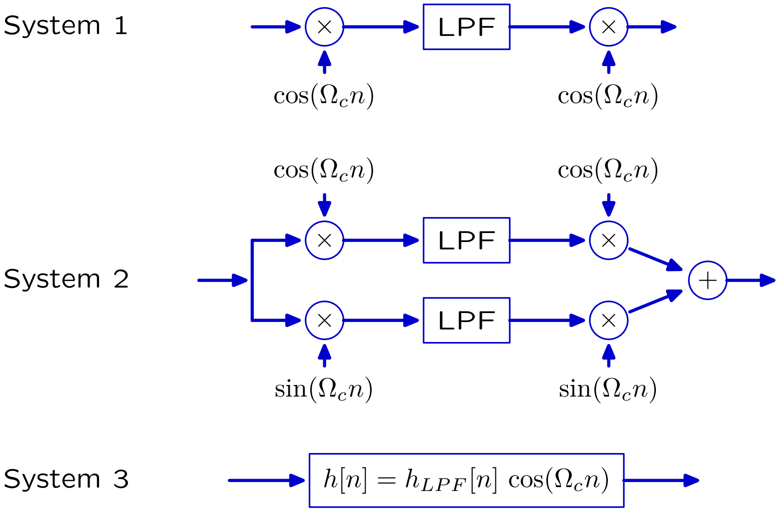
$$\begin{aligned}
y[n] &= (x * h)[n] \\
&= \sum_m x[m] h_{LPF}[n-m]\ \cos(\Omega_c(n-m)) \\
&= \sum_m x[m] h_{LPF}[n-m]\ (\cos(\Omega_c n)\cos(\Omega_c m) + \sin(\Omega_c n)\sin(\Omega_c m)) \\
&= \left(\sum_m x[m]\cos(\Omega_c m)\, h_{LPF}[n-m]\right)(\cos(\Omega_c n)) \\
&\quad + \left(\sum_m x[m]\sin(\Omega_c m)\, h_{LPF}[n-m]\right)(\sin \Omega_c n)) \\
&= \Big((x[n]\cos(\Omega_c n)) * h_{LPF}[n]\Big)\ \cos(\Omega_c n) \\
&\quad + \Big((x[n]\sin(\Omega_c n)) * h_{LPF}[n]\Big)\ \sin(\Omega_c n)
\end{aligned}$$

This formula represents system 2, so system 2 is also a bandpass filter.

Implementing a Bandpass Filter

How about system 1?

Do we really need both the sine and cosine paths in system 2?

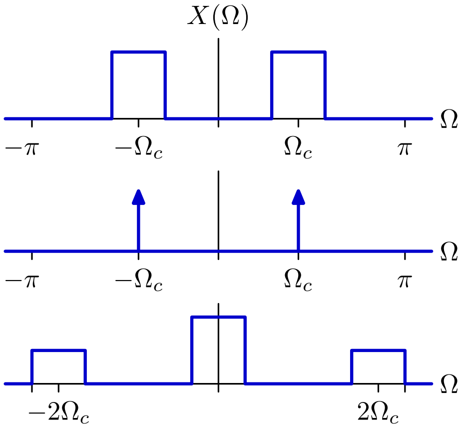


Implementing a Bandpass Filter

How about system 1?

Do we really need both the sine and cosine paths in system 2?

Apply the cosine path to the following input $X(\Omega)$.



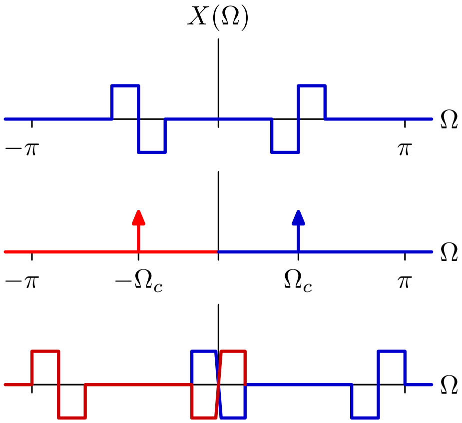
Multiplying $x[n]$ by $\cos(\Omega_c n)$ reproduces the components near Ω_c at $\Omega = 0$. If we then LPF and multiply again by $\cos(\Omega_c n)$, we get back the original $X(\Omega)$. The system works like a bandpass filter.

Implementing a Bandpass Filter

How about system 1?

Do we really need both the sine and cosine paths in system 2?

Now trace a different input $X(\Omega)$ through the cosine branch.



When we multiply by $\cos(\Omega_c n)$, the red and blue parts cancel near $\Omega = 0$.

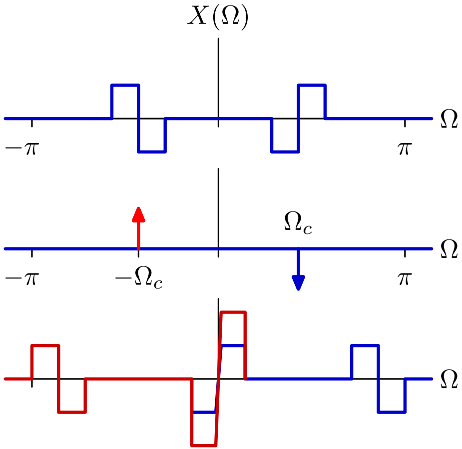
After the second multiplication by $\cos(\Omega_c n)$, the result near Ω_c is zero!

Implementing a Bandpass Filter

How about system 1?

Do we really need both the sine and cosine paths in system 2?

However, the red and blue parts reinforce each other in the sine path.

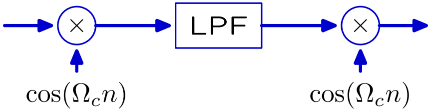


The cosine path preserves the part of the input $X(\Omega)$ that is symmetric about $\Omega = \Omega_c$. The sine path preserves the part of the input $X(\Omega)$ that is antisymmetric about $\Omega = \Omega_c$. Both are needed.

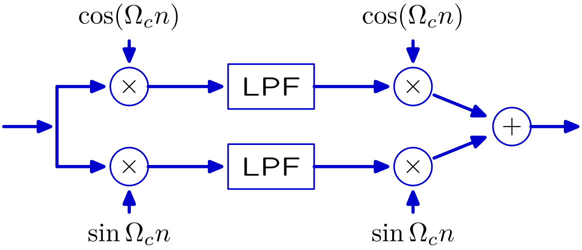
Implementing a Bandpass Filter

Which of following systems implement a bandpass filter?

System 1



System 2



System 3