6.3000: Signal Processing

2D Convolution

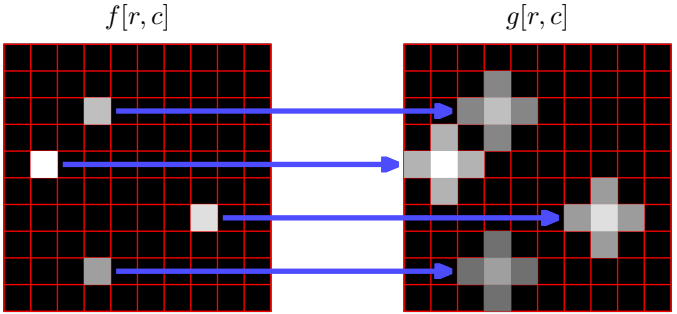
April 24, 2025

2D Convolution

We can represent a system that is linear and shift-invariant by its unit-sample response (its response to a unit-sample signal):

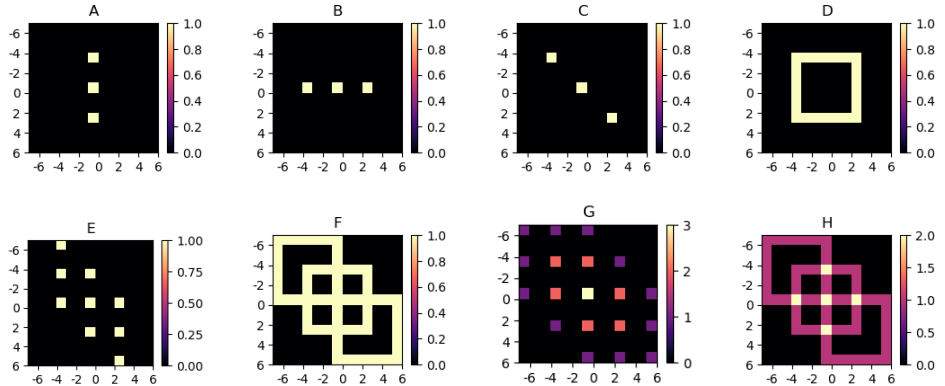
$$\delta[r, c] \rightarrow h[r, c]$$

The response of such a system to an input $f[r, c]$ is the superposition of shifted and scaled versions of the unit-sample response.



2D Convolution

Which of the following images can be constructed by convolving two of the other images?



2D Convolution

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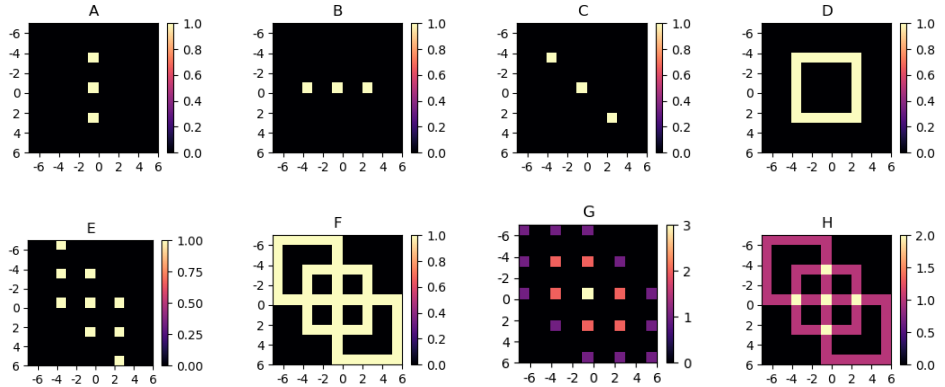


$E = A * C$

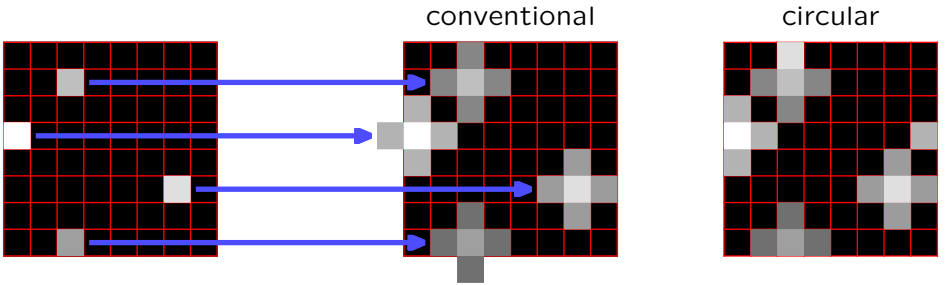
$G = B * E$

$H = C * D$

2D Circular Convolution

Convolution in space is equivalent to multiplication of DTFT's.

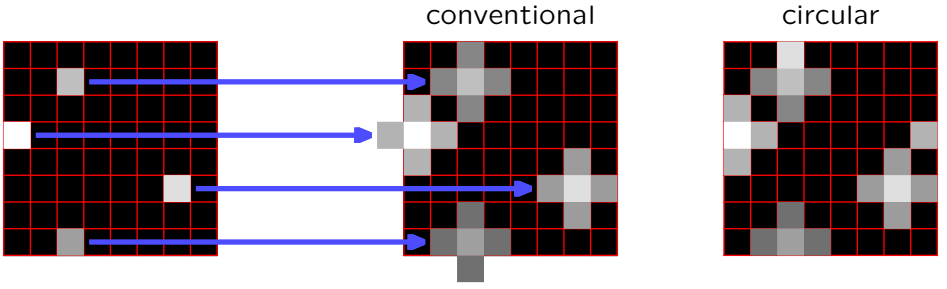
However, multiplication of DFT's (or DTFS's) is equivalent to **circular convolution** in space.



The domains of the input and output signals are limited by the dimensions of the DFTs.

2D Circular Convolution

Two perspectives.

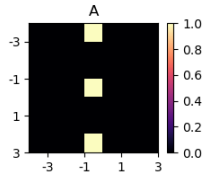


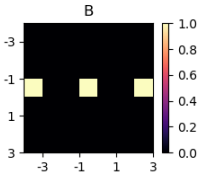
Focusing on the output: If part of the output image falls outside the region, move it back into the region by shifting that part by an integer number of widths or heights.

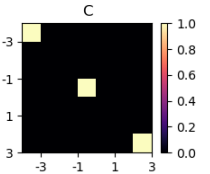
Focusing on the input: Start by periodically extending the input by repeating the region of interest to tile the entire plane. Then do conventional convolution.

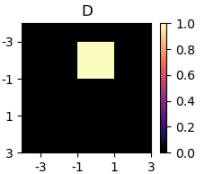
2D Circular Convolution

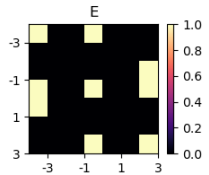
Which of the following images can be constructed by circularly convolving two of the other images?

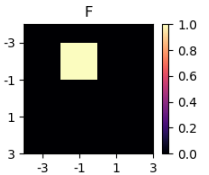


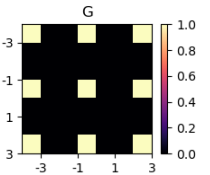


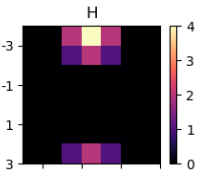












2D Circular Convolution

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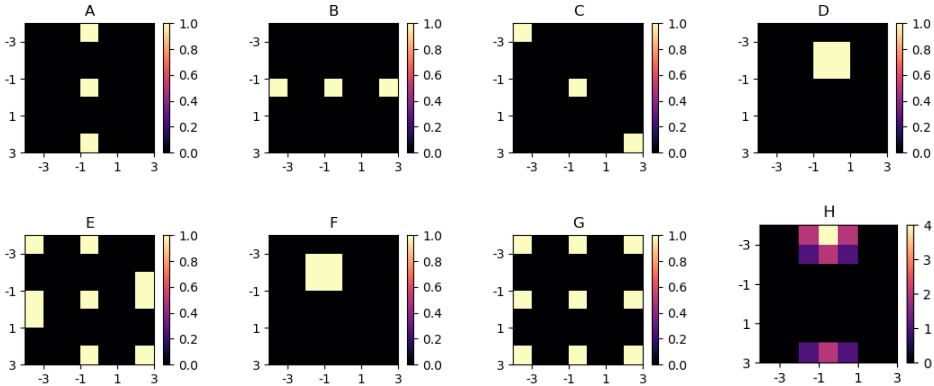


$E = A \circledast C$

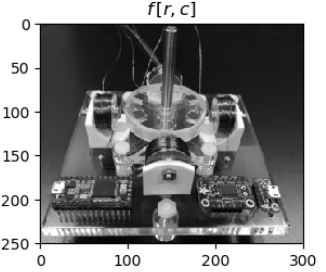
$G = A \circledast B$

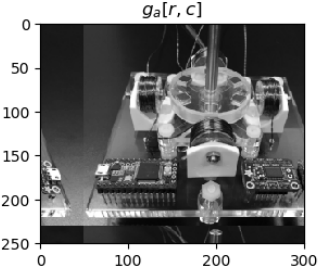
$H = D \circledast F$

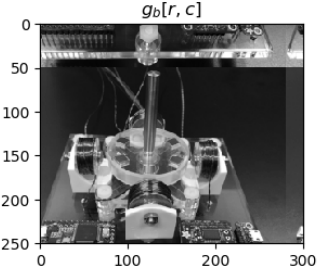
Picture Convolutions: Match an Expression to Each Image

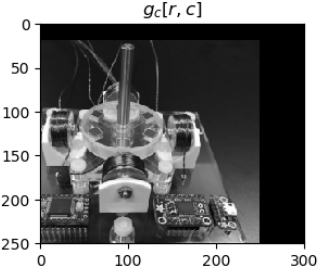
$$g_1[r,c] = f[r,c] * \delta[r-20, c+50]$$
$$g_2[r,c] = f[r,c] * \delta[r+20, c-50]$$
$$g_3[r,c] = f[r,c] * \delta[r-50, c+20]$$

$$g_4[r,c] = f[r,c] \circledast \delta[r-20, c+50]$$
$$g_5[r,c] = f[r,c] \circledast \delta[r+20, c-50]$$
$$g_6[r,c] = f[r,c] \circledast \delta[r-50, c+20]$$









Picture Convolutions: Match an Expression to Each Image

$$g_c[r,c] = f[r,c] * \delta[r-20, c+50]$$
$$g_2[r,c] = f[r,c] * \delta[r+20, c-50]$$
$$g_3[r,c] = f[r,c] * \delta[r-50, c+20]$$

$$g_4[r,c] = f[r,c] \circledast \delta[r-20, c+50]$$
$$g_a[r,c] = f[r,c] \circledast \delta[r+20, c-50]$$
$$g_b[r,c] = f[r,c] \circledast \delta[r-50, c+20]$$