6.3000 Final Exam Spring 2025

Name:

Kerberos (Athena) Username:

Please WAIT until we tell you to begin.

This exam is closed book, but you may use three 8.5×11 sheets of notes (six sides).

You may NOT use any electronic devices (such as calculators and phones).

If you have questions, please **come to us** at the front of the room to ask.

Please enter all solutions in the boxes provided.

Work on other pages with QR codes will be considered for partial credit.
Please provide a note if you continue work on worksheets at the end of the exam.

Please do not write on the QR codes at the bottom of each page.

We use those codes to identify which pages belong to each student.

Trigonometric Identities Reference

$$\cos(a+b) = \cos(a)\cos(b) - \sin(a)\sin(b)$$

$$\cos(a-b) = \cos(a)\cos(b) + \sin(a)\sin(b)$$

$$\sin(a+b) = \sin(a)\cos(b) + \cos(a)\sin(b)$$

$$\sin(a-b) = \sin(a)\cos(b) - \cos(a)\sin(b)$$

$$\cos(a) + \cos(b) = 2\cos\left(\frac{a+b}{2}\right)\cos\left(\frac{a-b}{2}\right)$$

$$\cos(a) - \cos(b) = -2\sin\left(\frac{a+b}{2}\right)\sin\left(\frac{a-b}{2}\right)$$

$$\sin(a) + \sin(b) = 2\sin\left(\frac{a+b}{2}\right)\cos\left(\frac{a-b}{2}\right)$$

$$\sin(a) - \sin(b) = 2\cos\left(\frac{a+b}{2}\right)\sin\left(\frac{a-b}{2}\right)$$

$$\cos(a+b)+\cos(a-b) = 2\cos(a)\cos(b)$$

$$\cos(a+b)-\cos(a-b) = -2\sin(a)\sin(b)$$

$$\sin(a+b)+\sin(a-b) = 2\sin(a)\cos(b)$$

$$\sin(a+b)-\sin(a-b) = 2\cos(a)\sin(b)$$

$$2\cos(a)\cos(b) = \cos(a-b)+\cos(a+b)$$

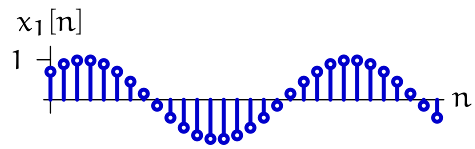
$$2\sin(a)\sin(b) = \cos(a-b)-\cos(a+b)$$

$$2\sin(a)\cos(b) = \sin(a+b)+\sin(a-b)$$

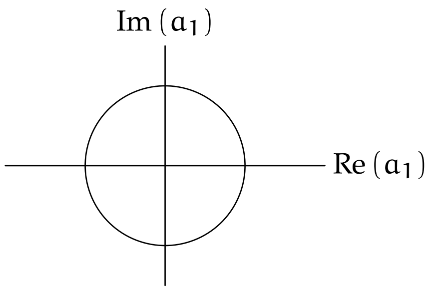
$$2\cos(a)\sin(b) = \sin(a+b)-\sin(a-b)$$

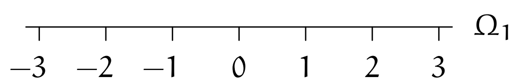
1 Describing Sinusoids (11 points)

Part a. Let $x_1[n] = a_1 e^{j\Omega_1 n} + a_1^* e^{-j\Omega_1 n}$ as shown in the following figure.

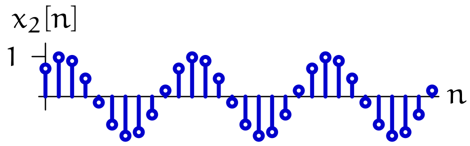


Estimate a_1 and Ω_1, where Ω_1 is real-valued and a_1 may be complex. Place an "×" on the complex plane shown below (where the circle has a radius of 1) to indicate the value of a_1. Also, place an "×" on the number line shown below to indicate the value of Ω_1.

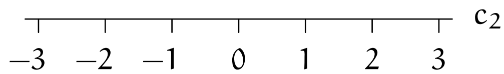


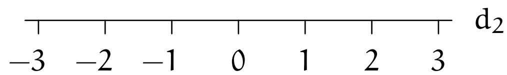


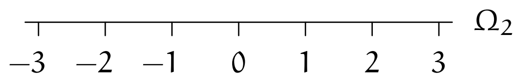
Part b. Let $x_2[n] = c_2 \cos(\Omega_2 n) + d_2 \sin(\Omega_2 n)$ as shown in the following figure.



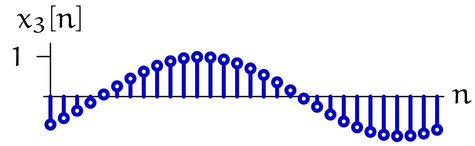
Estimate the real-valued constants c_2, d_2, and Ω_2. Place an "×" on each of the number lines shown below to indicate these values.



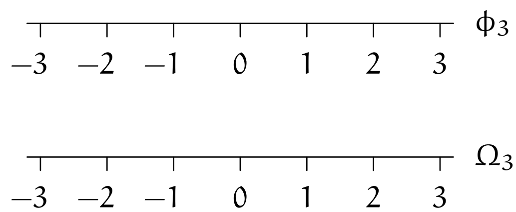




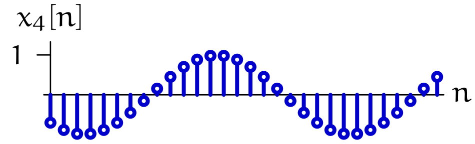
Part c. Let $x_3[n] = \cos(\Omega_3 n + \phi_3)$ as shown in the following figure.



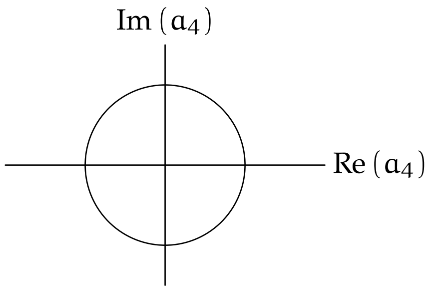
Estimate the real-valued constants ϕ_3 and Ω_3. Place an "$\times$" on each of the number lines shown below to indicate these values.

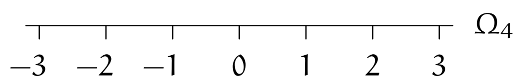


Part d. Let $x_4[n] = \text{Re}\left(a_4 e^{j\Omega_4 n}\right)$ as shown in the following figure.



Estimate a_4 and Ω_4, where Ω_4 is real-valued and a_4 may be complex. Place an "×" on the complex plane shown below (where the circle has a radius of 1) to indicate the value of a_4. Also, place an "×" on the number line shown below to indicate the value of Ω_4.





2 Periodic Extension (20 points)

Let $f[n]$ represent a real-valued, discrete-time signal, and let $g[n]$ represent a periodically extended version of $f[n]$, as follows:

$$g[n] = \begin{cases} f[n] & \text{if } 0 \leqslant n < N \\ f[n{-}N] & \text{if } N \leqslant n < 2N \\ f[n{-}2N] & \text{if } 2N \leqslant n < 3N \\ f[n{-}mN] & \text{if } mN \leqslant n < (m{+}1)N \end{cases}$$

where m represents an integer that is greater than or equal to 3.

Let $F[k]$ represent the DFT of the first N samples of $f[n]$.

Let $G[k]$ represent the DFT of the first $2N$ samples of $g[n]$.

Part a. Determine an expression for $G[0]$ in terms of the DFT coefficients $F[k]$ as well as familiar constants such as π and e. Simplify your expression as much as possible, and enter your result in the box below.

$G[0] =$ [ ]

Part b. Determine an expression for $G[1]$ in terms of the DFT coefficients $F[k]$ as well as familiar constants such as π and e. Simplify your expression as much as possible, and enter your result in the box below.

$G[1] =$ [ ]

Part c. Determine an expression for $G[k]$ in terms of the DFT coefficients $F[k]$ as well as familiar constants such as π and e. Simplify your expression as much as possible, and enter your result in the box below.

$G[k] =$

3 Impulsive Images (15 points)

Pixel values for three images ($f_1[r,c]$, $f_2[r,c]$, and $f_3[r,c]$) are shown below for $|r| \leqslant 2$ and $|c| \leqslant 2$. Pixel values outside the indicated regions are zero.

$f_1[r,c]$

r \ c	−2	−1	0	1	2
−2	0	0	0	1	0
−1	0	0	0	0	0
0	0	0	0	0	0
1	0	0	0	0	0
2	0	1	0	0	0

$\left|F_1[k_r,k_c]\right| =$ ☐

$f_2[r,c]$

r \ c	−2	−1	0	1	2
−2	0	0	0	0	0
−1	0	−1	0	1	0
0	0	−2	0	2	0
1	0	−1	0	1	0
2	0	0	0	0	0

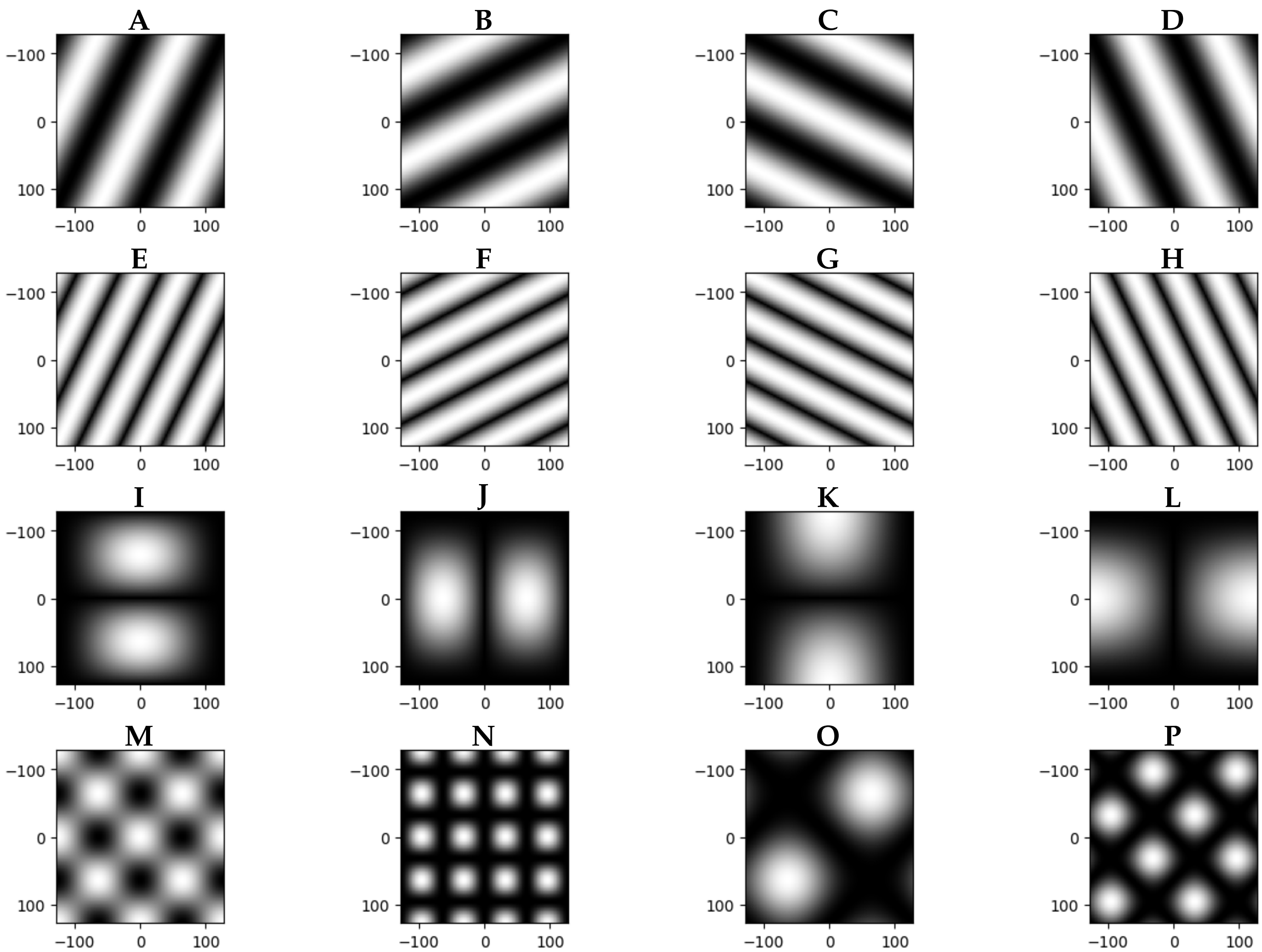
$\left|F_2[k_r,k_c]\right| =$ ☐

$f_3[r,c]$

r \ c	−2	−1	0	1	2
−2	0	0	1	0	0
−1	0	0	0	0	0
0	−1	0	4	0	1
1	0	0	0	0	0
2	0	0	−1	0	0

$\left|F_3[k_r,k_c]\right| =$ ☐

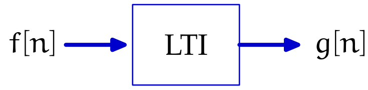
Each of the panels below show the magnitude of a 2D DFT calculated for $-128 \leqslant r, c < 128$ and displayed with black and white representing the minimum and maximum magnitude in each image. Determine which of the panels below shows the magnitude of the 2D DFT for each of the images shown above, and enter its label (A-P) in the corresponding box above.



Worksheet (intentionally blank)

4 Recurring Delay (22 points)

A discrete-time, linear, time-invariant system has input $f[n]$ and output $g[n]$



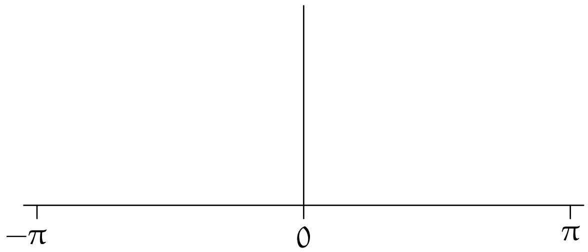
and these signals are related by the following difference equation:

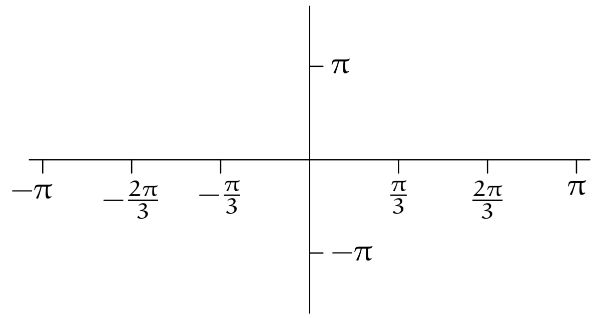
$$g[n] = f[n] - \frac{1}{2} g[n-3]$$

Part a. Assume that the output $g[n] = 0$ for $n < 0$, and find the output of the system when the input $f[n] = \delta[n]$. Enter an expression for your result in the following box.

$g[n] =$

Part b. Sketch the frequency response of this system on the axes below, with magnitude on the left and phase on the right. Label all of the important points on your sketches.





5 Tetra (16 points)

Let $f[r,c]$ represent the following image as an array of pixel brightnesses in the range $[-\frac{1}{2},\frac{1}{2}]$ where black corresponds to $-1/2$, white corresponds to $+1/2$, and the background grey corresponds to 0. The pixels are indexed by their row number r (with $-\frac{1}{2}R \leqslant r < \frac{1}{2}R$) and column number c (with $-\frac{1}{2}C \leqslant c < \frac{1}{2}C$), and $R = C = 200$.

Note: High-quality images of this figure as well as those on the following page have been provided on a separate sheet. Use the high-quality images to answer the questions below, but record your answers on this page (which has QR codes).

Also consider four additional images, two of which are defined in the spatial domain:

$$h_1[r,c] = \sin\left(\frac{40\pi r}{R}\right)$$

$$h_2[r,c] = \begin{cases} 1 & \text{if } r \text{ is even} \\ 0 & \text{otherwise} \end{cases}$$

and two defined by their two-dimensional Fourier transforms:

$$H_3[k_r,k_c] = j\sin\left(\frac{40\pi k_r}{R}\right)$$

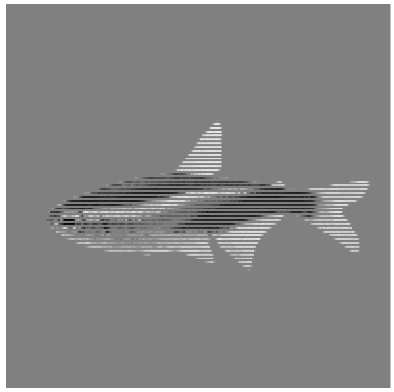
$$H_4[k_r,k_c] = \begin{cases} 1 & \text{if } k_r \text{ is even} \\ 0 & \text{otherwise} \end{cases}$$

Determine which of the images on the next page corresponds to each of the following image combinations, and enter the appropriate label (A-L) in each of the boxes below.

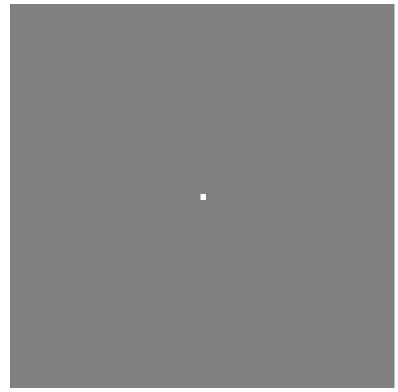
$(f \times h1)[r,c]$		$(f \circledast h1)[r,c]$	
$(f \times h2)[r,c]$		$(f \circledast h2)[r,c]$	
$(f \times h3)[r,c]$		$(f \circledast h3)[r,c]$	
$(f \times h4)[r,c]$		$(f \circledast h4)[r,c]$	

A:

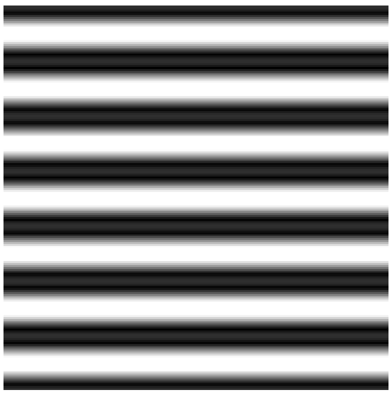
B:



C:



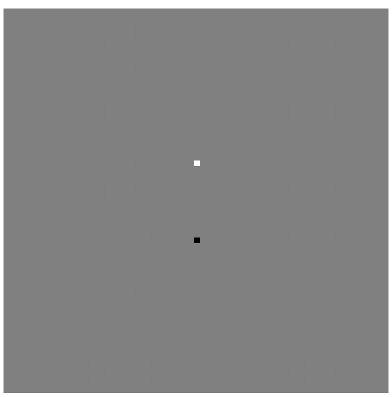
D:



E:

F:

G:



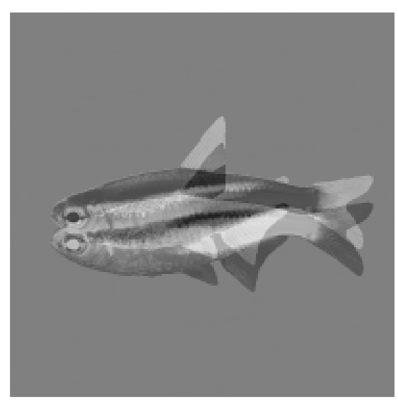
H:

I:

J:

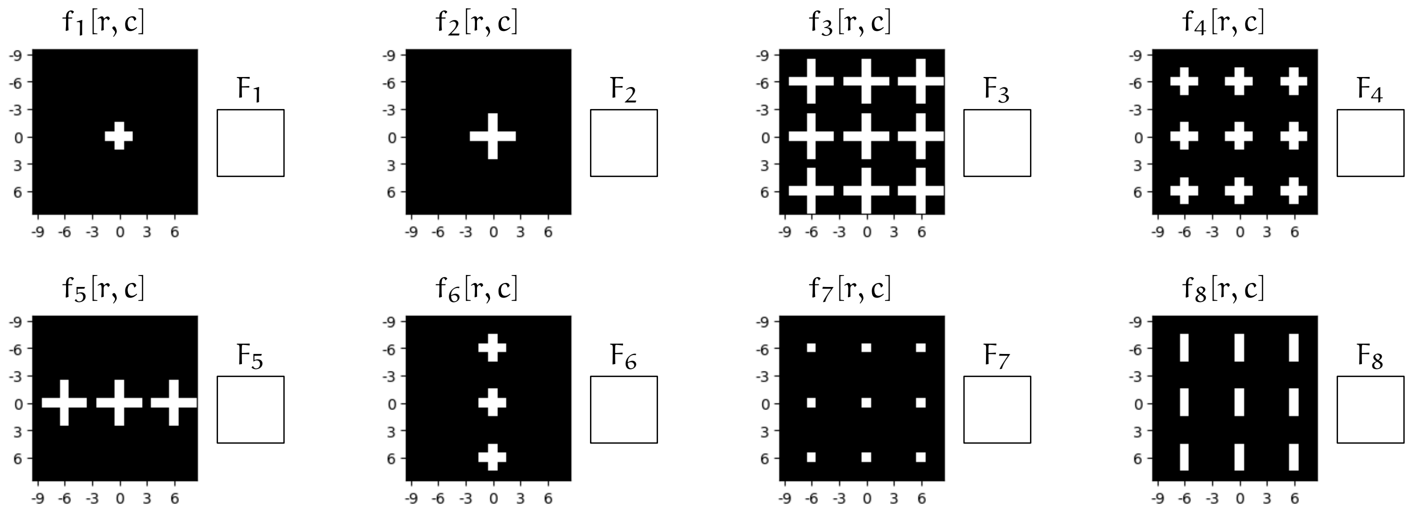
K:

L:

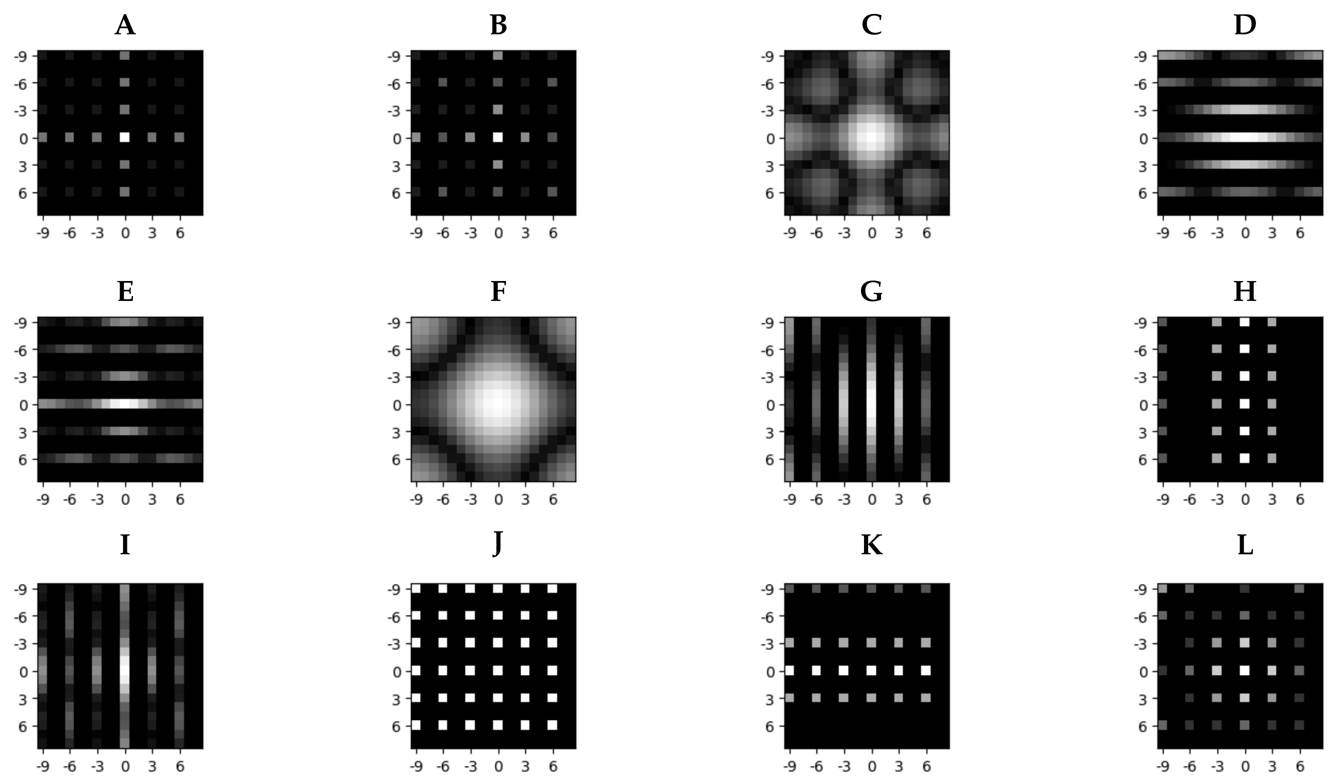


6 Pluses and Minuses (16 points)

The panels below show eight images ($f_1[r, c]$ to $f_8[r, c]$ with black (0) or white (1) pixels that are indexed by row number r and column number c where $-9 \leqslant r < 9$ and $-9 \leqslant c < 9$.



Each of the following panels shows the magnitude of a 2D DFT. For each panel, black represents a value of 0 and white represents the largest magnitude in that panel (which may be different for each panel). Determine which of the panels shows the magnitude of the 2D DFT of each of the images above, and enter the corresponding letter (A-L) in the appropriate box.



Worksheet (intentionally blank)

Worksheet (intentionally blank)

Worksheet (intentionally blank)

Worksheet (intentionally blank)

Worksheet (intentionally blank)

Worksheet (*intentionally blank*)