6.3000 Quiz 1 Spring 2025

Name:

Kerberos (Athena) username:

Please WAIT until we tell you to begin.

This quiz is closed book, but you may use one 8.5 × 11 sheet of notes (both sides).

You may NOT use any electronic devices (such as calculators and phones).

If you have questions, please **come to us** at the front of the room to ask.

Please enter all solutions in the boxes provided.

Work on other pages with QR codes will be considered for partial credit.
Please provide a note if you continue work on worksheets at the end of the exam.

Please do not write on the QR codes at the bottom of each page.

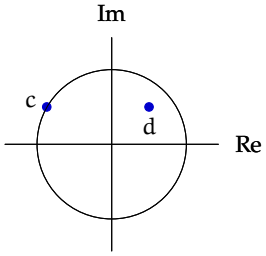
We use those codes to identify which pages belong to each student.

Trigonometric Identities Reference

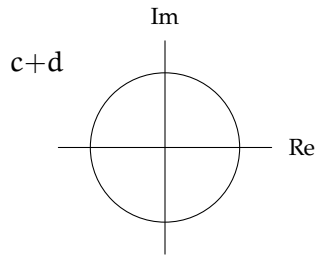
$$\cos(a+b) = \cos(a)\cos(b) - \sin(a)\sin(b)$$
$$\cos(a-b) = \cos(a)\cos(b) + \sin(a)\sin(b)$$
$$\sin(a+b) = \sin(a)\cos(b) + \cos(a)\sin(b)$$
$$\sin(a-b) = \sin(a)\cos(b) - \cos(a)\sin(b)$$
$$\cos(a) + \cos(b) = 2\cos\left(\frac{a+b}{2}\right)\cos\left(\frac{a-b}{2}\right)$$
$$\cos(a) - \cos(b) = -2\sin\left(\frac{a+b}{2}\right)\sin\left(\frac{a-b}{2}\right)$$
$$\sin(a) + \sin(b) = 2\sin\left(\frac{a+b}{2}\right)\cos\left(\frac{a-b}{2}\right)$$
$$\sin(a) - \sin(b) = 2\cos\left(\frac{a+b}{2}\right)\sin\left(\frac{a-b}{2}\right)$$
$$\cos(a+b)+\cos(a-b) = 2\cos(a)\cos(b)$$
$$\cos(a+b)-\cos(a-b) = -2\sin(a)\sin(b)$$
$$\sin(a+b)+\sin(a-b) = 2\sin(a)\cos(b)$$
$$\sin(a+b)-\sin(a-b) = 2\cos(a)\sin(b)$$
$$2\cos(a)\cos(b) = \cos(a-b)+\cos(a+b)$$
$$2\sin(a)\sin(b) = \cos(a-b)-\cos(a+b)$$
$$2\sin(a)\cos(b) = \sin(a+b)+\sin(a-b)$$
$$2\cos(a)\sin(b) = \sin(a+b)-\sin(a-b)$$

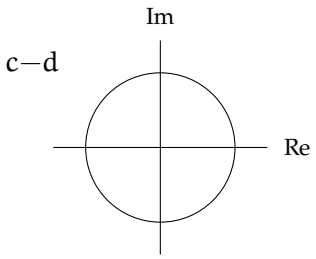
1 Complex Pair (24 points)

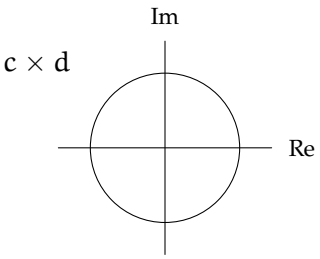
Let c and d represent the complex numbers shown by filled dots in the following diagram, where the real and imaginary parts of the complex numbers are shown on the horizontal and vertical axes, respectively, and the circle has a radius of 1.

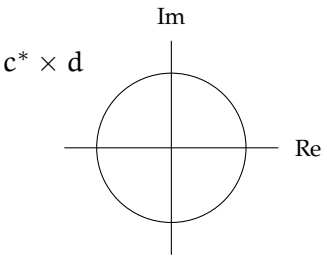


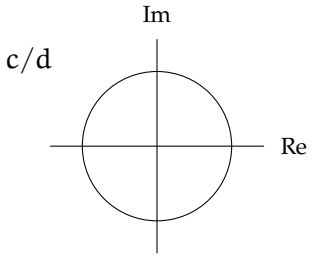
Below are eight complex-valued functions of c and d, each paired with a depiction of the complex plane demarked by the unit circle. Evaluate each expression and mark its value on the complex plane with a dot. Note that e represents Euler's number (2.71828...) and c^* represents the complex conjugate of c.

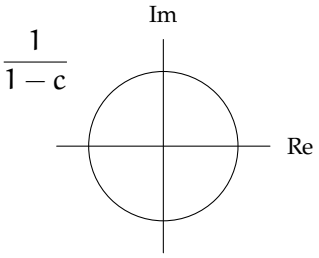


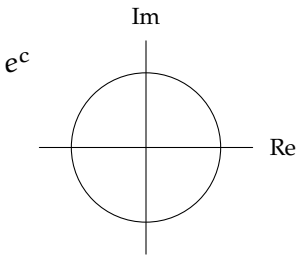


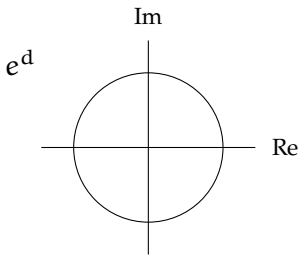












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2 Complex Exponentials (18 points)

For each of the following parts, find a complex number that satisfies the given constraint. You need only provide one possible answer, even if multiple answers exist. If there are no possible answers, write **none** in the answer box.

Part a. Find a complex constant c_1 so that $\text{Re}\left(c_1 e^{j\omega t}\right) = \sin(\omega t)$ for all real numbers ω and t.

$c_1 =$

Part b. Find a complex constant c_2 so that $\text{Im}\left(c_2 e^{j\omega t}\right) = \cos(\omega t)$ for all real numbers ω and t.

$c_2 =$

Part c. Find a complex constant c_3 so that $\text{Re}\left(c_3 e^{j\omega t}\right) = \cos(\omega t) + \sin(\omega t)$ for all real numbers ω and t.

$c_3 =$

Part d. Find a complex constant c_4 so that $\text{Re}\left(c_4 e^{j\omega t}\right) = A\cos(\omega t) + B\sin(\omega t)$ for all real numbers ω and t.

$c_4 =$

Part e. Find a complex constant c_5 so that $\text{Re}\left(c_5 e^{j\omega t}\right) = \cos(\omega t - \phi)$ for all real numbers ω and t.

$c_5 =$

Part f. Find a complex constant c_6 so that $\text{Re}\left(c_6 e^{j\omega t}\right) = \text{Im}\left(c_6^* e^{j\omega t}\right)$ for all real numbers ω and t.

$c_6 =$

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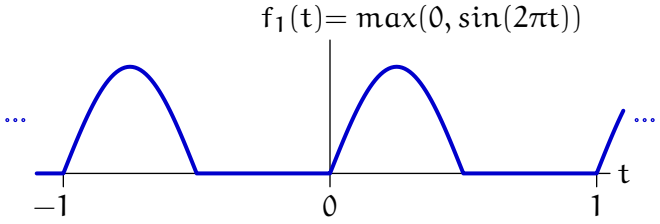
3 Peaks and Valleys (26 Points)

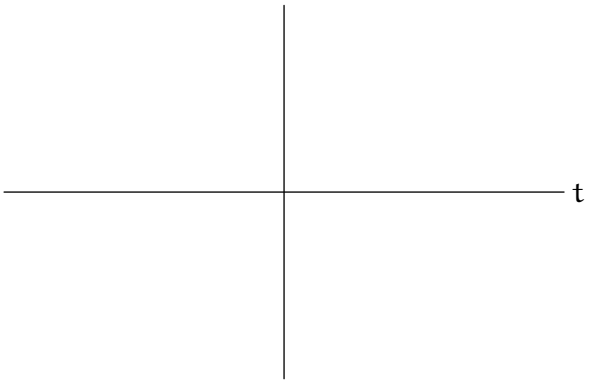
Let $f_1(t)$ represent the following periodic, continuous-time signal, with period $T = 1$:



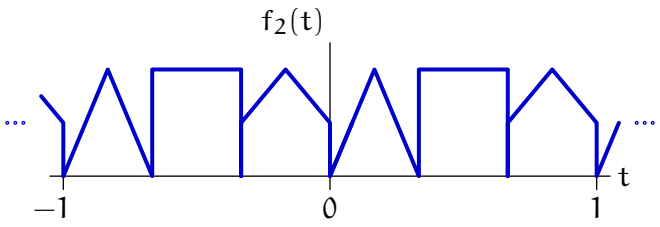
Let $g_1(t) = 1 - f_1\left(3t - \frac{1}{4}\right)$.

Sketch $g_1(t)$ on the following axes. Label the important parameters of your plot.

$$g_1(t) = 1 - f_1\left(3t - \frac{1}{4}\right)$$



Let $f_2(t)$ represent the following periodic, continuous-time signal with period $T = 1$:



Let $g_2(t) = 1 - f_2\left(3t - \frac{1}{4}\right)$.

Let $F_2[k]$ and $G_2[k]$ represent the Fourier series coefficients for $f_2(t)$ and $g_2(t)$, respectively, where both series are computed **with the same period** $T = 1$. Determine expressions for each of $G_2[0]$ through $G_2[15]$ in terms of the Fourier coeeficients $F_2[k]$. Your table entries can contain real and/or imaginary numbers and constants such as e and π. Your entries should not contain integrals or infinite sums.

k	$G_2[k]$
0	
1	
2	
3	
4	
5	
6	
7	

k	$G_2[k]$
8	
9	
10	
11	
12	
13	
14	
15	

4 Fourier Series Matching (32 Points)

Each of the signals $x_i[n]$ in the left column below is periodic with period $N = 16$. Find the Fourier series coefficients $X_i[k]$ for each signal and then identify which of plots M1 – M8 shows the magnitude of $X_i[k]$ and which of plots A1 – A8 shows the angle of $X_i[k]$ as functions of k. Enter your answers in the boxes provided.

$x_1[n]$ M: A:

$x_2[n]$ M: A:

$x_3[n]$ M: A:

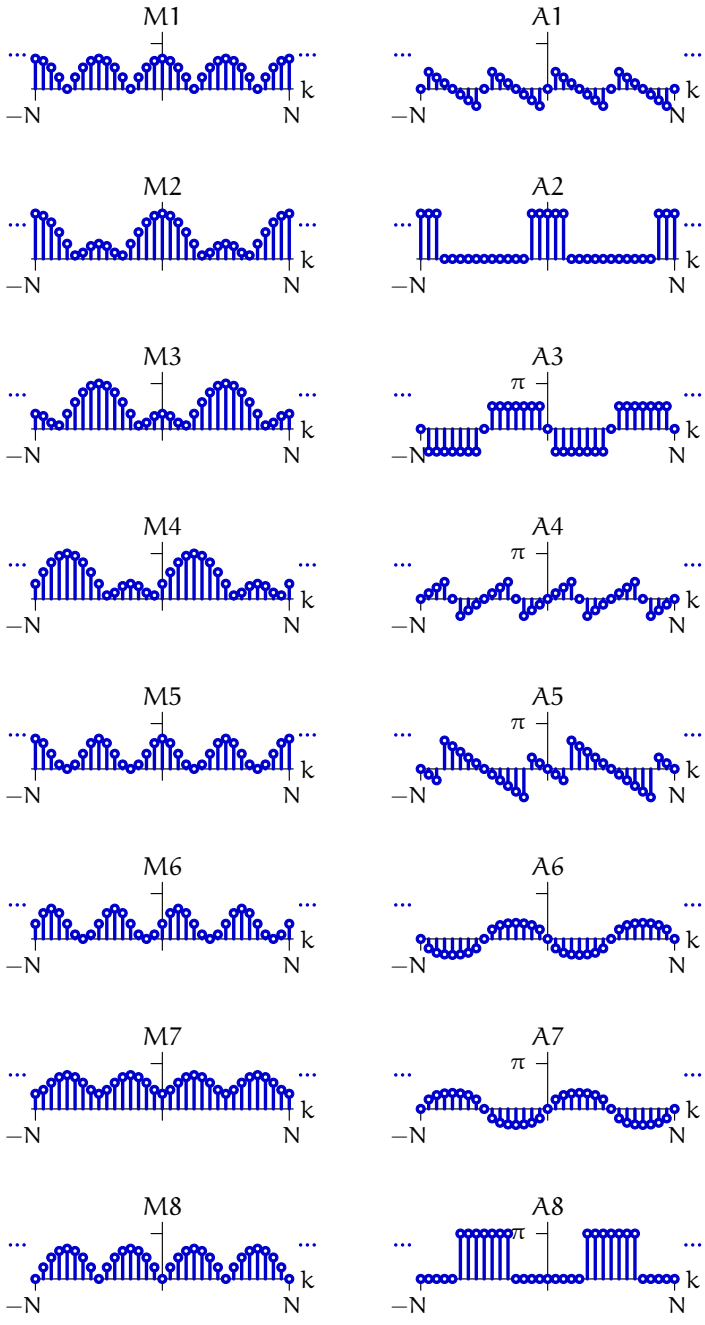
$x_4[n]$ M: A:

$x_5[n]$ M: A:

$x_6[n]$ M: A:

$x_7[n]$ M: A:

$x_8[n]$ M: A:



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