6.3000 Quiz 2 Fall 2023

Name:

Kerberos (Athena) username:

Please WAIT until we tell you to begin.

This quiz is closed book, but you may use two 8.5×11 sheets of notes (four sides).

You may NOT use any electronic devices (such as calculators and phones).

If you have questions, please **come to us** at the front of the room to ask.

Please enter all solutions in the boxes provided.

Work on other pages with QR codes will be considered for partial credit.
Please provide a note if you continue work on worksheets at the end of the exam.

Please do not write on the QR codes at the bottom of each page.

We use those codes to identify which pages belong to each student.

Trigonometric Identities Reference

$$\cos(a+b) = \cos(a)\cos(b) - \sin(a)\sin(b)$$
$$\sin(a+b) = \sin(a)\cos(b) + \cos(a)\sin(b)$$
$$\cos(a) + \cos(b) = 2\cos\left(\frac{a+b}{2}\right)\cos\left(\frac{a-b}{2}\right)$$
$$\sin(a) + \sin(b) = 2\sin\left(\frac{a+b}{2}\right)\cos\left(\frac{a-b}{2}\right)$$
$$\cos(a+b)+\cos(a-b) = 2\cos(a)\cos(b)$$
$$\sin(a+b)+\sin(a-b) = 2\sin(a)\cos(b)$$
$$2\cos(a)\cos(b) = \cos(a-b)+\cos(a+b)$$
$$2\sin(a)\cos(b) = \sin(a+b)+\sin(a-b)$$

$$\cos(a-b) = \cos(a)\cos(b) + \sin(a)\sin(b)$$
$$\sin(a-b) = \sin(a)\cos(b) - \cos(a)\sin(b)$$
$$\cos(a) - \cos(b) = -2\sin\left(\frac{a+b}{2}\right)\sin\left(\frac{a-b}{2}\right)$$
$$\sin(a) - \sin(b) = 2\cos\left(\frac{a+b}{2}\right)\sin\left(\frac{a-b}{2}\right)$$
$$\cos(a+b)-\cos(a-b) = -2\sin(a)\sin(b)$$
$$\sin(a+b)-\sin(a-b) = 2\cos(a)\sin(b)$$
$$2\sin(a)\sin(b) = \cos(a-b)-\cos(a+b)$$
$$2\cos(a)\sin(b) = \sin(a+b)-\sin(a-b)$$

1 Relating Transforms (18 points)

Let $F(\Omega)$ represent the discrete-time Fourier transform of the following discrete-time signal:

$$f[n] = \begin{cases} 1 & \text{if } 5 \leqslant n \leqslant 9 \\ 0 & \text{otherwise} \end{cases}$$

Part a. Determine numerical values (no sums or integrals) for $F(0)$, $F(\pi/2)$, and $F(\pi)$ and enter those values in the boxes below.

$F(0)$ =

$F(\pi/2)$ =

$F(\pi)$ =

Briefly explain your reasoning in the box below.

Part b. Let $F_{15}[k]$ represent the DFT of $f[n]$ computed with an analysis window $N = 15$. Let $g_{15}[n]$ represent the signal whose DFT is $F_{15}^2[k] = F_{15}[k] \times F_{15}[k]$. Determine the first five samples of $g_{15}[n]$ and enter those values in the boxes below (no sums or integrals).

$g_{15}[0] =$

$g_{15}[1] =$

$g_{15}[2] =$

$g_{15}[3] =$

$g_{15}[4] =$

Briefly explain your reasoning in the box below.

Part c. Let $F_{12}[k]$ represent the DFT of $f[n]$ computed with an analysis window $N = 12$. Let $g_{12}[n]$ represent the signal whose DFT is $F_{12}^2[k] = F_{12}[k] \times F_{12}[k]$. Determine the first five samples of $g_{12}[n]$ and enter those values in the boxes below (no sums or integrals).

$g_{12}[0] =$

$g_{12}[1] =$

$g_{12}[2] =$

$g_{12}[3] =$

$g_{12}[4] =$

Briefly explain your reasoning in the box below.

Part d. Let $F_{10}[k]$ represent the DFT of $f[n]$ computed with an analysis window $N = 10$. Let $g_{10}[n]$ represent the signal whose DFT is $F_{10}^2[k] = F_{10}[k] \times F_{10}[k]$. Determine the first five samples of $g_{10}[n]$ and enter those values in the boxes below (no sums or integrals).

$g_{10}[0] =$

$g_{10}[1] =$

$g_{10}[2] =$

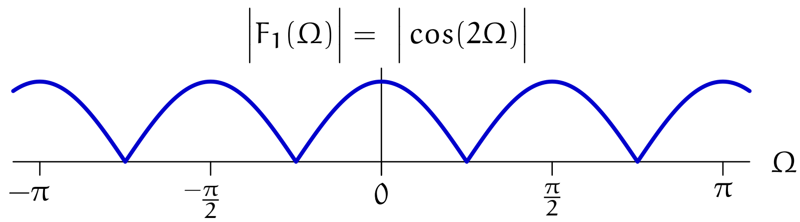
$g_{10}[3] =$

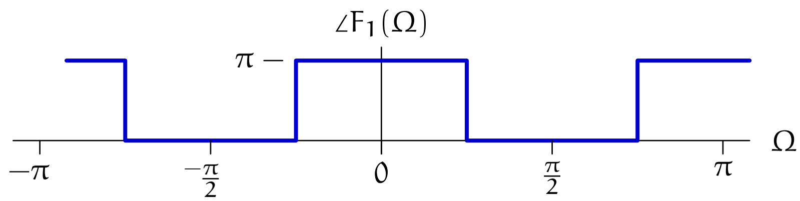
$g_{10}[4] =$

Briefly explain your reasoning in the box below.

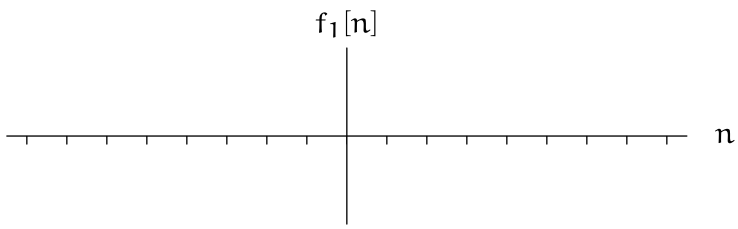
2 Finding Time (18 points)

Part a. Let $f_1[n]$ represent a discrete-time signal whose DTFT has the magnitude and angle shown below.

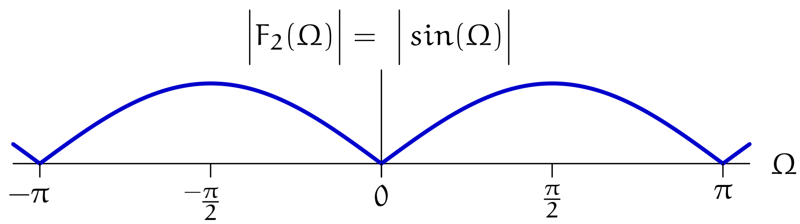


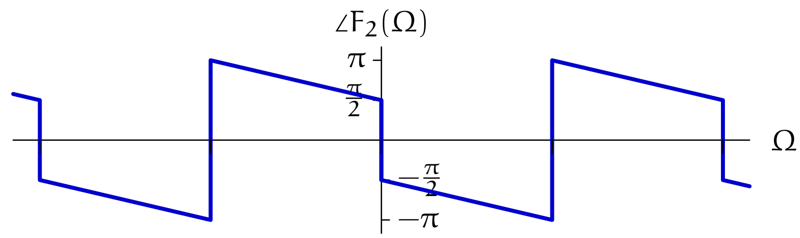


Determine $f_1[n]$ and plot it on the axes below. Label the values of each non-zero sample.

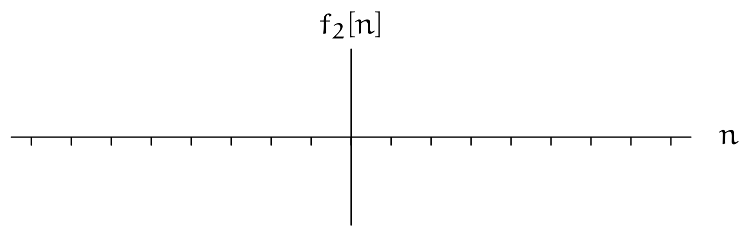


Part b. Let $f_2[n]$ represent a discrete-time signal whose DTFT has the magnitude and angle shown below.



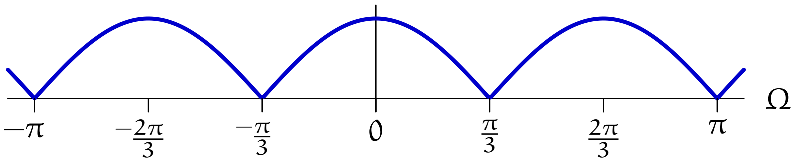


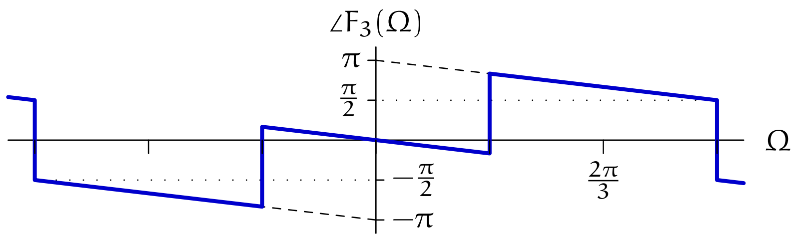
Determine $f_2[n]$ and plot it on the axes below. Label the values of each non-zero sample.



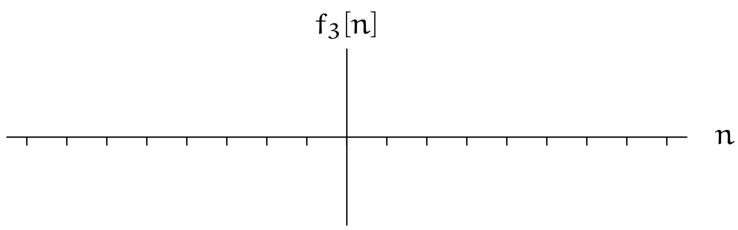
Part c. Let $f_3[n]$ represent a discrete-time signal whose DTFT has the magnitude and angle shown below.

$$\left|F_3(\Omega)\right| = \left|\cos\left(\tfrac{3}{2}\Omega\right)\right|$$





Determine $f_3[n]$ and plot it on the axes below. Label the values of each non-zero sample.



Worksheet (*intentionally blank*)

3 Find the System (18 points)

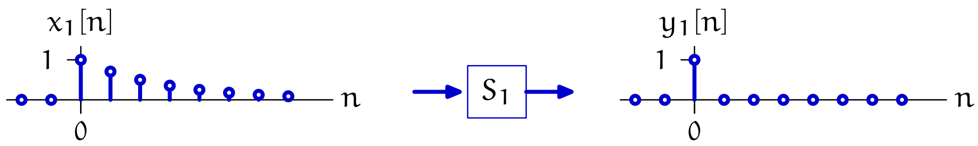
Part a. Let S_1 represent a discrete-time system that is linear and time invariant. When the input to S_1 is

$$x_1[n] = \begin{cases} 2^{-n/2} & \text{if } n \geqslant 0 \\ 0 & \text{otherwise} \end{cases}$$

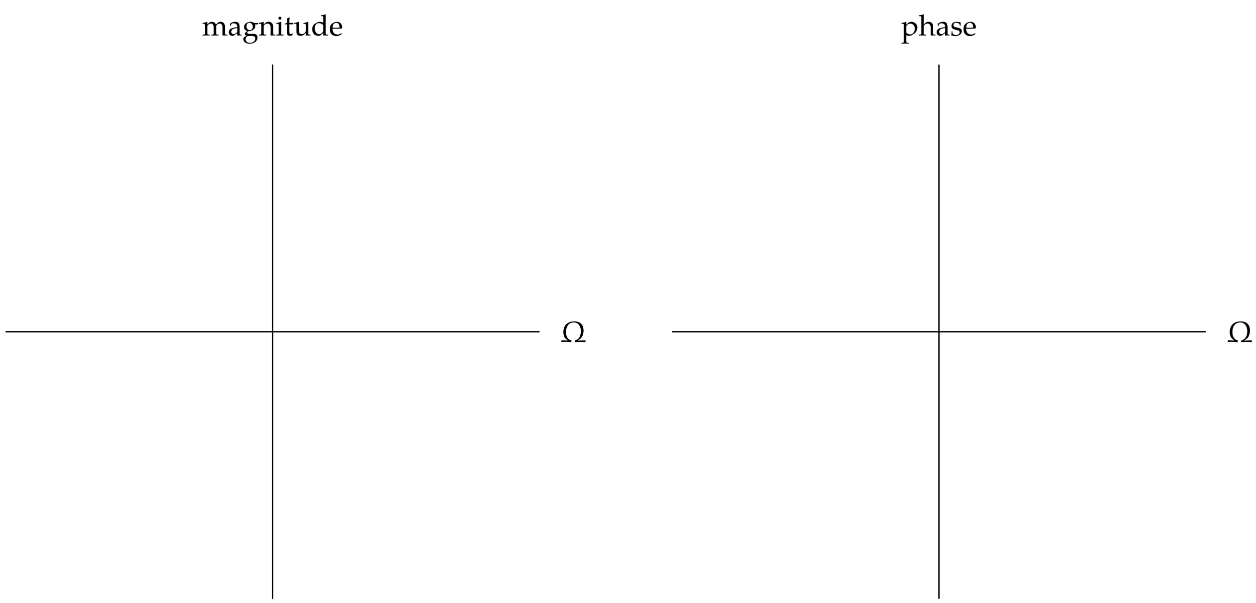
the output of the system is the unit-sample signal

$$y_1[n] = \begin{cases} 1 & \text{if } n = 0 \\ 0 & \text{otherwise} \end{cases}$$

as illustrated below.



On the axes below, plot the magnitude and phase of the frequency response of system S_1. Label the important values in each sketch.



Worksheet (*intentionally blank*)

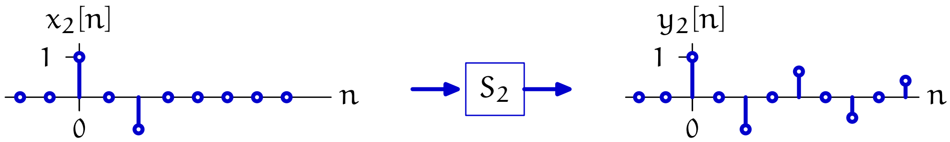
Part b. Let S_2 represent a discrete-time system that is linear and time invariant. When the input to S_2 is

$$x_2[n] = \begin{cases} 1 & \text{if } n = 0 \\ -0.8 & \text{if } n = 2 \\ 0 & \text{otherwise} \end{cases}$$

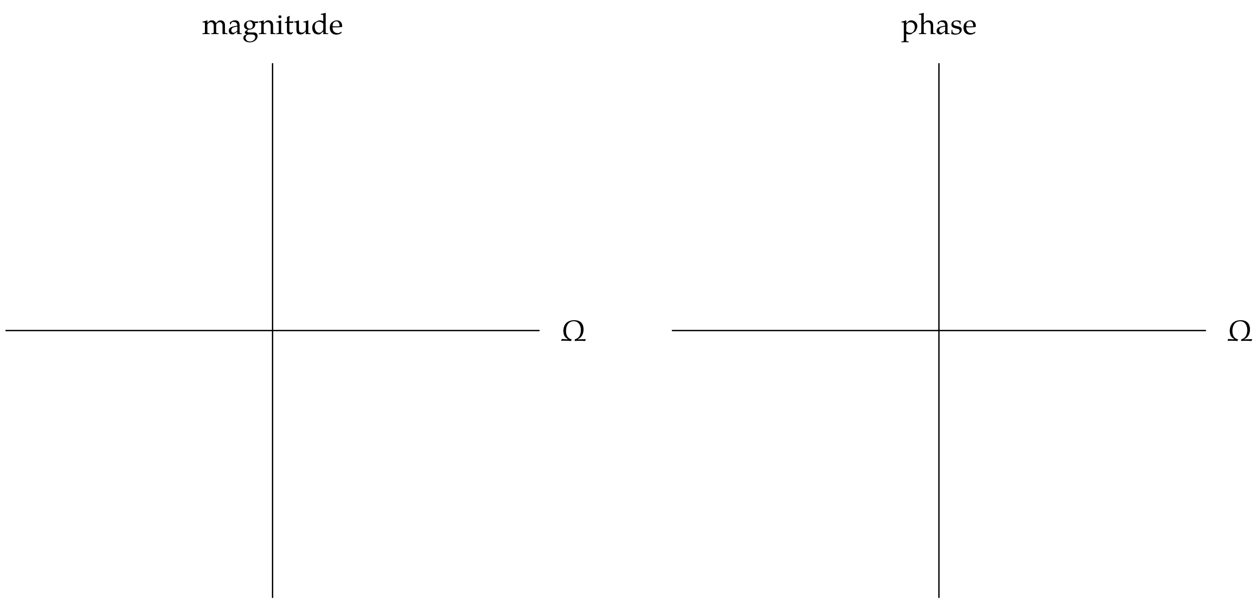
the output of the system is

$$y_2[n] = \begin{cases} (-0.8)^{n/2} & \text{if } n \text{ is even and } n \geqslant 0 \\ 0 & \text{otherwise} \end{cases}$$

as illustrated below.



On the axes below, plot the magnitude and phase of the frequency response of system S_2. Label the important values in each sketch.



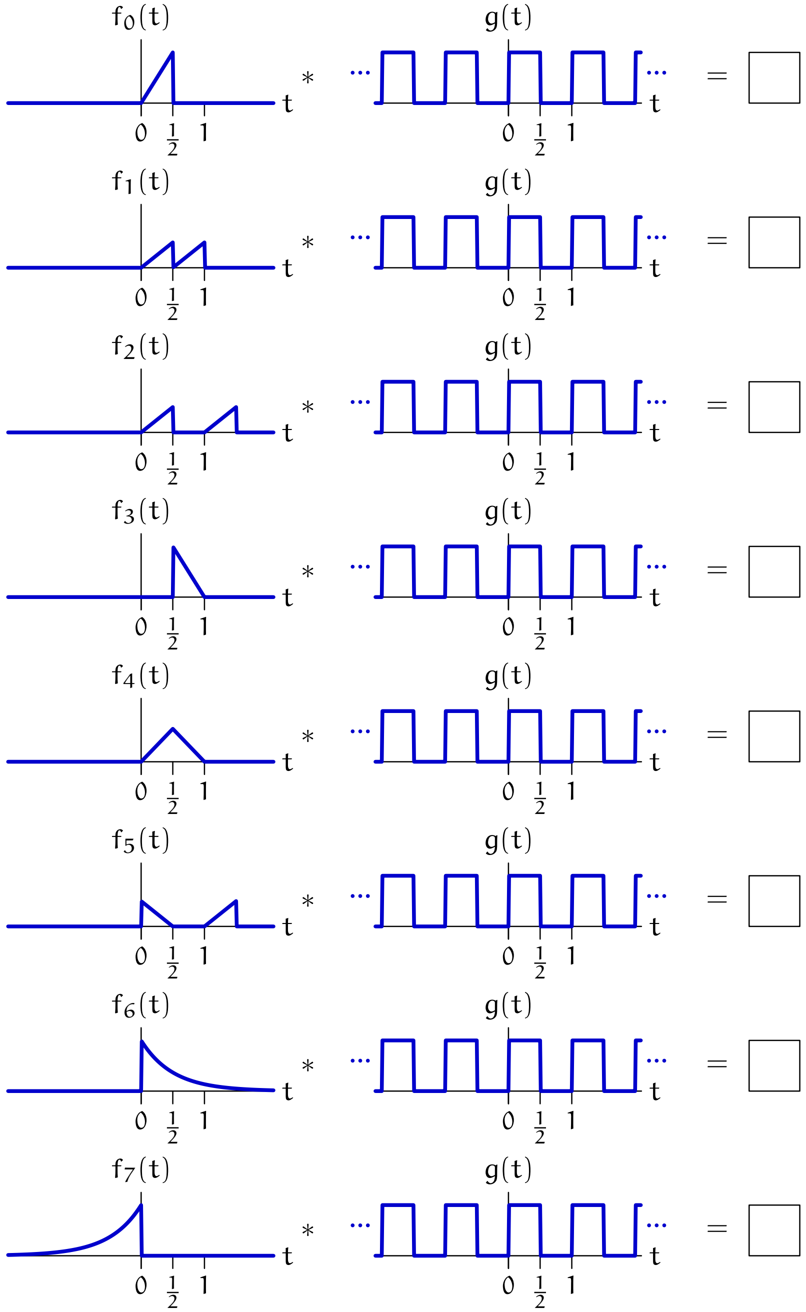
Worksheet (*intentionally blank*)

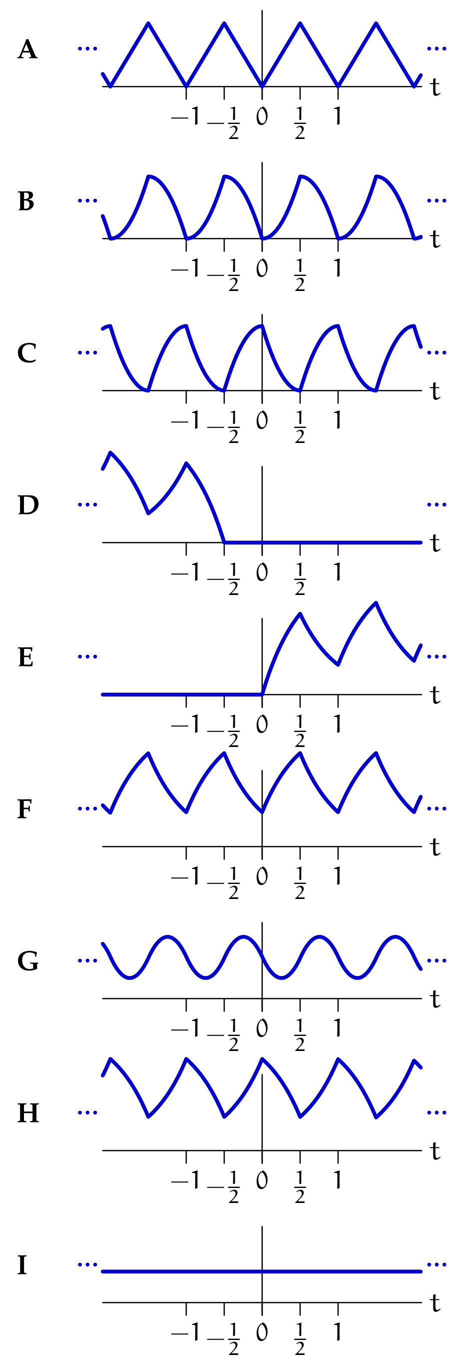
4 Mixed Transformations (16 points)

Each signal in the left column below ($f_0(t)$ to $f_7(t)$) goes to zero outside the regions shown in the plots. Determine the result of convolving each of these signals with a periodic train of rectangular pulses given by

$$g(t) = \begin{cases} 1 & \text{if } \sin(2\pi t) \geqslant 0 \\ 0 & \text{otherwise} \end{cases}$$

Determine which waveform (**A** to **I**) shows the result of each convolution and enter its label in the box provided.





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