6.3000: Signal Processing

Synthetic Aperture Optics

- Fourier Relations in Physics
- Fourier Optics
- Synthetic Aperture Microscopy

May 08, 2025

Why Focus on Fourier?

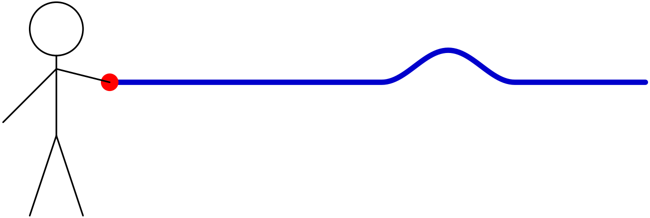
What's so special about sines and cosines?

Sinusoidal functions have interesting **mathematical properties**.
$\rightarrow$ harmonically related sinusoids are **orthogonal** to each other over $[0, T]$.

Sines and cosines also play important roles in **physics**
– nuclear magnetic spins underlie MRI imaging
– physics of waves underlie many applications

Physical Example: Vibrating String

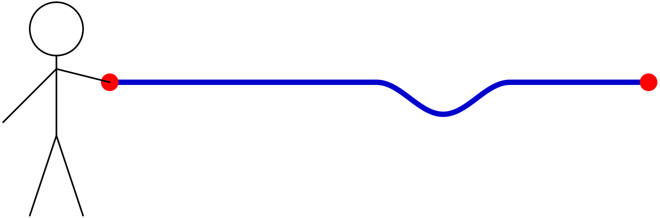
A taut string supports wave motion.



The speed of the wave depends on the tension on and mass of the string.

Physical Example: Vibrating String

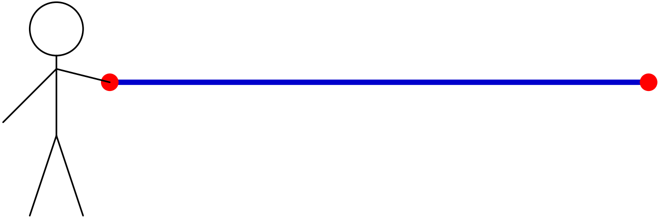
The wave will reflect off a rigid boundary.



The amplitude of the reflected wave is opposite that of the incident wave.

Physical Example: Vibrating String

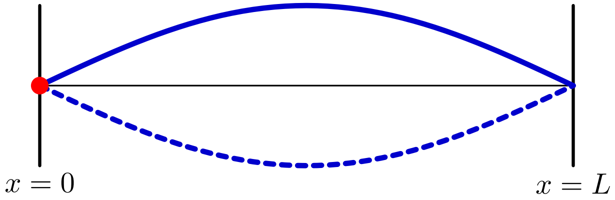
Reflections can interfere with excitations.



The interference can be constructive or destructive depending on the frequency of the excitation.

Physical Example: Vibrating String

We get constructive interference if round-trip travel time equals the period.

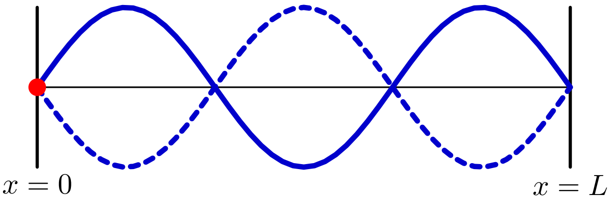


Round-trip travel time $= \dfrac{2L}{v} = T$

$$\omega_o = \frac{2\pi}{T} = \frac{2\pi}{2L/v} = \frac{\pi v}{L}$$

Physical Example: Vibrating String

In fact, we also get constructive interference if round-trip travel time is kT.



Round-trip travel time $= \dfrac{2L}{v} = kT$

$$\omega = \frac{2\pi}{T} = \frac{2\pi}{2L/kv} = \frac{k\pi v}{L} = k\omega_o$$

Only certain frequencies persist: harmonics of $\omega_o = \pi v/L$.

This is the basis of stringed instruments.

Fourier Optics

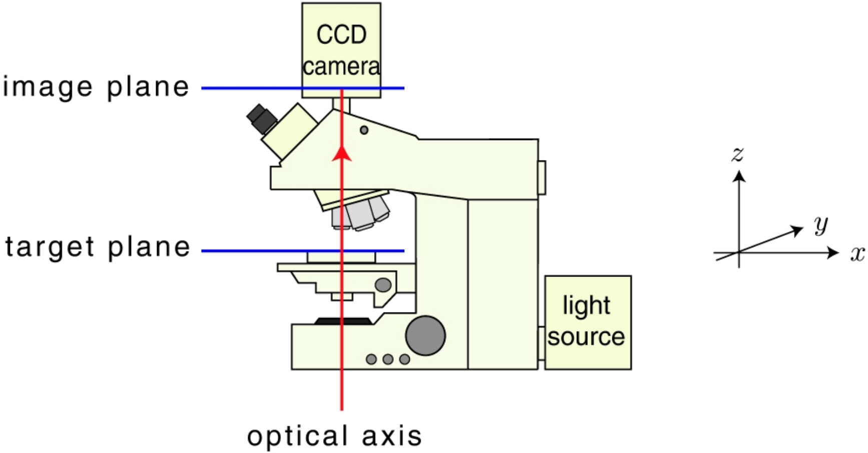
Fourier relations play important roles in many branches of physics – especially those concerning wave phenomena.

Today: Fourier relations in optics.

Optical Imaging

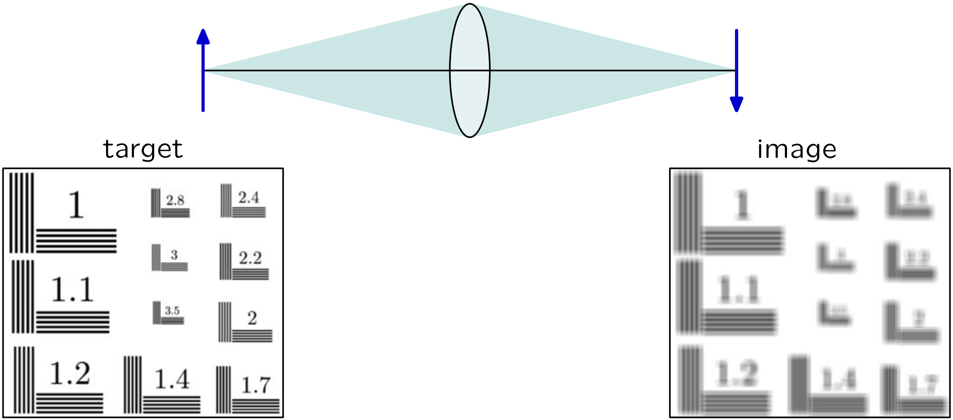
Images from even the best microscopes are blurred.

Blurring is a fundamental property of lenses.



Optical Imaging

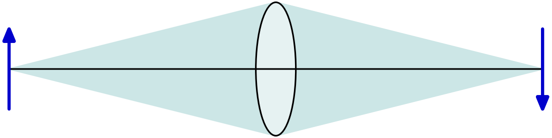
A perfect lens transforms a spherical wave of light from a target into a spherical wave that converges to an image of the target.



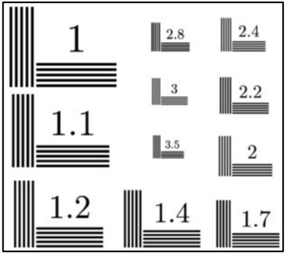
Blurring is inversely related to the diameter of the lens.

Optical Imaging

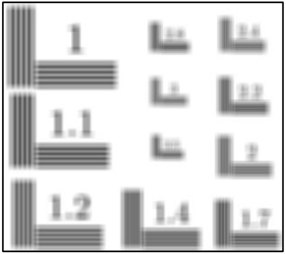
Today's lecture is on how the size of a lens affects image resolution, and how Fourier representations can be used to understand (and even overcome some of) these limitations.



target

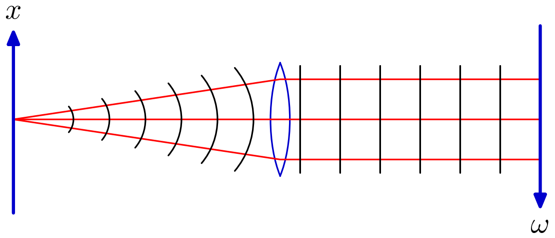


image



Fourier Optics

If a target is located in the focal plane of a lens, light from a point on the target forms a plane wave as it passes through the lens.



If the target point lies on the axis of the lens, then the plane wave is perpendicular to the imaging plane.

Fourier Optics

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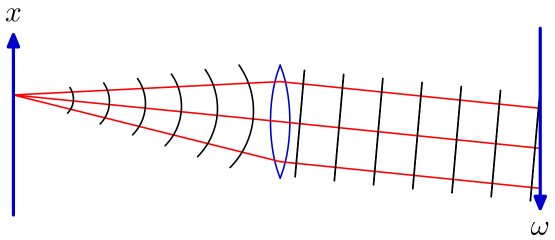
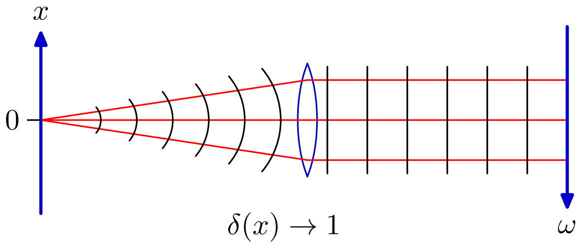


If the target point lies on the axis of the lens, then the plane wave is perpendicular to the imaging plane.

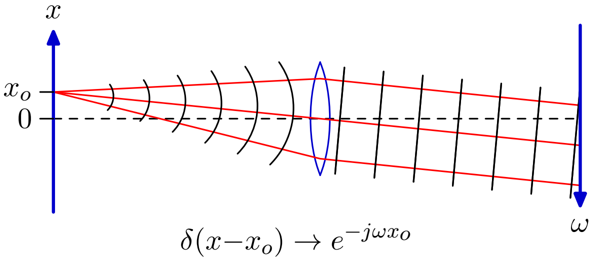
If the target point lies off the axis of the lens, then the plane wave is no longer perpendicular to the image plane. The light striking the image plane has linearly increasing phase delay with distance.

Fourier Optics

Light from the point $x=0$ generates a plane wave, that is everywhere in phase at the imaging plane.



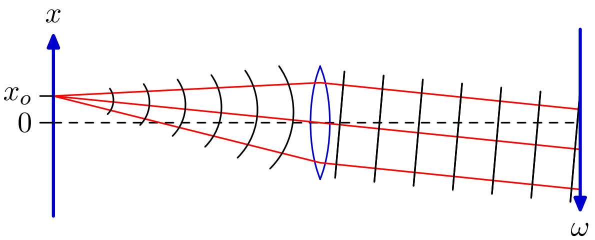
Light from $x=x_o$ generates a plane wave with linearly increasing phase lag.



Fourier Optics

The target can be described as a collection of point sources of light

$$f(x) = \int f(x_o)\delta(x - x_o)\, dx_o$$



and the result in the image plane is a superposition of plane waves, one for each point in the target.

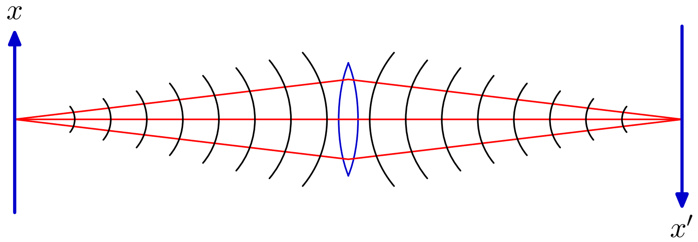
$$g(\omega) = \int f(x)\, e^{-j\omega x}\, dx = F(\omega)$$

Notice that $g(\omega) = F(\omega)$ is the Fourier transform of $f(x)$.

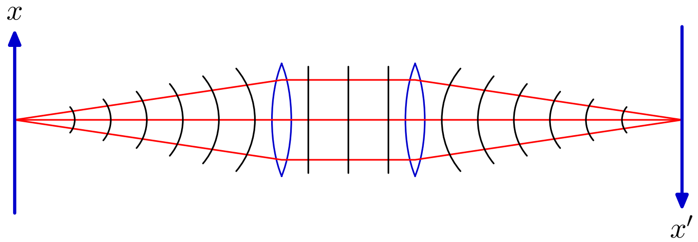
Fourier Optics: $f(x) \stackrel{\text{CTFT}}{\Longrightarrow} F(\omega)$

Fourier Optics

If an object is more than one focal distance from the lens, then the light converges to create an image of the object in the image plane.

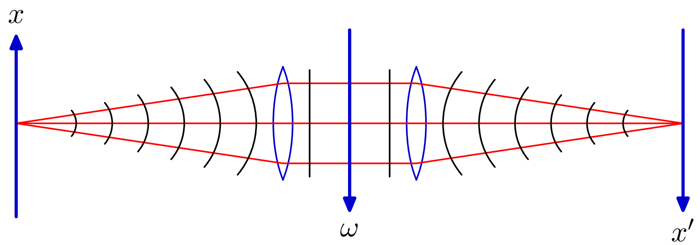


This is equivalent to two lenses: one located a focal distance from the object and one located a focal distance from the image.



Fourier Optics

Now the Fourier transform relation holds for both halves of the system.



$$F(\omega) = \int f(x) e^{-j\omega x} dx$$

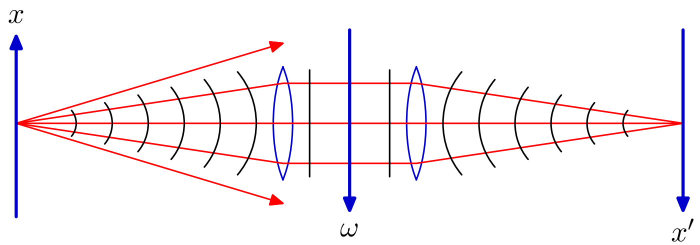
$$f'(x') = \frac{1}{2\pi} \int F(\omega) e^{j\omega x'} d\omega$$

Ideally, both limits of integration would be infinite.

However the finite diameter of the lens limits the highest frequencies $|\omega|$.

Fourier Optics

Light emanating from the target at large angles is not captured by the lens.

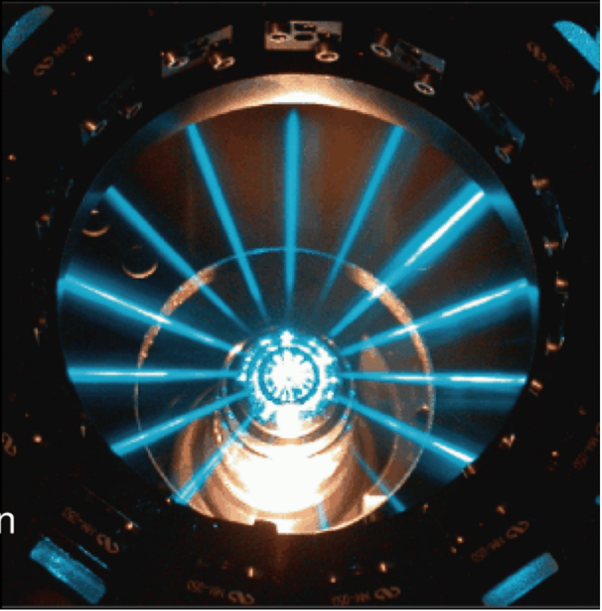


$$F(\omega) = \int f(x) e^{-j\omega x} dx$$

$$f'(x') = \frac{1}{2\pi} \int F(\omega) e^{j\omega x'} d\omega$$

As a result, the image at x' is a lowpass version of the target at x.

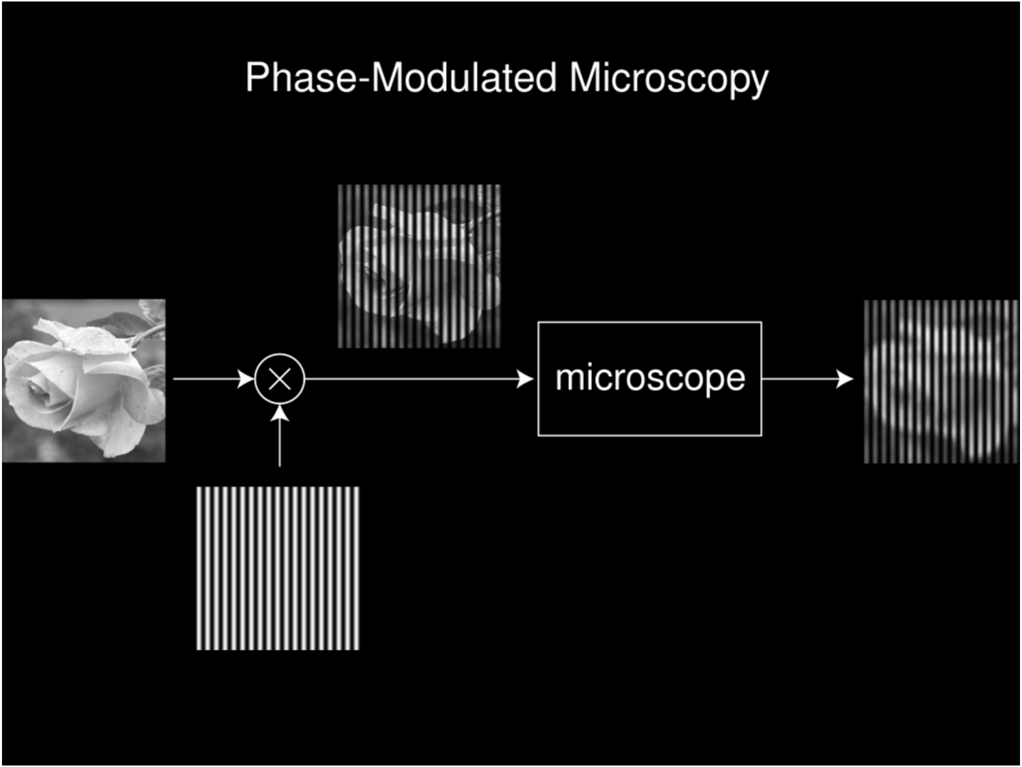
Microscopy with 6.003



Dennis M. Freeman
Stanley S. Hong
Jekwan Ryu
Michael S. Mermelstein
Berthold K. P. Horn

6.003 Model of a Microscope

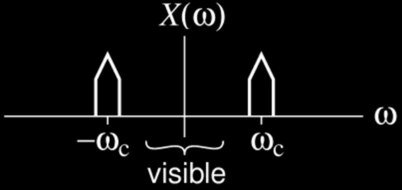
Microscope = low-pass filter


Phase-Modulated Microscopy
microscope

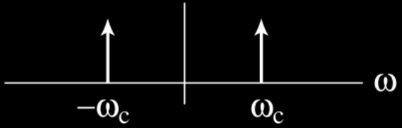
Demonstration

Phase-Modulated Microscopy

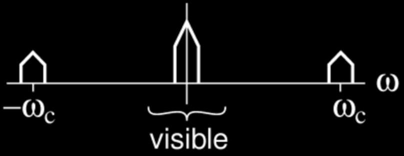
Poster: $\cos(\omega_c y + f(x,y))$



Projector: $\cos(\omega_c y)$

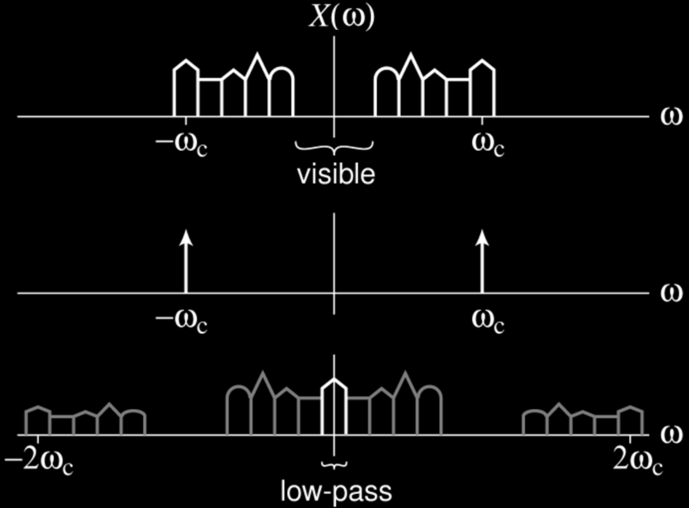


Poster with Projector: $\cos(\omega_c y)\cos(\omega_c y + f(x,y))$



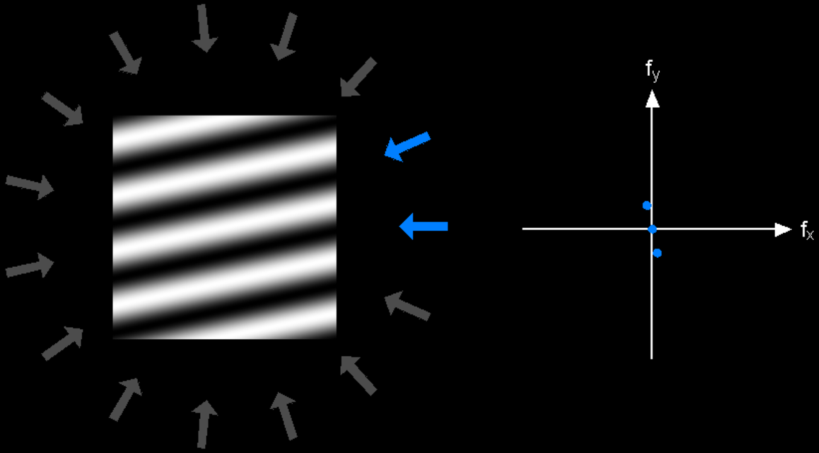
Modulated illumination enables low-pass system (eyes) to detect high spatial frequencies

Phase-Modulated Microscopy



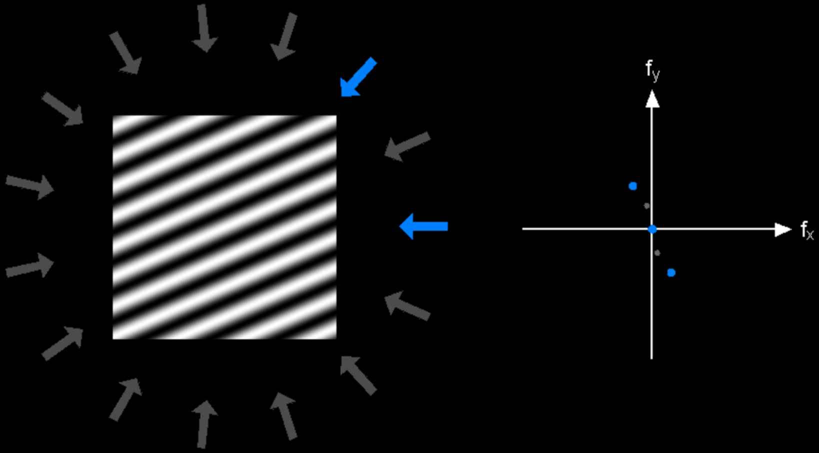
Modulated illumination enables low-pass system (eyes) to detect high spatial frequencies

Standing-wave illumination spectrum



Thanks to M. Mermelstein

Standing-wave illumination spectrum



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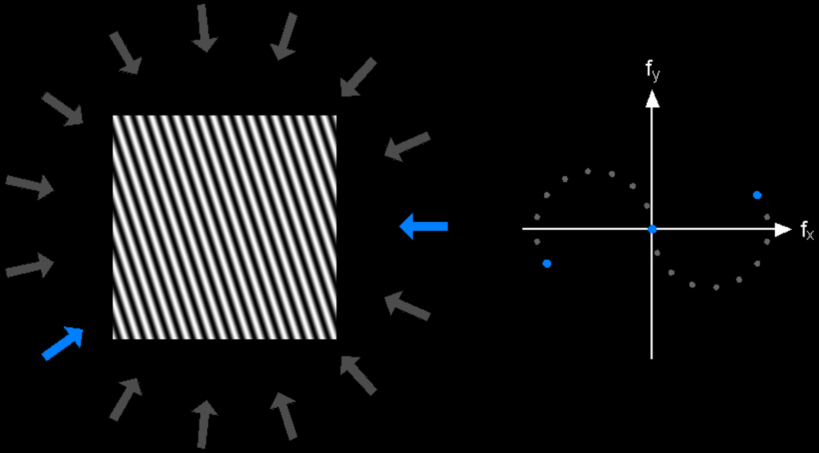
Standing-wave illumination spectrum



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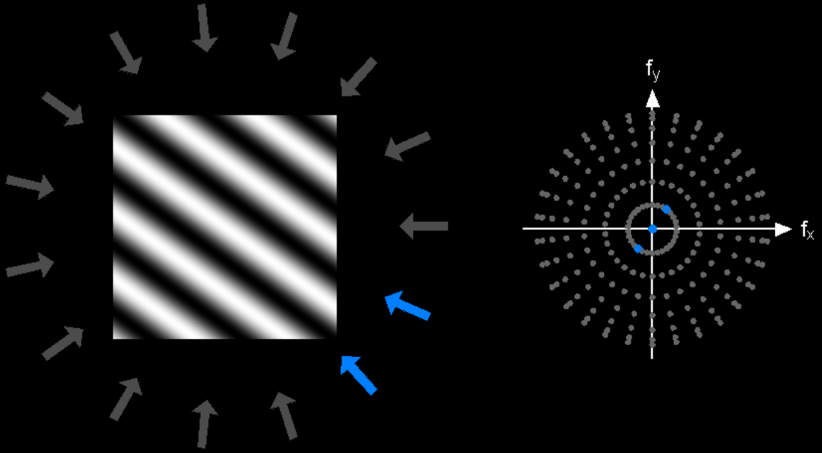
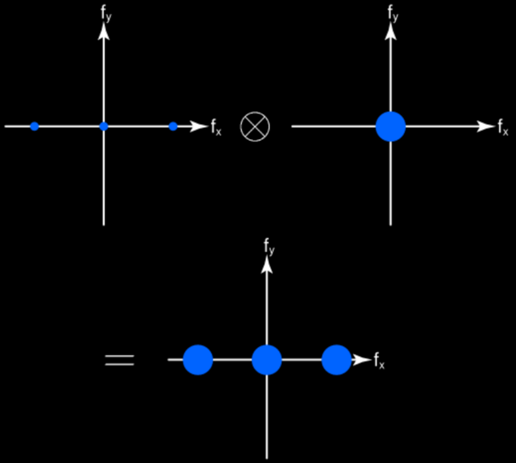
Standing-wave illumination spectrum



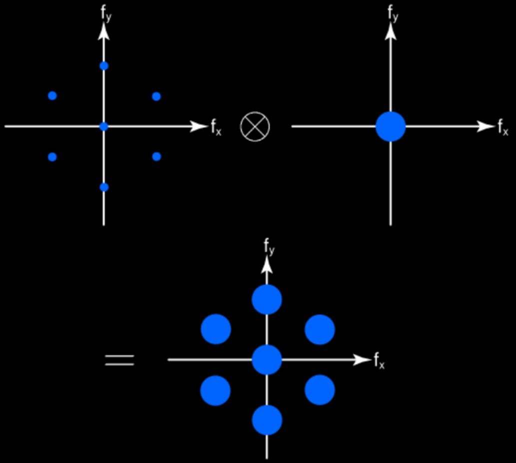
Thanks to M. Mermelstein

Optical transfer function



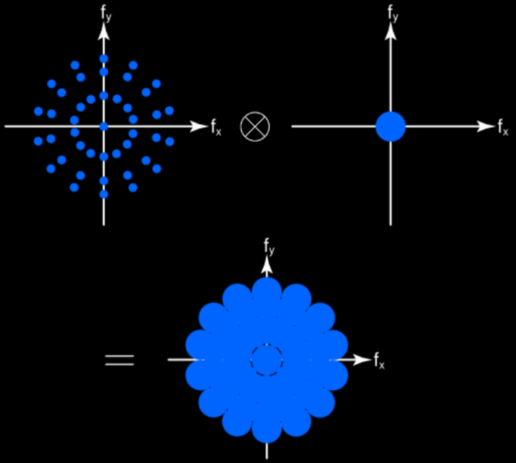
2 beams

Optical transfer function



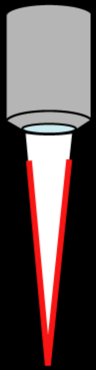
3 beams

Optical transfer function

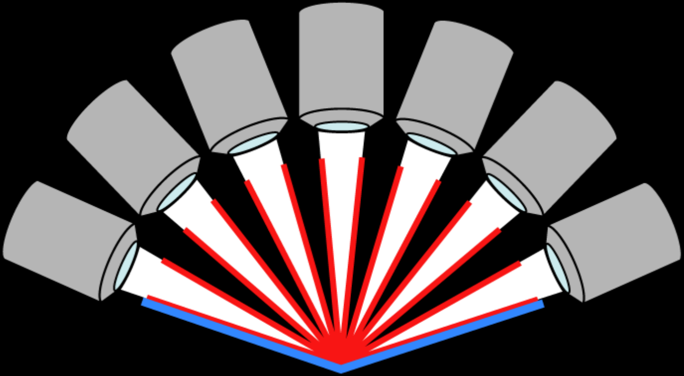


7 beams

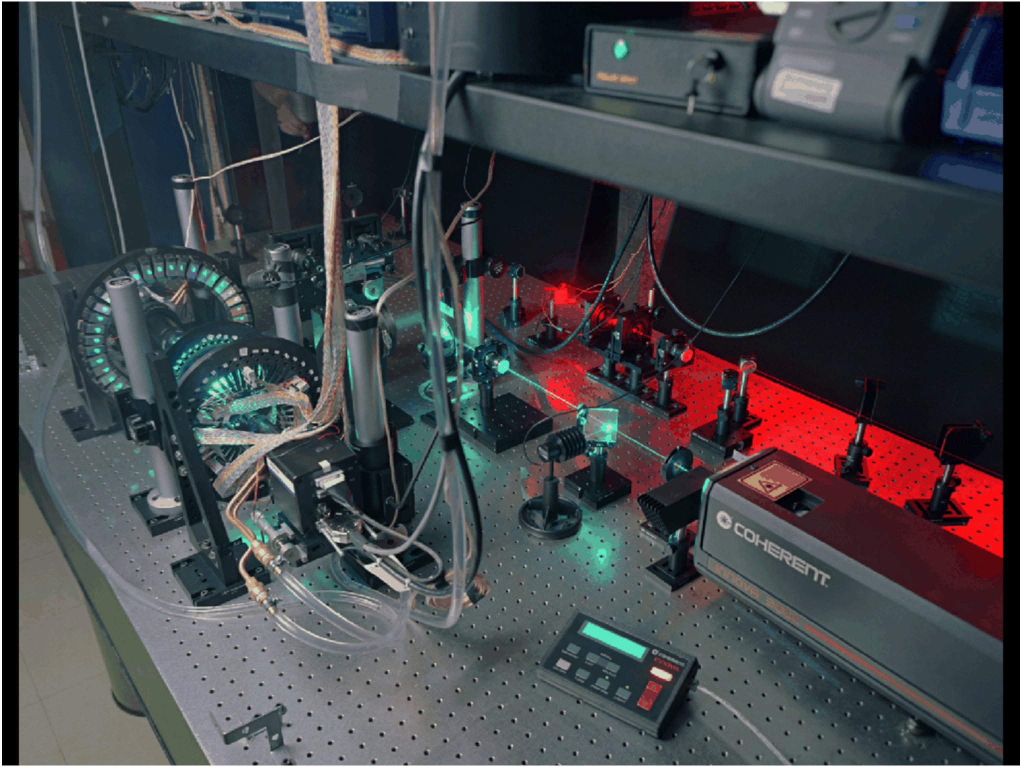
Aperture synthesis

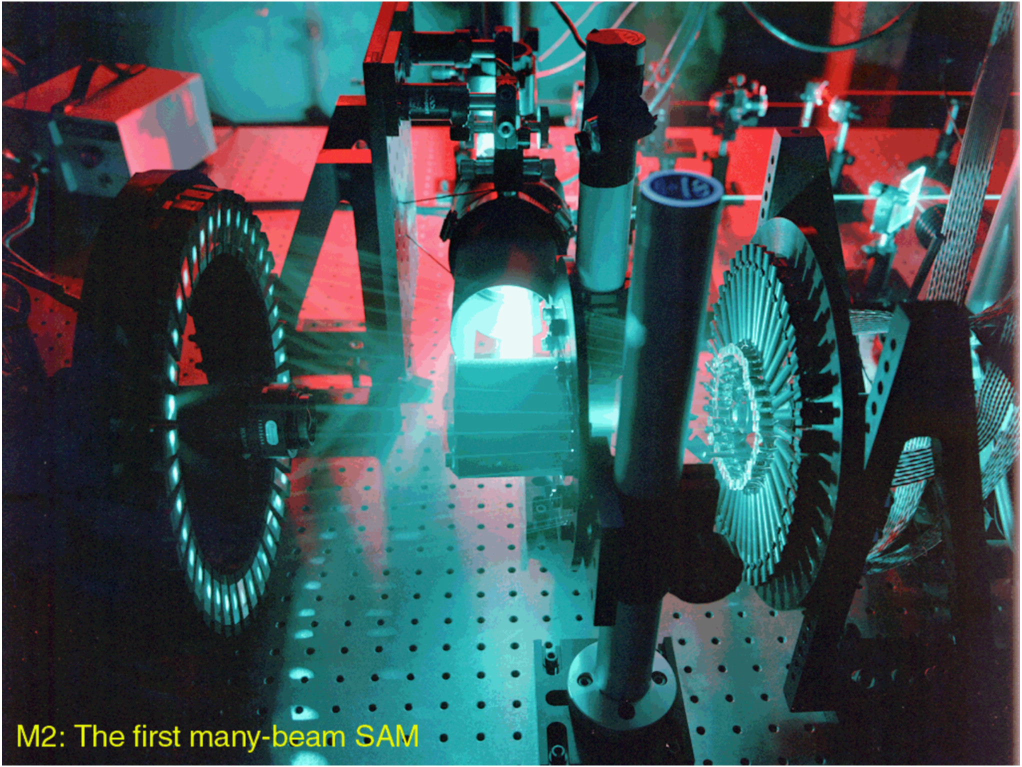


Aperture synthesis



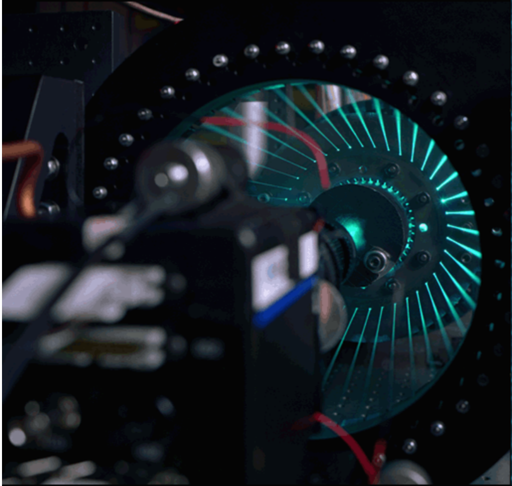
Combine multiple low-NA optics to *synthesize* high NA


COHERENT

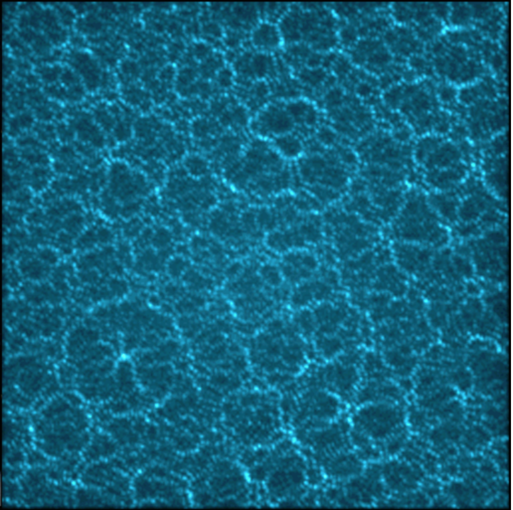


M2: The first many-beam SAM

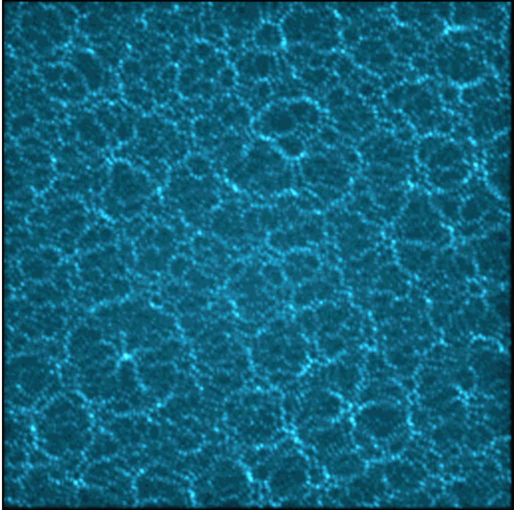
41 BEAMS IN A RING



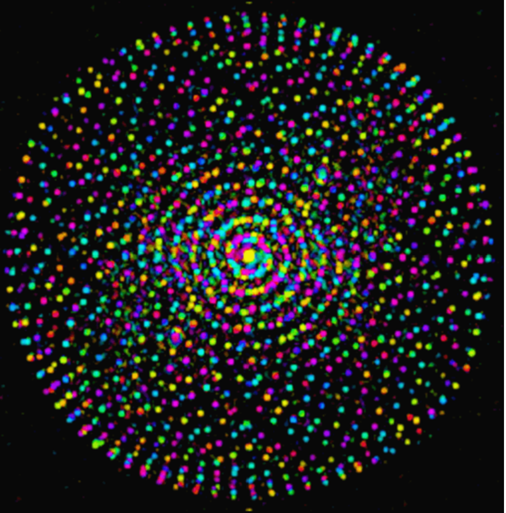
MAKE PATTERNS LIKE THIS



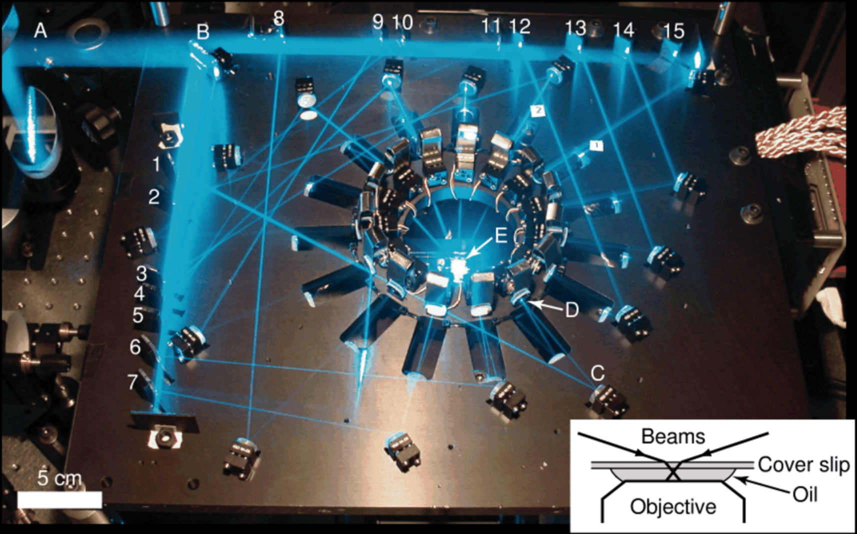
PATTERNS LIKE THIS



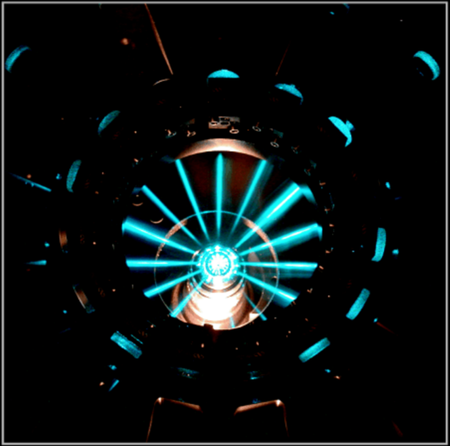
HAVE TRANSFORMS LIKE THIS



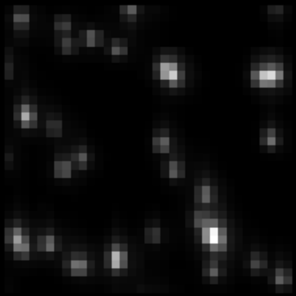
Experimental apparatus



Stanley S. Hong



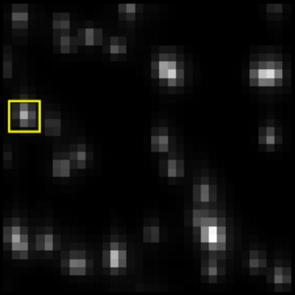
Uniform Illumination



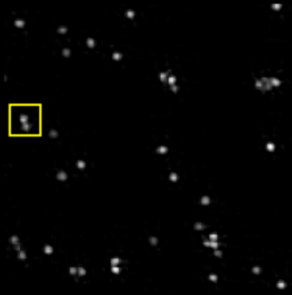
Structured Illumination

Jekwan Ryu

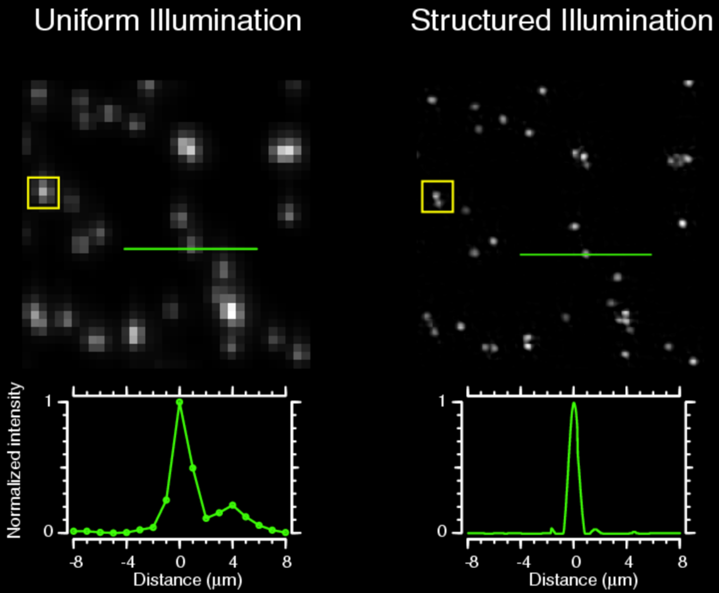
Uniform Illumination



Structured Illumination



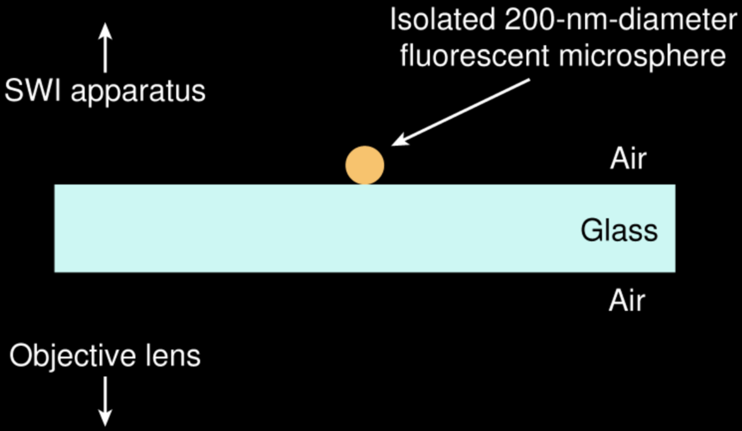
Jekwan Ryu



Uniform Illumination
Structured Illumination
Normalized intensity
1
0
-8
-4
0
4
8
Distance (µm)
1
0
-8
-4
0
4
8
Distance (µm)

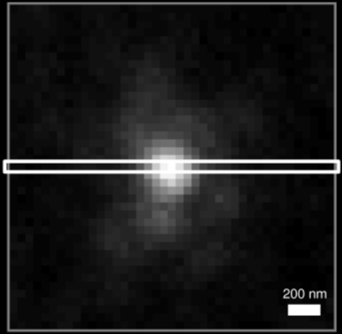
Jekwan Ryu

Measurement of PSF

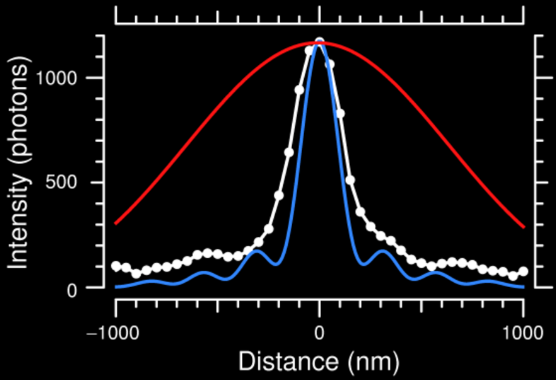


(Cross section, not to scale)

Measurement of PSF



Measurement of PSF

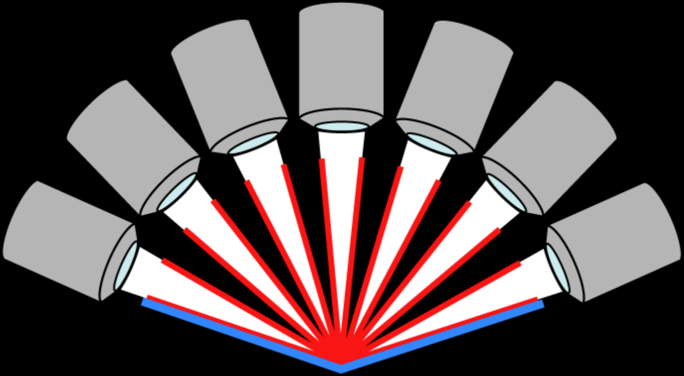


Measured diameter = 290 nm

Predicted diameter = 250 nm

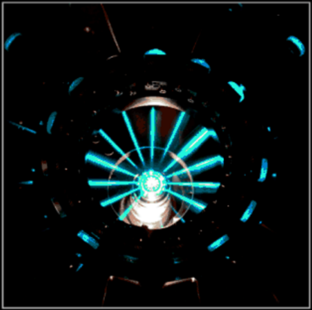
Diameter lens alone = 1,500 nm

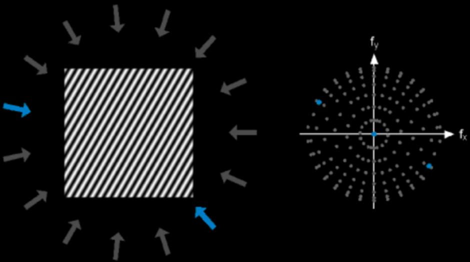
Aperture synthesis



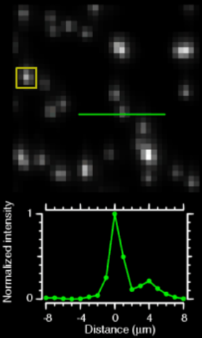
Combine multiple low-NA
optics to *synthesize* high NA

6.003 Approach to Increased Resolution

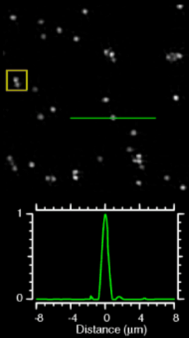




Uniform Illumination



Structured Illumination



Summary

Fourier transforms are important in many branches of physics, mathematics, electrical engineering, and computer science.

Today we saw how **Fourier optics** helps us to understand why optical systems blur.

We also introduced **Synthetic Aperture Optics** as a way to overcome some limitations of conventional optics.

– greatly reduced the blurring in conventional microscopy

This new method of optical imaging is directly inspired by Fourier transforms – and especially by the application of **modulation** to optics.