6.3000: Signal Processing

MRI

Please give us feedback on 6.3000:

- End-of-Term **Subject Evaluations** now until Friday, Dec. 16, 9am
- `http://registrar.mit.edu/subjectevaluation`

Final Exam: Wednesday, May 21, 1:30pm-4:30pm in Johnson Ice Rink
Conflict Exams are assigned by Registrar.

May 06, 2025

Magnetic Resonance Imaging[1]

Magnetic Resonance Imaging (MRI): enormous impact on health care.

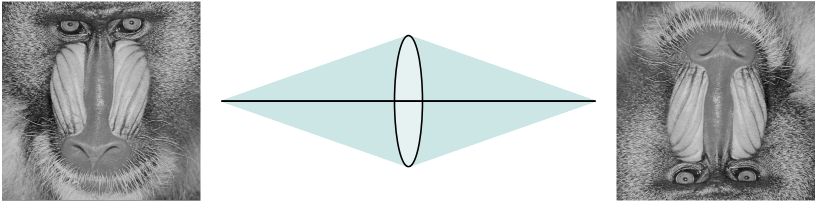
MRI works very differently from other more familiar imaging systems
$\rightarrow$ MRI imaging is deeply dependent on Fourier methods.

Today's goal is to motivate how MRI works and why MR images are different, so that we can understand how these unique signals are processed.

[1] Prof. Elfar Adalsteinsson provided data/examples used in today's lecture.

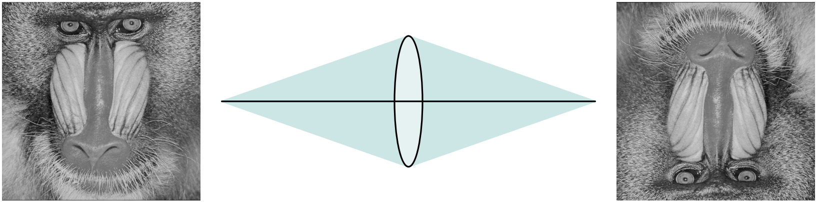
Magnetic Resonance Imaging

Each pixel in a conventional camera reports the amount of light at a particular position in **space.** The collection of pixels represents a spatial mapping of light intensity and produces an **image of space.**



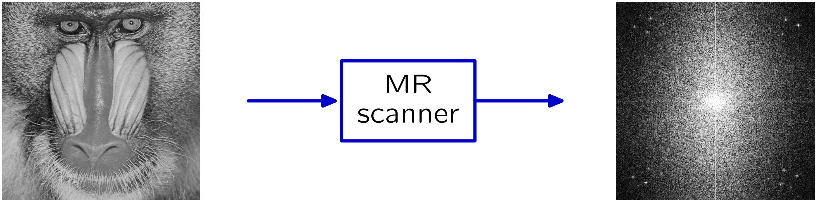
Magnetic Resonance Imaging

Each pixel in a conventional camera reports the amount of light at a particular position in **space.** The collection of pixels represents a spatial mapping of light intensity and produces an **image of space.**



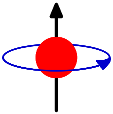
MRI is different.

An MR scanner collects data that represent samples in **Fourier space.** The collection of measurements provides the **Fourier transform** of an image.



Magnetic Resonance

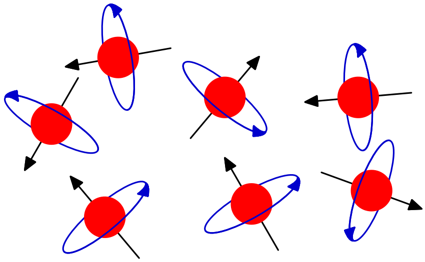
Magnetic resonance can be understood in terms of how the spin angular momentum of a hydrogen nucleus (i.e., a proton) and its associated magnetic dipole moment interact with an external magnetic field.[1]



[1] Although spin angular momentum only arises in quantum mechanics, we consider a classical model that captures many important features of magnetic resonance.

Magnetic Resonance

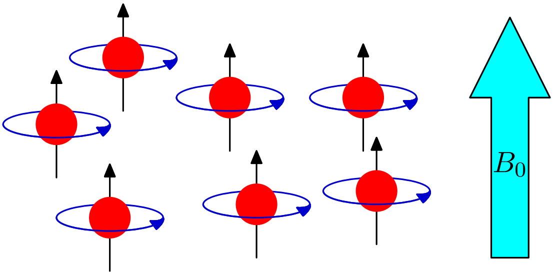
Normally, spins are randomly oriented.



There is no net magnetization.

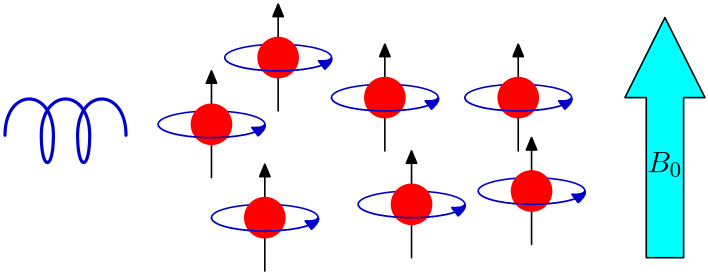
Magnetic Resonance

Spins align with a strong (3–7 T (tesla)) external magnetic field B_0.



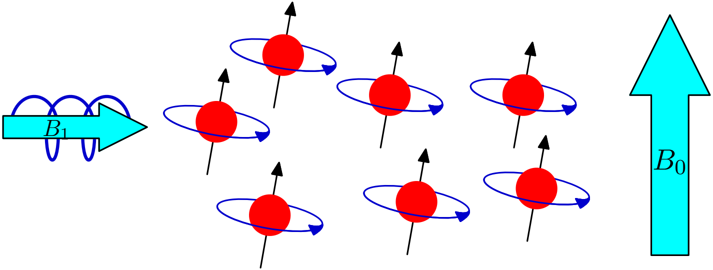
Magnetic Resonance

Spins are tipped by brief excitation B_1 from a transversely oriented coil.



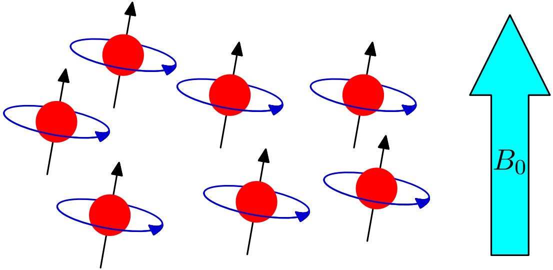
Magnetic Resonance

Spins are tipped by brief excitation B_1 from a transversely oriented coil.



Magnetic Resonance

The spins "precess" as they relax back to their previous alignment.

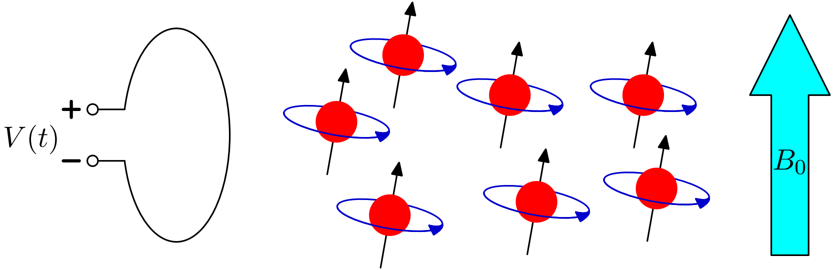


Precession frequency ω is proportional to external field strength B.

The constant of proportionality $\gamma = 42.58\,$MHz/T for water.

Magnetic Resonance

The changing magnetic field is detected with a coil.

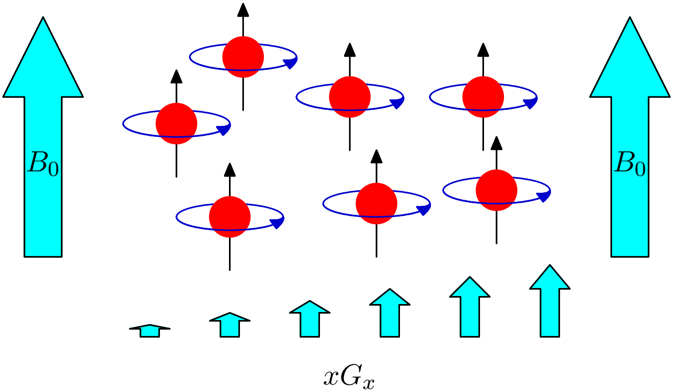


The voltage $V(t)$ is the signal from which an MR image is derived.

Magnetic Resonance Imaging

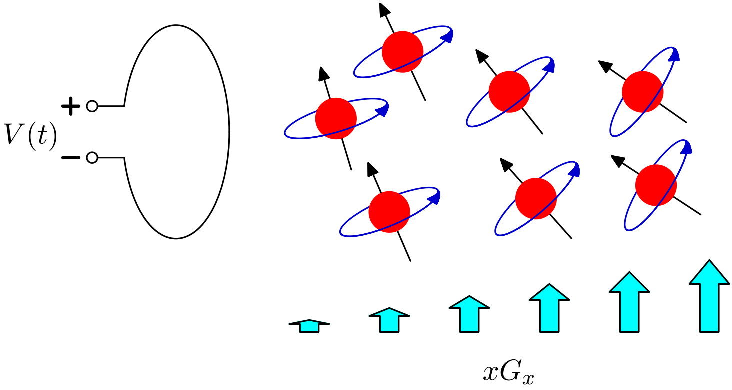
Imaging requires information about space, which is provided by the addition of gradient fields G_x and G_y so the total longitudinal field is

$$B_z(x,y)\,\hat{z} = (B_0 + xG_x + yG_y)\,\hat{z}\,.$$



Magnetic Resonance Imaging

Now dipoles at different locations precess at different frequencies, and the total magnetic field $\Phi(t)$ is the sum of contributions from each dipole.



Magnetic Resonance Imaging

Precession frequency ω is the derivative of phase $\phi(t)$, and is proportional to the local (low-frequency) magnetic field strength:

$$\frac{d\phi(t)}{dt} = \omega = \gamma(B_0 + xG_x + yG_y)$$

Each dipole contributes $e^{-j\phi(x,y,t)}$ to total (high-frequency) magnetic field

$$\Phi(t) = \iint m(x,y)\, e^{-j\phi(x,y,t)} dx\, dy$$

where $m(x,y)$ represents the density of dipoles in location (x,y).
Subsituting the top equation for $\phi(t)$

$$\Phi(t) = \iint m(x,y)\, e^{-j\int_0^t \gamma\left(B_0 + xG_x(\tau) + yG_y(\tau)\right)d\tau}\, dx\, dy$$

which has the form of a Fourier transform (times a modulation term):

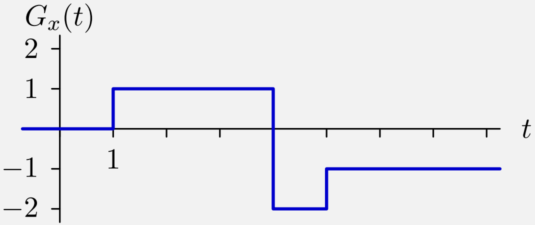
$$\Phi(t) = \underbrace{e^{-j\gamma B_0 t}}_{\text{modulation}} \iint \underbrace{m(x,y)\, e^{-j2\pi(k_x x + k_y y)}\, dx\, dy}_{\text{Fourier form}}$$

Gradient fields G_x,G_y determine which basis function (k_x, k_y) is measured.

$$k_x = \frac{\gamma}{2\pi}\int_0^t G_x(\tau)d\tau \qquad \text{and} \qquad k_y = \frac{\gamma}{2\pi}\int_0^t G_y(\tau)d\tau$$

Check Yourself

Assume that the gradient field $G_x(t)$ of an MR scanner is zero at $t = 0$ and that the scanner is sampling k-space at $k_x = 0$ at that time.

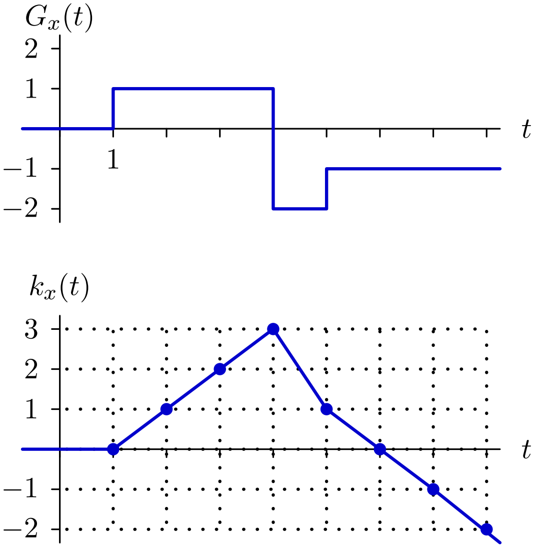


Which of the following sequences of k_x would be sampled by the MR scanner at the 8 clock ticks shown above for $t \geq 1$?

1:	1,	1,	1,	-2,	-1,	-1,	-1	-1
2:	0,	1,	1,	1,	-2,	-1,	-1	-1
3:	1,	0,	0,	-3,	1,	0,	0	0
4:	0,	1,	2,	3,	1,	0,	-1	-2
5:	0,	1,	2,	3,	0,	-1,	-2	-3

Check Yourself

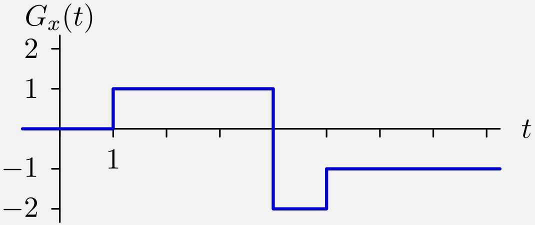
Discrete frequency k_x is proportional to the integral of gradient field G_x.



$k_x : 0, 1, 2, 3, 1, 0, -1, -2$ answer 4.

Check Yourself

Assume that the gradient field $G_x(t)$ of an MR scanner is zero at $t = 0$ and that the scanner is sampling k-space at $k_x = 0$ at that time.

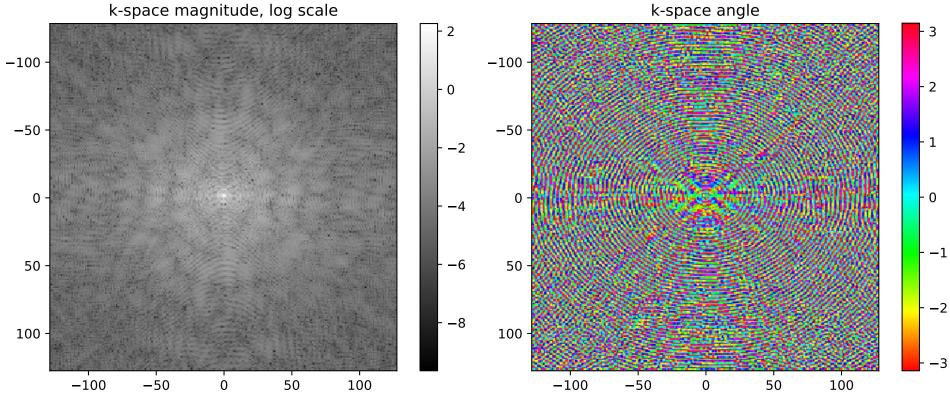


Which of the following sequences of k_x would be sampled by the MR scanner at the 8 clock ticks shown above for $t \geq 1$? 4

1:	1,	1,	1,	-2,	-1,	-1,	-1	-1
2:	0,	1,	1,	1,	-2,	-1,	-1	-1
3:	1,	0,	0,	-3,	1,	0,	0	0
4:	0,	1,	2,	3,	1,	0,	-1	-2
5:	0,	1,	2,	3,	0,	-1,	-2	-3

Example Image

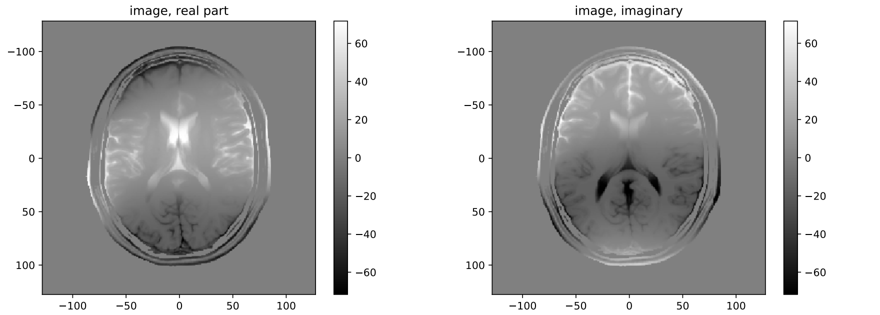
By sampling $V(t)$ as $G_x(t)$ and $G_y(t)$ are varied, we can assemble an array of **k-space** data $M[k_x, k_y]$.



Notice that these direct measurements do NOT represent an image. They represent the Fourier transform of the image.

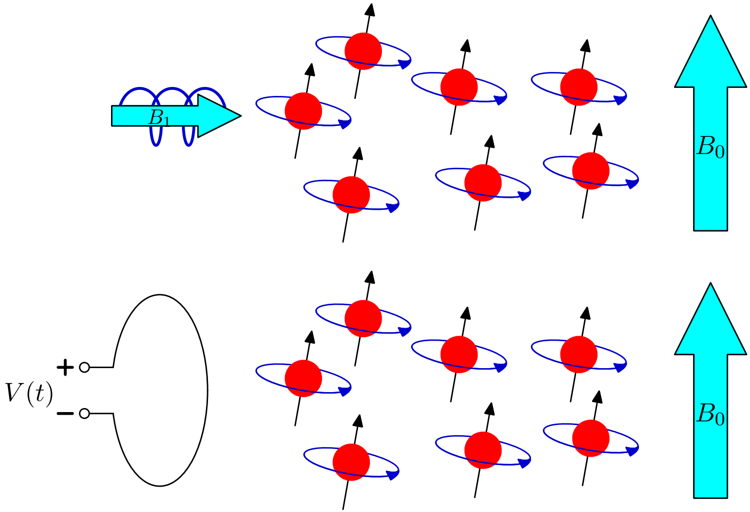
Example Image

The reconstructed image has both real and imaginary parts because of phase delays in the signal path from $B_1(t)$ to the reception of $V(t)$.



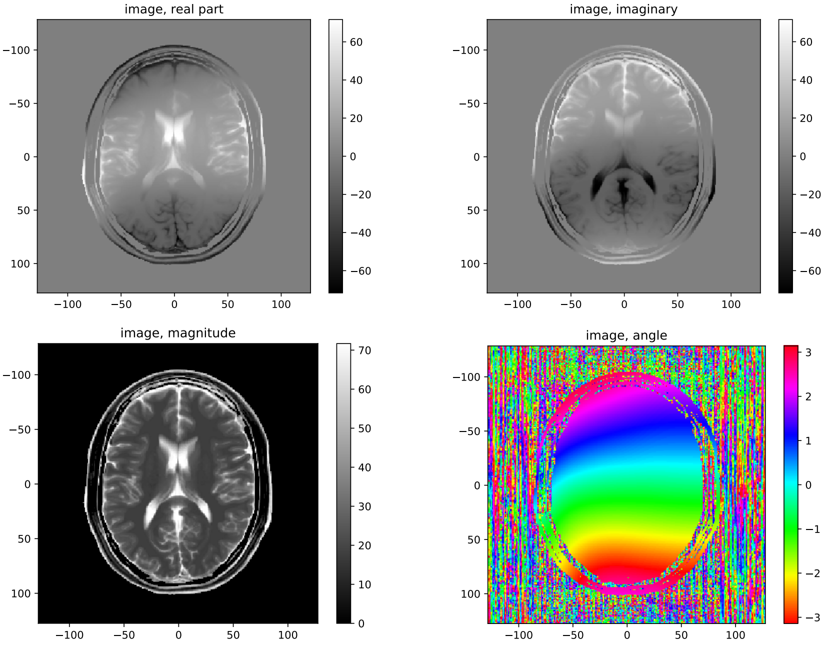
Example Image

The reconstructed image has both real and imaginary parts because of phase delays in the signal path from $B_1(t)$ to the reception of $V(t)$.



Example Image

The magnitude of the inverse Fourier transform provides a better image.



Scan Time

How long does it take to obtain an image like the one on the previous slide?

Typically, one can measure an entire row or column of data as the $\int_0^t G_x(\tau)d\tau$ and/or $\int_0^t G_y(\tau)d\tau$ ramps up after a single RF excitation.

If RF excitation occurs once every 2 seconds, then the total acquisition time would be 256×2 seconds, which is approximately 8.5 minutes.

This is a long time even for a healthy young adult. What about a child? Or a patient with uncontrolled tremors?

Reducing scan time is an active area of research.

Accelerating Imaging

An important area of current research is in decreasing imaging time.

While CD quality audio is recorded at $44,100$ kHz, we can downsample such signals and still get high-quality audio – provided we anti-alias properly.

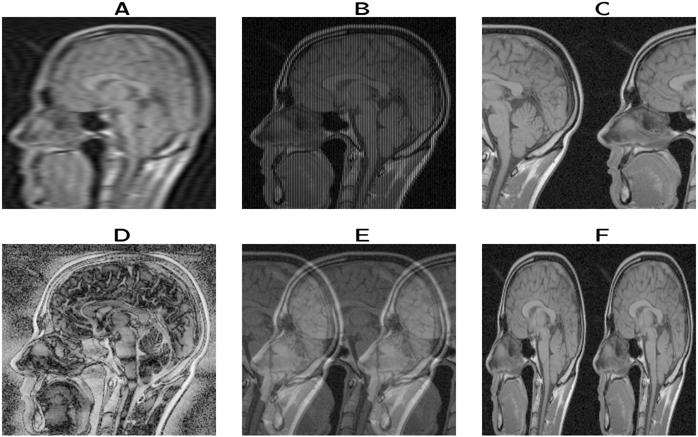
Is there a way to downsample MRI images?

What is the effect of measuring $F[k_r, k_c]$ at only even values of k_c?

Check Yourself

Full reconstruction: using all samples $-128 < k_x, k_y < 128$.

Which is reconstructed from just even-numbered columns (odds set to 0)?



Check Yourself

Which image is reconstructed from just even-numbered columns?

Setting the odd-numbered columns of the transform to zero is equivalent to multiplying the DFT by

$$H[k_r,k_c]=\frac{1}{2}\Big(1+(-1)^{k_c}\Big)=\frac{1}{2}\Big(1+e^{-j\pi k_c}\Big)$$

Multiplying in frequency is equivalent to convolving in space by

$$\begin{aligned}
h[r,c] &= \sum_{k_r,k_c} H[k_r,k_c]\,e^{j2\pi\left(\frac{k_r r}{R}+\frac{k_c c}{C}\right)} \\
&= \sum_{k_r,k_c} \frac{1}{2}\Big(1+e^{-j\pi k_c}\Big)\;e^{j2\pi\left(\frac{k_r r}{R}+\frac{k_c c}{C}\right)} \\
&= \frac{1}{2}\sum_{k_r,k_c} e^{j2\pi\left(\frac{k_r r}{R}+\frac{k_c c}{C}\right)}+\frac{1}{2}\sum_{k_r,k_c} e^{j2\pi\left(\frac{k_r r}{R}+\frac{k_c(c-C/2)}{C}\right)} \\
&= \frac{RC}{2}\delta[r,c]+\frac{RC}{2}\delta\left[r,c-\frac{C}{2}\right]
\end{aligned}$$

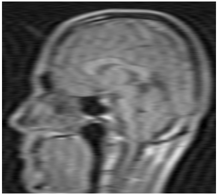
Convolving by $h[r,c]$ adds a half-frame circular shift of the image to the original image $\rightarrow$ panel E.

Check Yourself

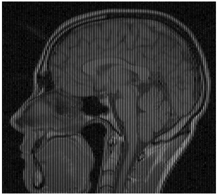
Reconstruction from $-128 < k_x, k_y < 128$:

Which image is reconstructed from just even-numbered columns? E

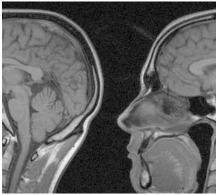
A



B

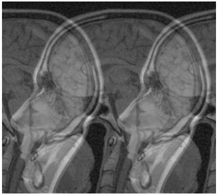


C

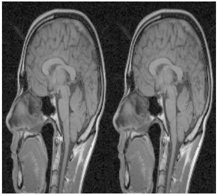


D

E

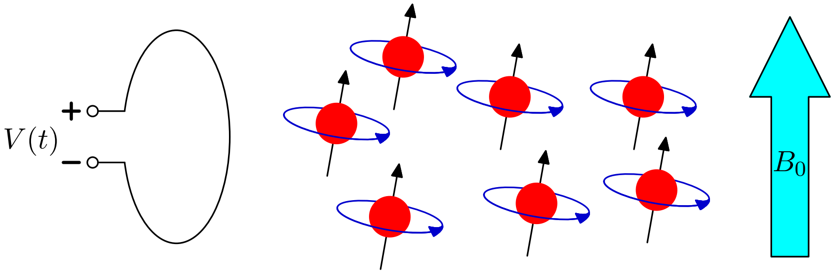


F



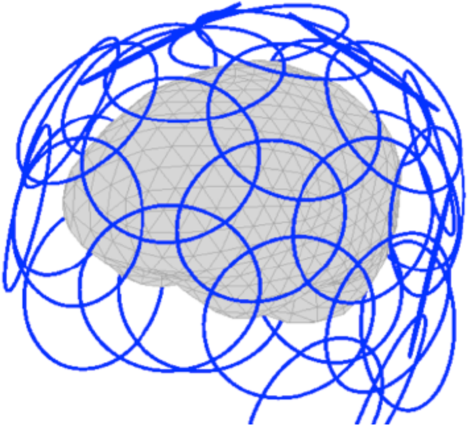
Multi-Coil Imaging

Multiple readout coils can be read in parallel, and thereby provide additional data without increasing imaging time.



Multi-Coil Imaging

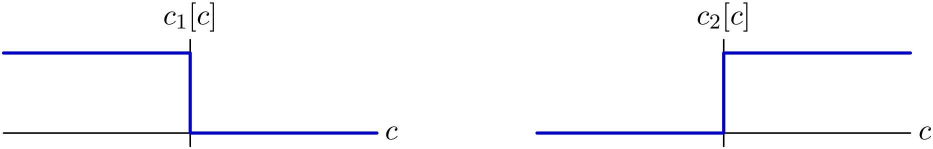
“Helmets” with as many as 16 to 32 readout coils have been used to increase the resolution of brain images.



Reconstructing Images from Multi-Coil Data

Consider two coils, one on each side of the head. The left coil will be more sensitive to the left portions of the image, and vice versa.

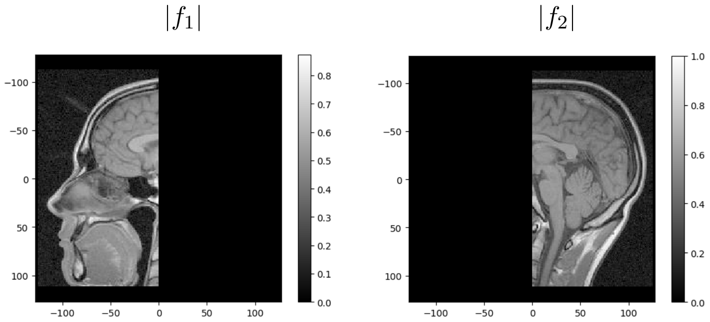
Characterize the **sensitivity** of each coil by specifying a number between 0 (insensitive) and 1 (sensitive) for each pixel in the image.
In 1D, these sensitivities could have the following form:



What would be the effect of these coils on the resulting image?

Images From Coils 1 and 2

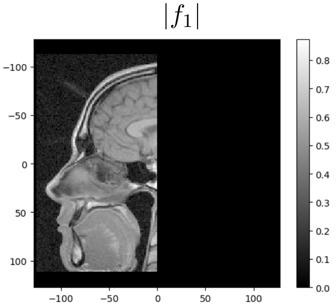
Since coil 1 is only sensitive to half of the head, the image produced with data from coil 1 shows just that half. Similarly, the image produced with data from coil 2 shows just that half.

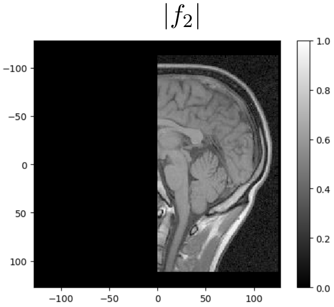


If we only measure $F_1[k_r, k_c]$ at even-numbered k_c, then the image from coil 1 will be added to a circularly shifted version of itself.
The same holds for coil 2.

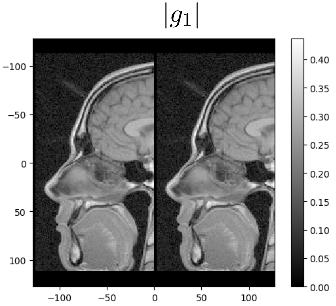
Images From Coils 1 and 2

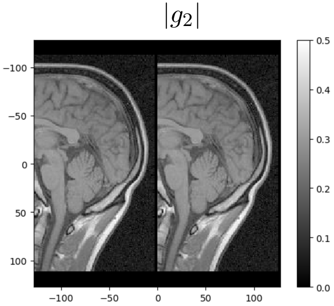
Images f_1 and f_2 are derived from full-resolution data F_1 and F_2.





Images g_1 and g_2 are derived from just the even-numbered k_c.





Images From Coils 1 and 2

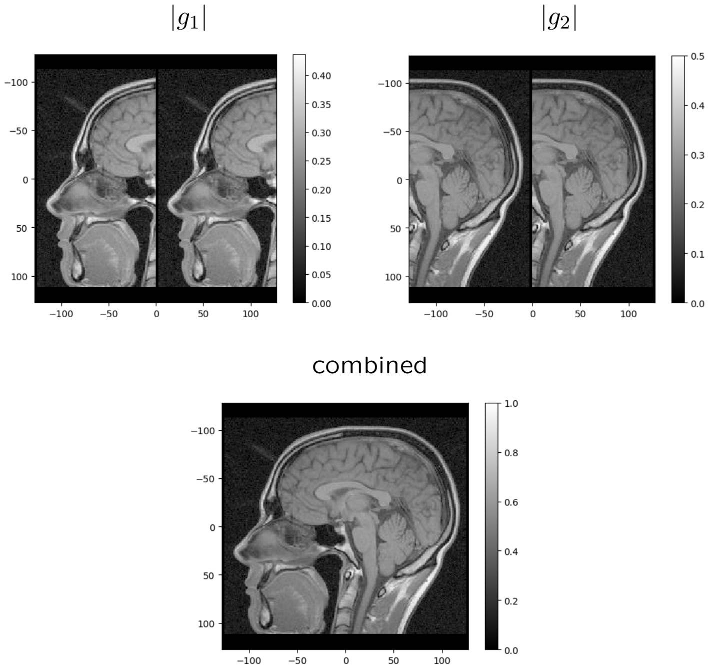
Could you construct a full-frame full-resolution image from these data?

Yes. Combine the left part of $|g_1|$ with the right part of $|g_2|$.

Advantage:
$|g_1|$ was acquired in half the time required for a full-frame full-resolution image. Similar with $|g_2|$.

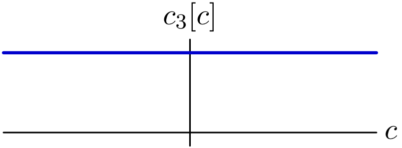
But $|g_1|$ and $|g_2|$ data can be acquired simultaneously!

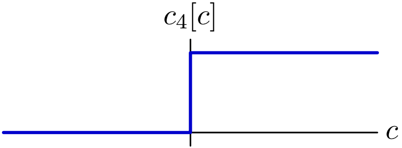
Constructing Full-Frame Image From Coil 1 and 2 Data



Multi-Coil MRI

What if the coils had the following sensitivities?

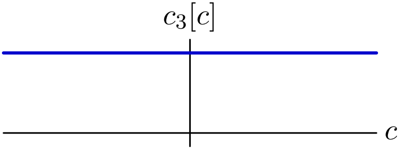


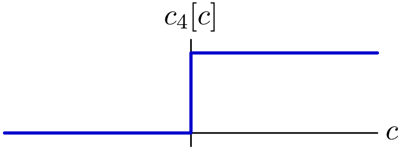


What would be the effect of each of these coils on the image?

Multi-Coil MRI

What if the coils had the following sensitivities?

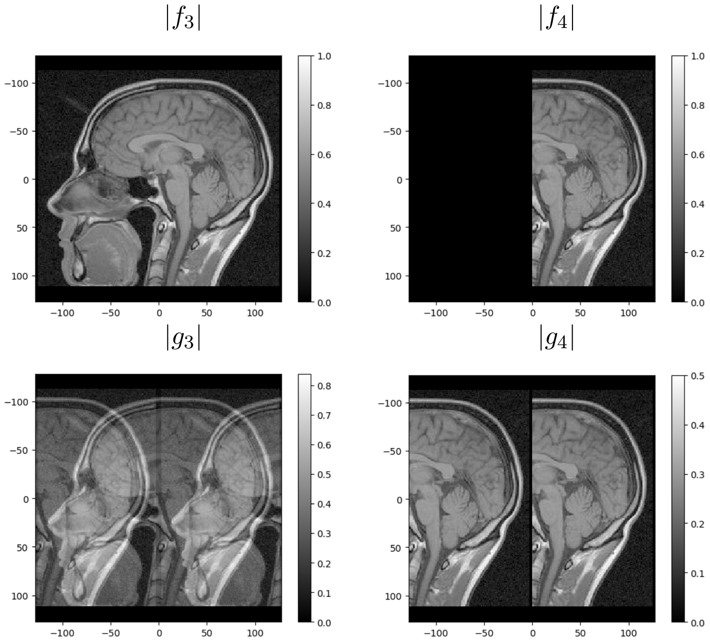




What would be the effect of each of these coils on the image?

c_3 is a full-frame image. Omitting the odd number columns from G_3 will produce the aliased image we started with.
c_4 is the same as the previous c_2, so $|g_4|$ is the same as $|g_2|$.

Images From Coils 3 and 4



Can we create a full-frame full-resolution image from this data?

Images From Coils 3 and 4

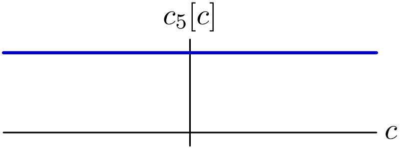
The $|f_3|$ image can be viewed as the sum of results for the left and right sides of the image (as in the c_1 and c_2 example).

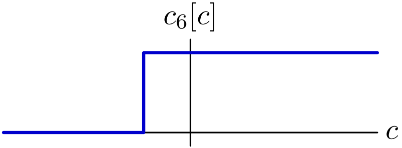
Subtracting $|g_4|$ from $|g_3|$ would generate the previous $|g_1|$ image.

Algorithm:
Combine the left part of $|g_3| - |g_4|$ with the right part of $|g_4|$.

Multi-Coil MRI

What if the coils had the following sensitivities?

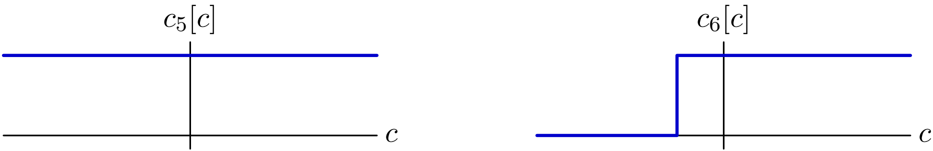




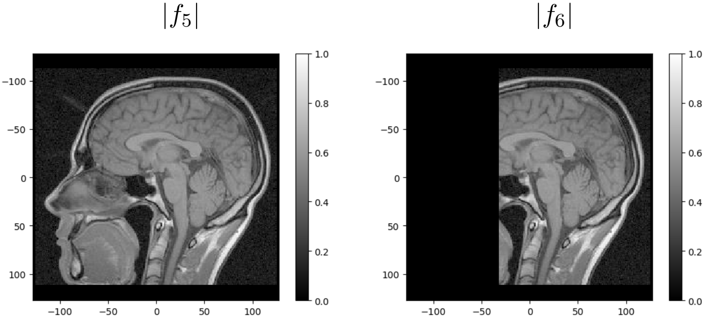
What would be the effect of each of these coils on the image?

Multi-Coil MRI

What if the coils had the following sensitivities?

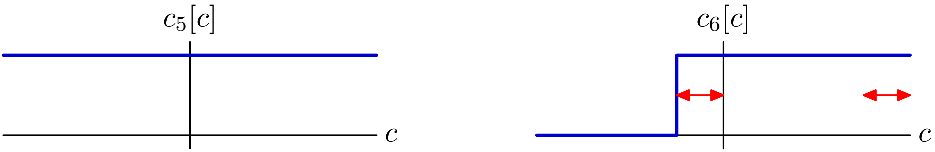


What would be the effect of each of these coils on the image?



Multi-Coil MRI

What if the coils had the following sensitivities?



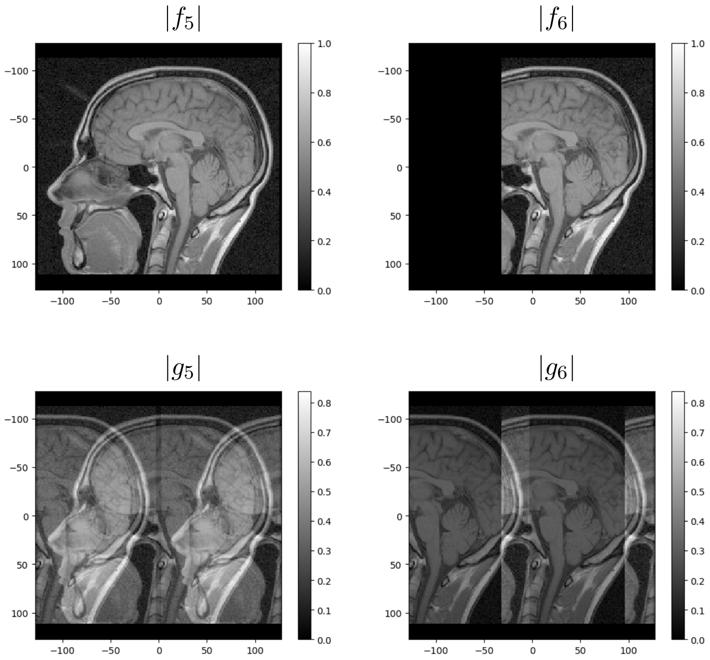
Notice that c_6 weights contributions from pixels in the range $-32 \leq c < 0$ exactly the same as those in $96 \leq c < 128$.

Therefore the c_6 image contains no information that is useful for separating these two bands of pixels.

Similar statements apply for c_5.

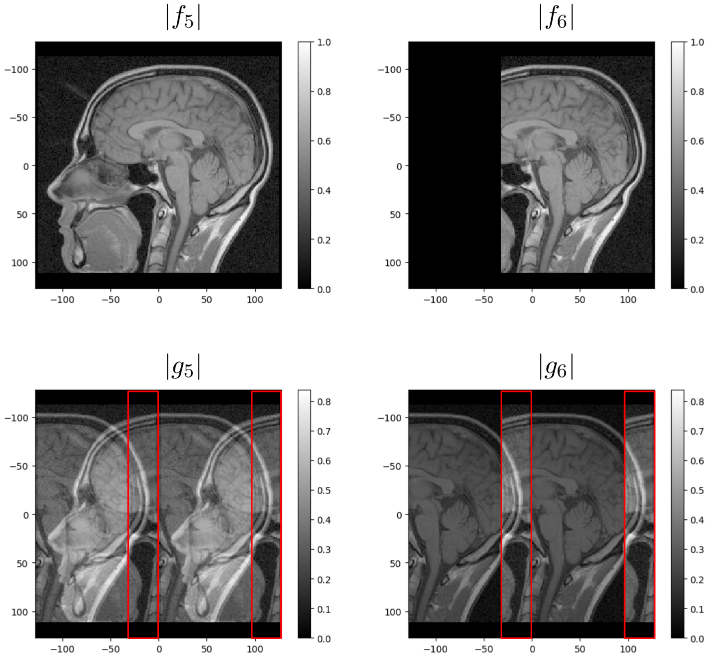
Images From Coils 5 and 6

g_5, g_6 are after omitting odd numbered columns from $|f_5|$, $|f_6|$.



Images From Coils 5 and 6

Highlighted regions are identical: both represent sum $f[r,c]+f[r,c+128]$.



Conclusions

Magnetic Resonance Images are amazing – revealing deep tissue structure while being completely non-invasive.

Magnetic Resonance Images are acquired by sampling the Fourier representation of the proton density function.

Improving the imaging speed is an area of active research.

Magnetic Resonance Imaging can be made faster using multiple readout coils, which enables parallel acquisition of undersampled k-space data.

Question of the Day

How many pairs of frequencies (k_x,k_y) would be needed to reconstruct an image of a human brain with millimeter resolution?

Assume that the field of view should be a 10'' square.