6.3000: Signal Processing

Communication Systems

- Matching Signals to Communications Media
- Amplitude Modulation
- Frequency-Division Multiplexing
- Radio Receivers

Quiz 2: April 15, 2-4pm, 50-340 (Walker).

- Closed book except for two pages of notes (four sides 8.5ẍ11)̈.
- No electronic devices. (No headphones, cellphones, calculators, ...)
- Coverage up to and including classes on April 10 and HW 8.

We have posted a **practice quiz** as a study aid for quiz 2 (no HW 9).

Additional quiz review slides are posted under Thursday's lecture. Students are encouraged to look over them before Thursday. These slides will not be presented during class on Thursday. Instead, Thursday's class will center around solving problems from past quizzes and highlight practical problem-solving strategies.

April 08, 2025

Communications Systems

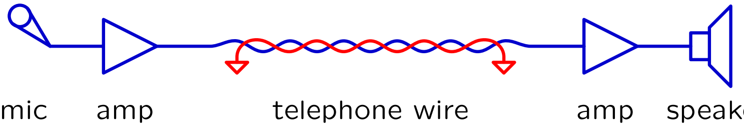
Beginning with commercial radio (1900s), communications technologies continue to be among the **fastest growing** applications of signal processing.

Examples:

- cellular communications
- wifi
- bluetooth
- GPS (Global Positioning System)
- broadband
- IOT (Internet of Things)
  - smart house / smart appliances
  - smart car
  - medical devices
- cable
- private networks: fire departments, police
- radar and navigation systems

Telephone

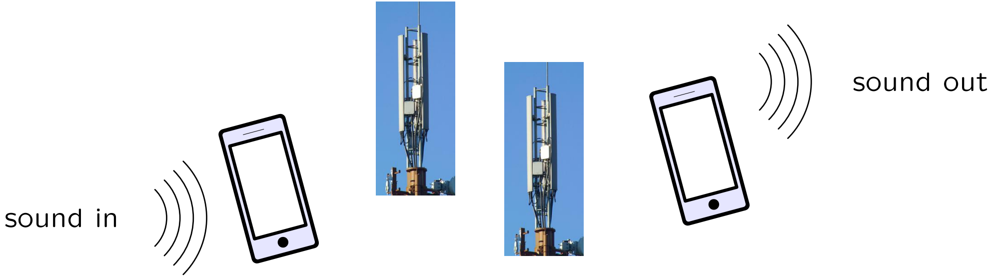
Popular thirst for communications has been evident since the early days of telephony.

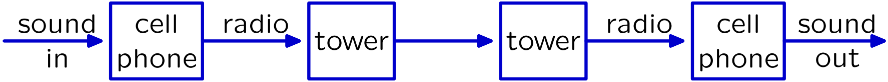


Patented by Alexander Graham Bell (1876), this technology flourished first as a network of **copper wires** and later as **optical fibers** (“long-distance” network) connecting virtually every household in the US by the 1980s.

Cellular Communication

First demonstrated by Motorola (1973), cellular communications quickly revolutionized the field. There are now more cell phones than people in the world.

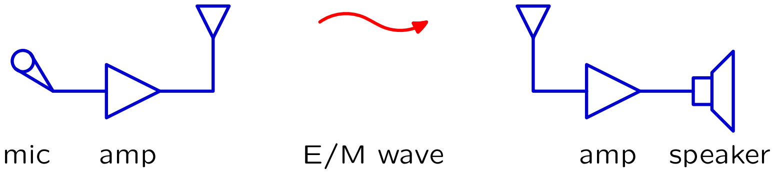




Much of the popularity and convenience of cellular communications is that the communication is **wireless** (at least to the local tower).

Wireless Communication

Wireless signals are transmitted via electromagnetic (E/M) waves.

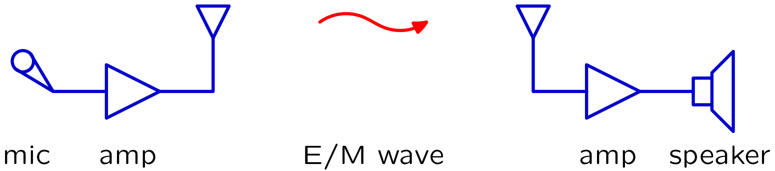


What is possible? (physics)

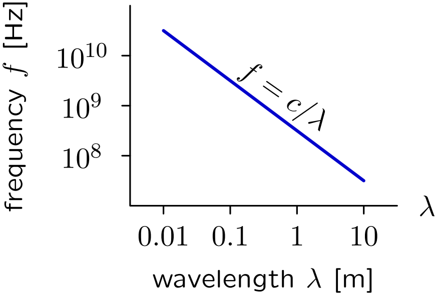
What is desireable? (FCC)

Wireless Communication

Wireless signals are transmitted via electromagnetic (E/M) waves.



For energy-efficient transmission and reception, the dimensions of the antenna should be on the order of the wavelength. But the wavelength of an electromagnetic wave depends on frequency.



Check Yourself

A key problem in the design of any communications system is matching characteristics of the signal to those of the media.

Telephone-quality speech contains frequencies from 200 Hz to 3000 Hz.

How long must the antenna be for efficient transmission and reception of E/M waves at audio frequencies?

1. < 1 mm
2. $\sim$ cm
3. $\sim$ m
4. $\sim$ km
5. > 100 km

Check Yourself

What frequency E/M wave is well matched to an antenna with a length of 10 cm (about 4 inches)?

1. < 100 kHz
2. 1 MHz
3. 10 MHz
4. 100 MHz
5. > 1 GHz

Wireless Communication

Speech is not well matched to the wireless medium.

Many applications require the use of signals that are not well matched to the required media.

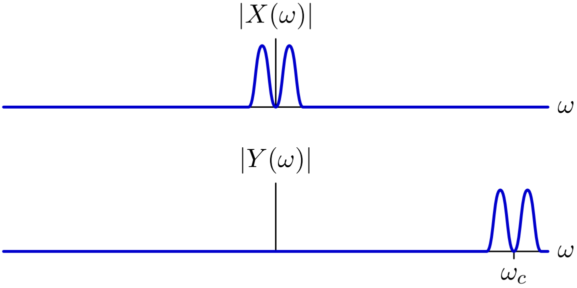
signal	applications
audio	radio, CD, cellular, optical fiber (TOSLINK), flash memory, MP3
video	DVD, flash memory, MP4
internet	twisted pair (Cat 5), optical fiber (backbone), wifi, bluetooth

Much of current research in communications focuses on modifying the intended signals to better accomodate constraints imposed by the media.

Today we will introduce simple matching strategies based on **modulation**, which underlie virtually all matching schemes.

Check Yourself

Construct a signal Y that codes the audio frequency information in X using frequency components near 2 GHz.



Determine an expression for Y in terms of X.

1. $y(t) = x(t)\, e^{j\omega_c t}$
2. $y(t) = x(t) * e^{j\omega_c t}$
3. $y(t) = x(t) \cos(\omega_c t)$
4. $y(t) = x(t) * \cos(\omega_c t)$
5. none of the above

Matching Signals to Communications Media

This transformation is based on the frequency-shift property of Fourier transforms.

If

$$x(t) \quad \overset{\text{CTFT}}{\Longrightarrow} \quad X(\omega)$$

then

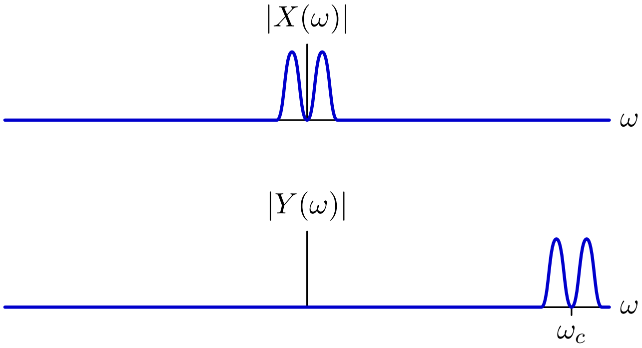
$$y(t) = e^{j\omega_c t}x(t) \quad \overset{\text{CTFT}}{\Longrightarrow} \quad Y(\omega) = X(\omega - \omega_c)$$

$$Y(\omega) = \int \left(e^{j\omega_c t}x(t)\right)e^{-j\omega t}\,dt = \int x(t)e^{-j(\omega-\omega_c)t}\,dt$$

$$Y(\omega) = X(\omega - \omega_c)$$

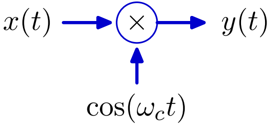
Matching Signals to Communications Media

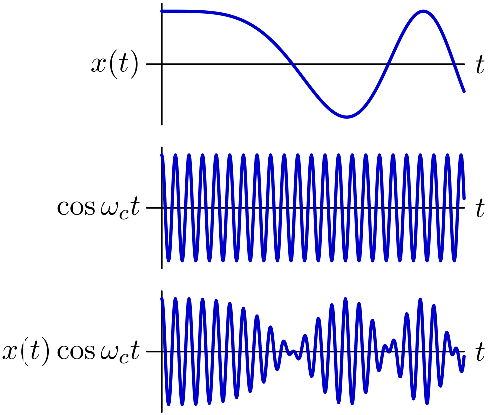
This scheme cannot be implemented **physically**. Why?



Amplitude Modulation (Time Domain)

Multiplying by a sinusoidal "carrier" is called **amplitude modulation.**
The signal "modulates" the amplitude of the carrier.





Multiplication Property of Fourier Transform

Multiplication in time corresponds to convolution in frequency.

Let

$$z(t) = x(t)y(t)$$

then

$$Z(\omega) = \int x(t)y(t)e^{-j\omega t}\,dt$$

Substitute $y(t) = \frac{1}{2\pi}\int Y(\omega)e^{j\omega t}d\omega = \frac{1}{2\pi}\int Y(\lambda)e^{j\lambda t}d\lambda$.

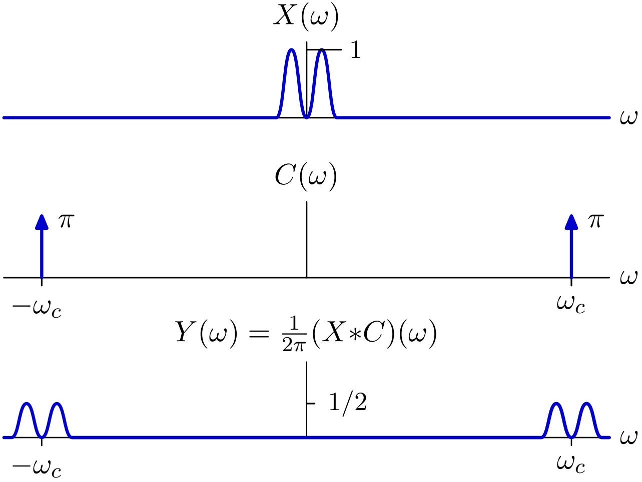
$$\begin{aligned}Z(\omega) &= \int x(t)\left(\frac{1}{2\pi}\int Y(\lambda)e^{j\lambda t}\,d\lambda\right)e^{-j\omega t}\,dt \\ &= \frac{1}{2\pi}\int Y(\lambda)\left(\int x(t)e^{-j(\omega-\lambda)t}dt\right)d\lambda \\ &= \frac{1}{2\pi}\int Y(\lambda)X(\omega-\lambda)\,d\lambda = \frac{1}{2\pi}(X * Y)(\omega)\end{aligned}$$

This result is the **dual** of filtering, where convolution in time corresponds to multiplication in frequency.

Amplitude Modulation (Frequency Domain)

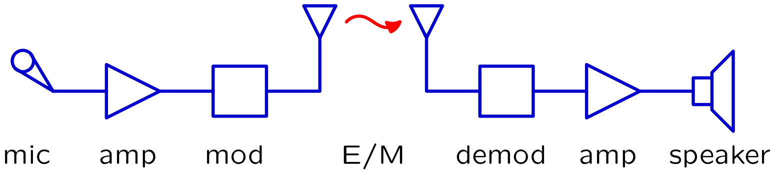
Multiplication in time corresponds to convolution in frequency.

Let $X(\omega)$ represent the Fourier transform of the signal to be transmitted.
Let $C(\omega)$ represent the Fourier transform of $\cos(\omega_c t) = \frac{1}{2}\left(e^{j\omega_c t} + e^{-j\omega_c t}\right)$.
Then $Y(\omega)$ is the result of convolving $X(\omega)$ with $C(\omega)$.



Demodulating the Received Signal

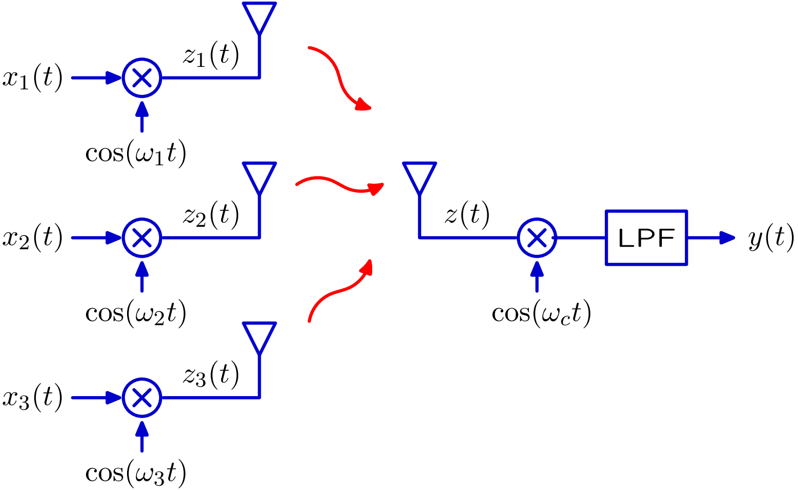
We can match the signal to the medium with a modulator as shown by the **mod** box below. But then we must demodulate the received signal to get back our original message (the **demod** box below).



How can we recover the original message from the modulated signal?

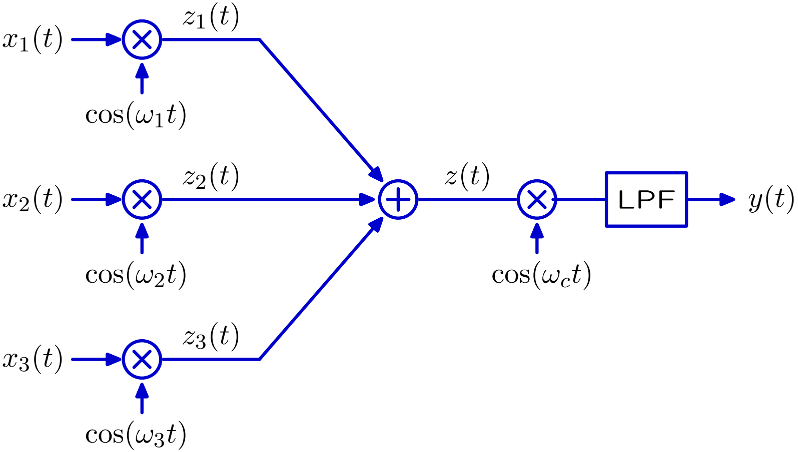
Frequency-Division Multiplexing

Multiple transmitters can co-exist, as long as the frequencies that they transmit do not overlap.



Frequency-Division Multiplexing

To first order, multiple transmitters simply sum.



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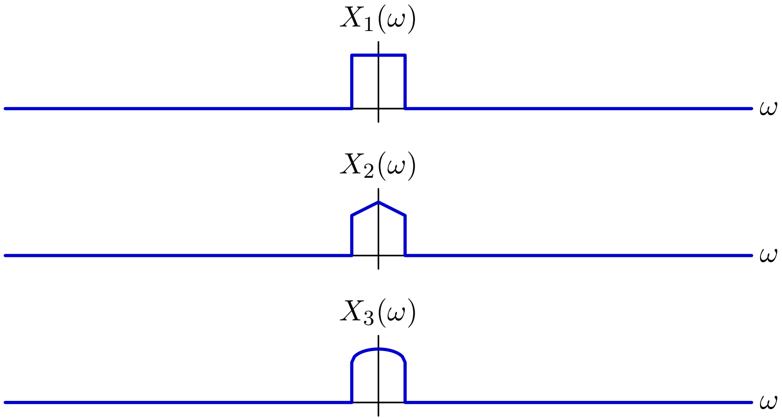

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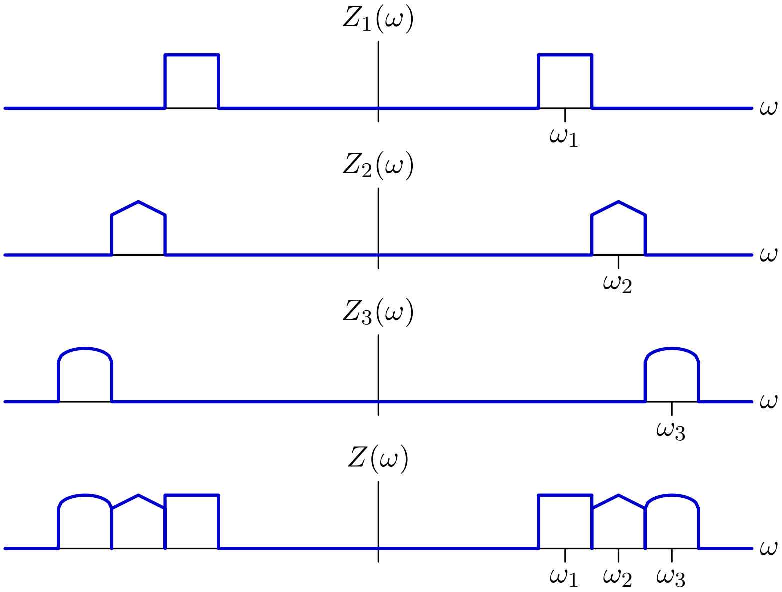
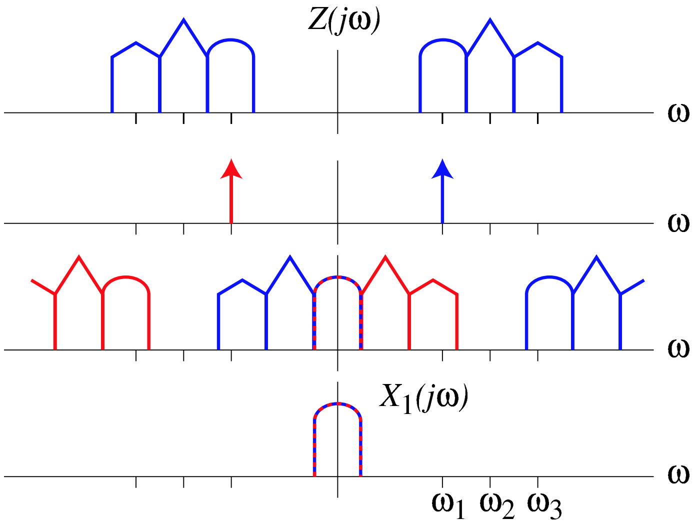


Frequency-Division Multiplexing

The receiver can select the transmitter of interest by choosing the corresponding demodulation frequency.



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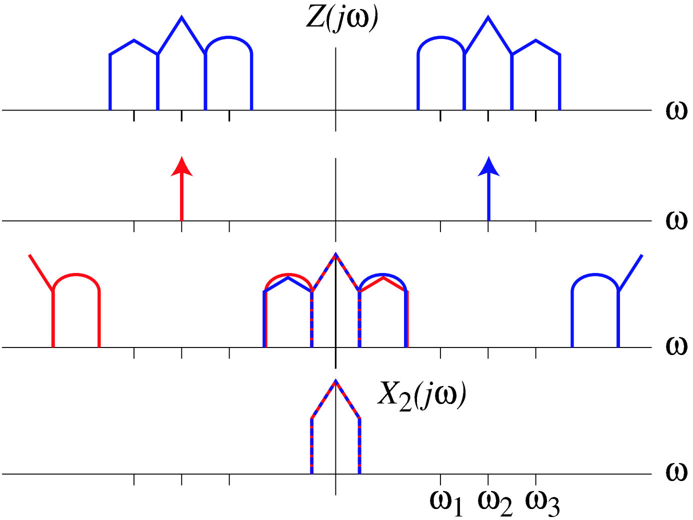

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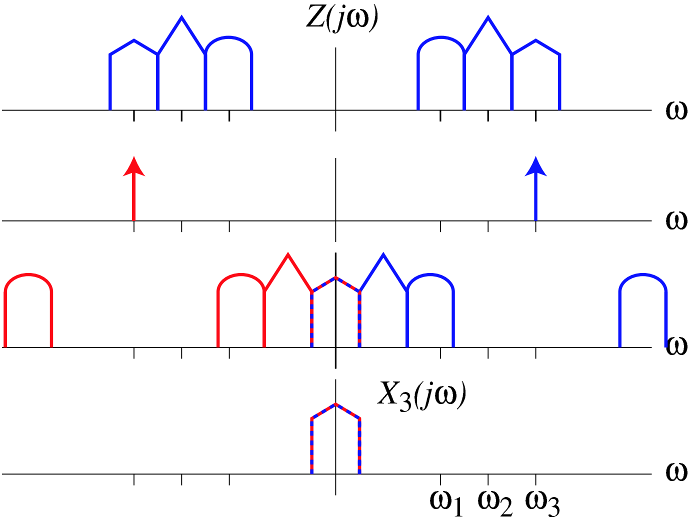
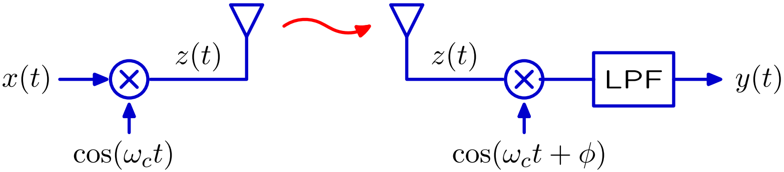


Radio Receiver

Synchronous demodulation requires knowing the carrier signal exactly.



What happens if there is a phase shift ϕ between the signal used to modulate and the one used to demodulate?

Radio Receiver

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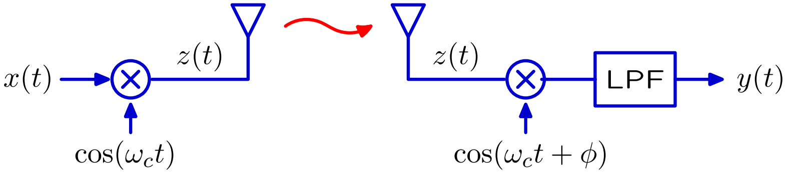


What happens if there is a phase shift ϕ between the signal used to modulate and the one used to demodulate?

$$
\begin{aligned}
y(t) &= x(t)\cos(\omega_c t)\cos(\omega_c t + \phi) \\
&= \frac{1}{2}x(t)\Big(\cos\phi + \cos(2\omega_c t{+}\phi)\Big) \\
&= \frac{1}{2}x(t)\cos\phi + \frac{1}{2}x(t)\cos(2\omega_c t{+}\phi)
\end{aligned}
$$

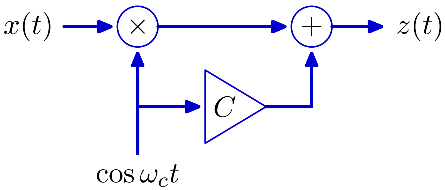
The second term is at a high frequency, so we can filter it out.

But multiplying by $\cos\phi$ in the first term is a problem: the signal "fades."

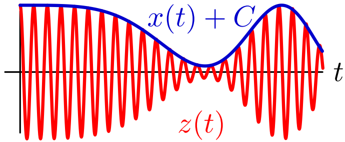
For example, if $\phi = \frac{\pi}{2}$, there is no output at all!

AM with Carrier

One way to know the carrier exactly is to send it along with the message.



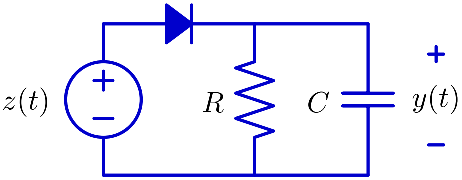
$$z(t) = x(t)\cos\omega_c t + C\cos\omega_c t = (x(t) + C)\cos\omega_c t$$

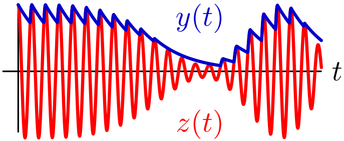


Adding carrier is equivalent to shifting the DC value of $x(t)$.
If we shift the DC value sufficiently, the message is easy to decode: it is just the envelope (minus the DC shift).

Radio Receiver

If the carrier frequency is much greater than the highest frequency in the message, AM with carrier can be demodulated with a peak detector.





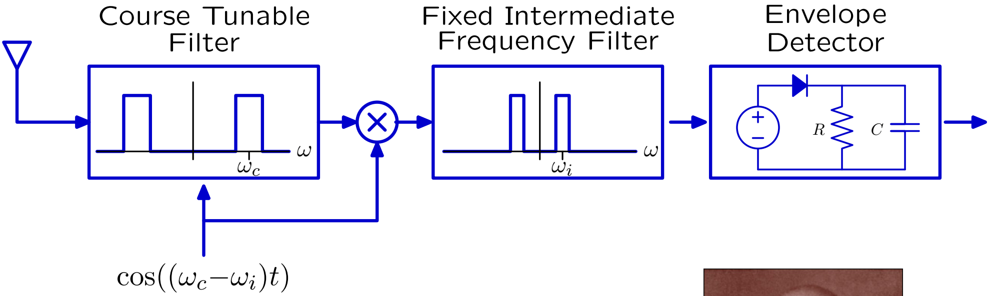
In AM radio, the highest frequency in the message is 5 kHz and the carrier frequency is between 500 kHz and 1500 kHz.

This circuit is simple and inexpensive.

But there is a problem.
Envelope detection cannot separate multiple senders.

Superheterodyne Receiver

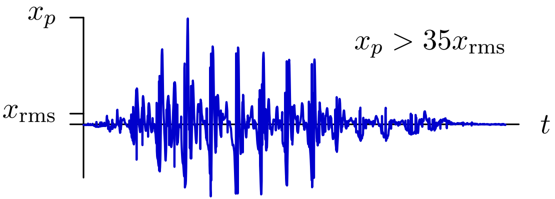
Edwin Howard Armstrong invented the superheterodyne receiver, which made broadcast AM practical.



Edwin Howard Armstrong also invented and patented the "regenerative" (positive feedback) circuit for amplifying radio signals (while he was a junior at Columbia University). He also invented wideband FM.

Radio Transmission

AM with carrier requires more power to transmit the carrier than to transmit the message!



Speech sounds have high crest factors (peak value divided by rms value). Envelope detection will only work if the DC offset C is larger than x_p.

The power needed to transmit the carrier can be $35^2 \approx 1000\times$ that needed to transmit the message.

Okay for broadcast radio (WBZ: 50 kwatts).
Not for point-to-point (cell phone batteries wouldn't last long!).

Broadcast Radio

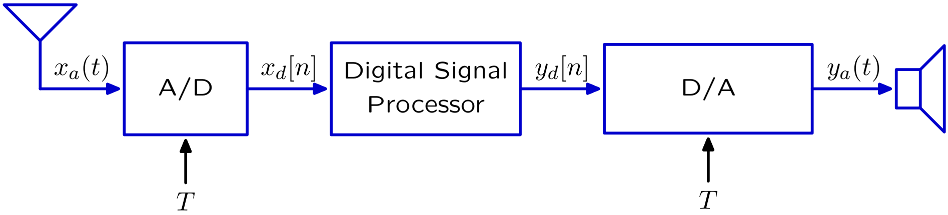
"Broadcast" radio was championed by David Sarnoff, who previously worked at Marconi Wireless Telegraphy Company (point-to-point).

- envisioned "radio music boxes"
- analogous to newspaper, but at speed of light
- receiver must be cheap (as with newsprint)
- transmitter can be expensive (as with printing press)

Sarnoff (left) and Marconi (right)

Digital Radio

Today's radios are very different from those that launched broadcast radio.



Some issues remain the same:

- power utilization
- bandwidth limitations

Other issues are newer:

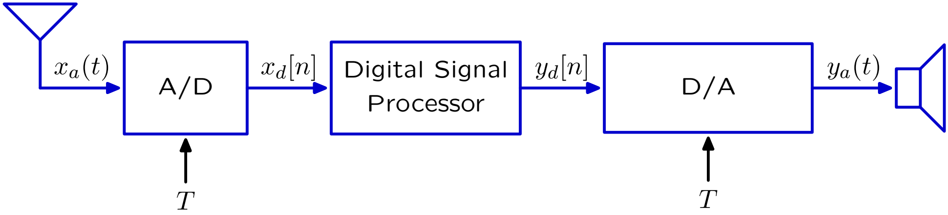
- more users
- more messages per user
- more different kinds of messages (audio, video, data)
- privacy and security

Signal processing plays an important role in all of these areas.

Question of the Day

Commercial AM radio

- 106 channels
- each channel is allocated 10 kHz bandwidth
- center frequencies from 540 kHz to 1600 kHz



We would like to choose the sampling period T so that $y_a(t)$ can be tuned to receive any of the 106 possible channels by simply changing the program in the digital signal procesor (without changing T).

What is the maximum value of T for which this is possible?