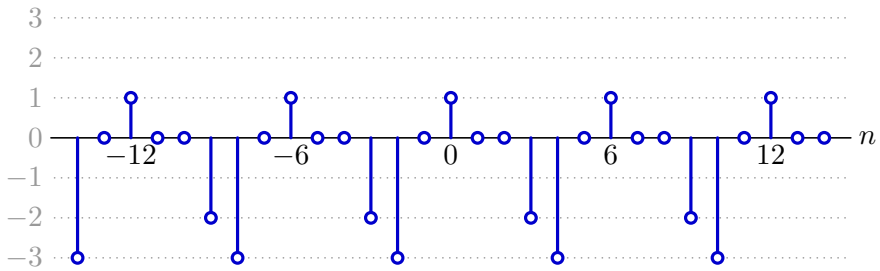
Signal Changes

Let $x[n]$ represent a periodic, discrete-time signal with a period of 6, a portion of which is shown below:



In this problem, we will explore alternative signals that we can make by changing a small number of samples per period in the signal above. Consider the related signals below (where $x_1[\cdot]$ through $x_5[\cdot]$ represent time-domain signals and $X_1[\cdot]$ through $X_5[\cdot]$ represent the associated Fourier series).

For each, we want to know whether it is possible to create a signal with the given properties by modifying **at most two samples per period** of $x[n]$. If it is possible, specify which value(s) you wish to change, and what you wish to change them to. If it is not possible, put an **X** in each box.

There may be multiple solutions to some of the following parts. You need only find one solution for full credit.

Part a. x_1

We would like to create $x_1[\cdot]$ by modifying **at most two** samples in $x[\cdot]$ so that $X_1[\cdot]$ is a symmetric function of k.

Which index should be changed?	2	What should the value be changed to?	-3
Which index should be changed?	**X**	What should the value be changed to?	**X**

If $X[k]$ is the DFT of $x[n]$, then $X[-k]$ will be the DFT of $x[-n]$. It follows that the antisymmetric part of $X[k]$ (which is $(X[k] - X[-k])/2$) is the DFT of the antisymmetric part of $x[n]$ (which is $(x[n] - x[-n])/2$). If the former is 0 for all k, then the latter will be zero for all n. Thus if we make $x_1[n]$ a symmetric function of n, $X_1[k]$ will be a symmetric function of k.

Part b. x_2

We would like to create $x_2[\cdot]$ by modifying **at most two** samples in $x[\cdot]$ so that $e^{j5\pi k/3}X_2[\cdot]$ is purely imaginary.

Which index should be changed?	0	What should the value be changed to?	3
Which index should be changed?	1	What should the value be changed to?	2

The DFT will be purely imaginary if the signal $x[n]$ is real and antisymmetric in n. The phase term $e^{j5\pi/3} = e^{-j2pik/6}$ corresponds to a delay of one sample. Therefore $X_2[k]$ will be purely imaginary if $x_2[n-1]$ is antisymmetric in n. antisymmetric.

Part c. x_3

We would like to create $x_3[\cdot]$ by modifying **at most two** samples in $x[\cdot]$ so that $\sum_{m=0}^{17} X_3[m] = 0$.

Which index should be changed?	0	What should the value be changed to?	0
Which index should be changed?	X	What should the value be changed to?	X

This sum is over three periods. Setting this sum to zero is the same as setting $x[0]$ to zero.

Part d. x_4

We would like to create $x_4[\cdot]$ by modifying **at most two** samples in $x[\cdot]$ so that $X_4[0] = 0$.

Which index should be changed?	0	What should the value be changed to?	5
Which index should be changed?	X	What should the value be changed to?	X

$X[0]$ will be zero if the average value of $x[n]$ is zero.

Part e. x_5

We would like to create $x_5[\cdot]$ by modifying **at most two** samples in $x[\cdot]$ so that $X_5[k] = -X_5[k+1]$ for all k.

Which index should be changed?	0	What should the value be changed to?	0
Which index should be changed?	4	What should the value be changed to?	0

The DFT will alternate in sign if $x[n] = 0$ for $n \neq 3$.