Sampling Sinusoids

Let $f(t)$ represent the following continuous-time signal:

$$f(t) = 4\cos(300\pi t) + 2\sin(400\pi t) + \cos(600\pi t)$$

Part a. Let $f_a[n]$ represent a discrete-time signal that is obtained by sampling $f(t)$ with sampling frequency $f_{sa} = 100\,\text{Hz}$, so that

$$f_a[n] = f(n/f_{sa})$$

Determine the fundamental (shortest) period of $f_a[n]$ (if one exists). Briefly explain.

$$f_a[n] = f(n/100) = 4\cos(3\pi n) + 2\sin(4\pi n) + \cos(6\pi n) = 4(-1)^n + 0 + 1$$

This function is periodic in n with periods $N = 2,\ 4,\ 6,\ \ldots$.
Therefore the fundamental period is $N = 2$.

Part b. Let $f_b[n]$ represent a discrete-time signal that is obtained by sampling $f(t)$ with sampling frequency $f_{sb} = 200\,\text{Hz}$, so that

$$f_b[n] = f(n/f_{sb})$$

Determine the fundamental (shortest) period of $f_b[n]$ (if one exists). Briefly explain.

$$f_b[n] = f(n/200) = 4\cos(3\pi n/2) + 2\sin(2\pi n) + \cos(3\pi n) = 4\cos(3\pi n/2) + 0 + (-1)^n$$

The fundamental period of this function is $N = 4$.

Part c. Let $f_c[n]$ represent a discrete-time signal that is obtained by sampling $f(t)$ with sampling frequency $f_{sc} = 300\,\text{Hz}$, so that

$$f_c[n] = f(n/f_{sc})$$

Determine the fundamental (shortest) period of $f_c[n]$ (if one exists). Briefly explain.

$$f_c[n] = f(n/300) = 4\cos(\pi n) + 2\sin(4\pi n/3) + \cos(2\pi n) = 4(-1)^n + 2\sin(4\pi n/3) + 1$$

The fundamental period of this function is $N = 6$.

Part d. Determine a sampling frequency f_{sd} for which

$$f_d[n] = f(n/f_{sd})$$

is not periodic (if such a frequency exists). Briefly explain.

If f_s is an irrational number, then $300\pi/f_s$, $400\pi/f_s$, and $600\pi/f_s$ will not be integer multiples of 2π.
Therefore f_s can be any irrational number, e.g., $f_s = \pi$.