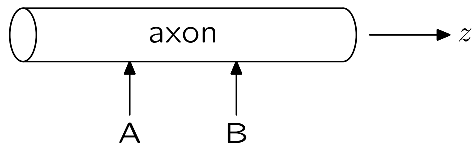
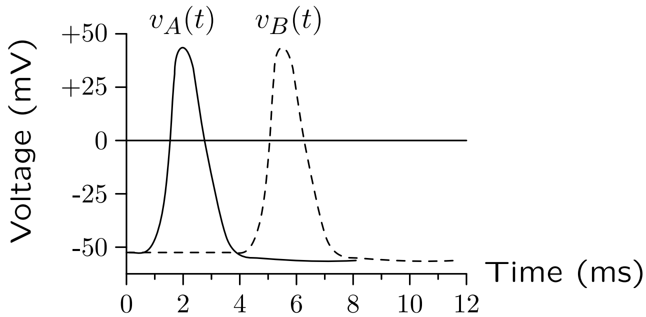
Neural Transmission

The axon of an unmyelinated nerve fiber is cylindrical in shape and conducts neural messages known as action potentials. These action potentials have a characteristic shape that propagates with uniform speed along the length of the axon, so that action potentials measured at two different locations, labeled A and B below,



differ by a shift in time, as shown in the following plots of voltage versus time.



Throughout this question, you can assume that time t is measured in milliseconds (ms) and distance z is measured in millimeters (mm).

Part a. Assume that measurement site A is located at $z = 0$ and measurement site B is located at $z = 5\,\text{mm}$. How fast is the action potential propagating?

The peak of the action potential occurs at approximately $t = 2\,\text{ms}$ at measurement site A and at approximately $t = 5.5\,\text{ms}$ at measurement site B. Thus, the action potential travels 5 mm in approximately 3.5 ms, so the speed ν is given by the following expression:

$$\nu = \frac{5\,\text{mm}}{3.5\,\text{ms}} \approx 1.43\,\text{m/s}$$

Part b. Let $v(z, t)$ represent the voltage of the action potential as a function of distance z and time t. Determine an expression for $v(z, t)$ of the form

$$v(z, t) = v_A(az + bt)\,.$$

where a and b are constants.

Substitute $z = 0$ into the general expression

$$v(z, t) = v_A(az + bt)$$

to find that

$$v(0, t) = v_A(bt)\,.$$

But $v(0, t)$ must be $v_A(t)$. Therefore $b = 1$. Next, substitute $z = 5\,\text{mm}$ to get

$$v(5, t) = v_A(5a + t)\,.$$

But we know that $v(5, t)$ is $v_B(t)$, and $v_B(t)$ is just a delayed version of $v_A(t)$:

$$v(5, t) = v_B(t) = v_A(t - 3.5\,\text{ms})\,.$$

To make this expression match the given form, a must equal $-3.5\,\text{ms}/5\,\text{mm} = -0.7\,\text{s/m}$. The final answer is

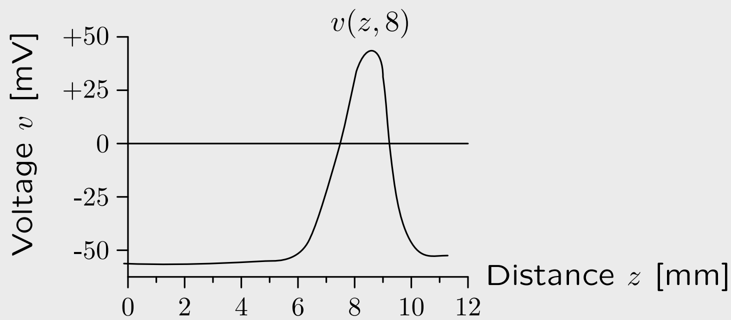
$$v(z, t) = v_A(t - 0.7z)\,.$$

Part c. Sketch voltage as a function of space (z) when $t = 8\,\text{ms}$.

Substitute $t = 8$ into the solution to the previous part to find that

$$v(z, 8) = v_A(8 - 0.7z)\,.$$

There are three important differences between the functional dependence of $v_A(t)$ on t and the functional dependence of $v_A(8 - 0.7z)$ on z. First, the minus sign in the latter means that the argument to $v_A(\cdot)$ decreases as z increases. Thus the z axis is flipped relative to the t axis. Second, the factor 0.7 means that the z axis is stretched relative to the t axis. Third, the constant 8 shifts the horizontal axis. The function $v_A(t)$ peaks when its argument t is 2. Therefore, $v(z, 8) = v_A(8 - 0.7z)$ will peak when $8 - 0.7z = 2$, which is at $z = 6/0.7 \approx 8.57$. Putting these factors together results in the following plot.



We could alternatively think about this problem using fiducials, which is a term used by surveyors to relate points in space to points on a map. To use this method, we would translate the coordinates of a few "defining" points on the $v_A(t)$ plot to points on the $v(z, 8)$ plot. For example, $v_A(t)$ peaks at approximately $2\,\text{ms}$ and has zero crossings at approximately $t = 1.5\,\text{ms}$ and $2.8\,\text{ms}$. We then translate those points to equivalent points in z using the relation that t should be replaced by $8 - 0.7z$.

t (ms)	z (mm)
1.5	9.29
2.0	8.57
2.8	7.43

These values fit well with the preceding mathematical analysis, and they also have a simple interpretation.