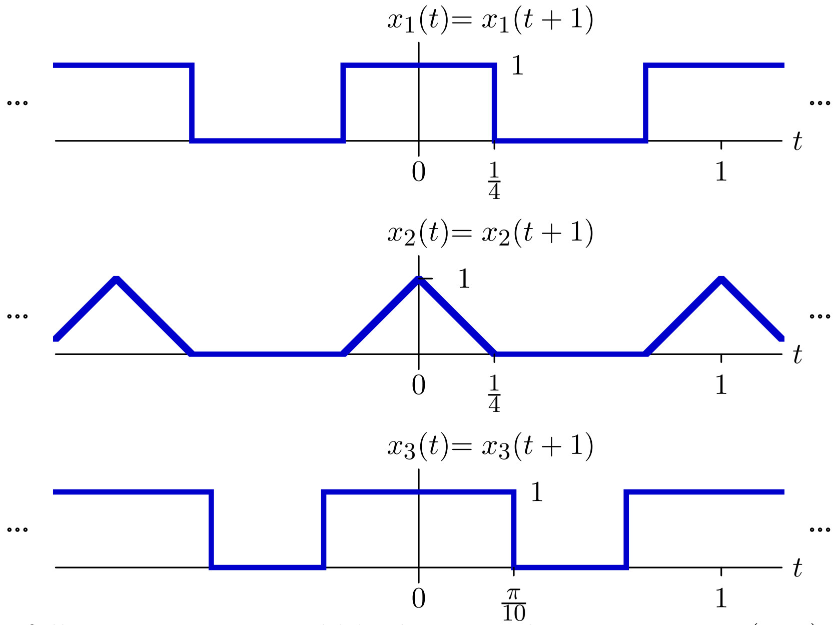
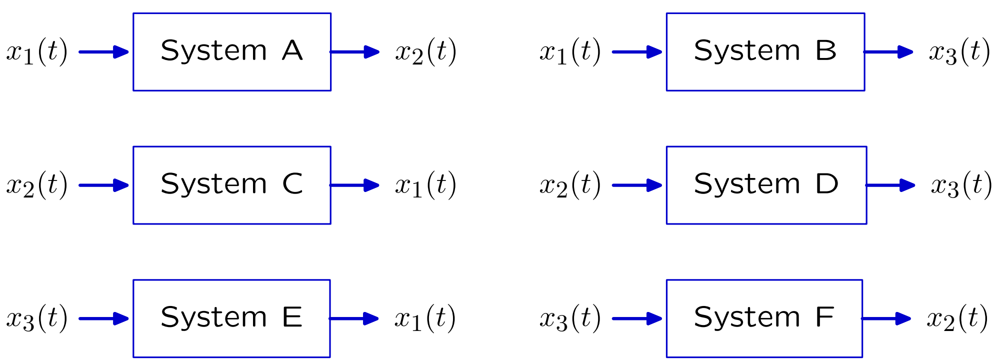
Input-Output Pairs

The following signals are periodic in time t with period $T = 1$.



Determine whether the following systems could be linear and time-invariant (LTI).

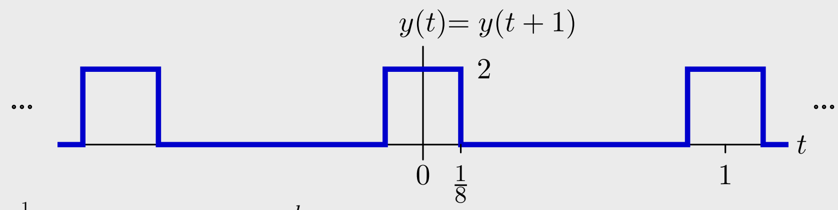


We can use the "filter" idea as follows. First calculate the Fourier series coefficients. Then ask if each Fourier series coefficient in the output is a scaled version of the corresponding coefficient in the input.

$$x_1(t) \leftrightarrow X_1[k] = \frac{1}{1}\int_{\frac{-1}{4}}^{\frac{1}{4}} e^{-j\frac{2\pi}{1}kt}dt = \frac{\sin\frac{\pi k}{2}}{\pi k} = \begin{cases} \frac{1}{2} & \text{if } k=0 \\ \frac{1}{\pi k} & \text{if } |k| = 1,5,9,13,\ldots \\ -\frac{1}{\pi k} & \text{if } |k| = 3,7,11,15,\ldots \\ 0 & \text{if } |k| = 2,4,6,8,\ldots \end{cases}$$

$$x_2(t) = (y * y)(t)$$

where $y(t)$ is the following signal:



$$y(t) \leftrightarrow Y[k] = \frac{1}{1}\int_{\frac{-1}{8}}^{\frac{1}{8}} 2e^{-j\frac{2\pi}{1}kt}dt = \frac{2\sin\frac{\pi k}{4}}{\pi k}$$

$$x_2(t) \leftrightarrow X_2[k] = TY^2[k] = 1 \times \frac{4\sin^2\frac{\pi k}{4}}{\pi^2 k^2} = \begin{cases} \frac{1}{4} & \text{if } k=0 \\ \frac{2}{\pi^2 k^2} & \text{if } |k| = 1,3,5,7,9,11,13,\ldots \\ \frac{4}{\pi^2 k^2} & \text{if } |k| = 2,6,10,14,\ldots \\ 0 & \text{if } |k| = 4,8,12,16,\ldots \end{cases}$$

$$x_3(t) \leftrightarrow X_3[k] = \frac{1}{1}\int_{-\frac{\pi}{10}}^{\frac{\pi}{10}} e^{-j\frac{2\pi}{1}kt}dt = \frac{\sin\frac{2\pi^2 k}{10}}{\pi k}$$

System A: The Fourier series coefficients at $k = 2, 6, 10, \ldots$ are zero in x_1 but these are not zero in x_2. Therefore the system could not be LTI.

System B: The Fourier series coefficients at $k = 2, 4, 6, 8, 10, \ldots$ are zero in x_1 but these are not zero in x_3. Therefore the system could not be LTI.

System C: All of the nonzero Fourier coefficients in x_1 are also present in x_2. Therefore the system could be LTI.

System D: The Fourier series coefficients at $k = 4, 8, 12, 16, \ldots$ are zero in x_2 but these are not zero in x_3. Therefore the system could not be LTI.

System E: All of the nonzero Fourier coefficients in x_1 are also present in x_3. Therefore the system could be LTI.

System F: All of the nonzero Fourier coefficients in x_2 are also present in x_3. Therefore the system could be LTI.