Fourier Pieces Let $f_1(t)$ represent a function of continuous time t that is represented by a trigonometric Fourier series:

$$f_1(t) = \sum_{k=1}^{\infty} \frac{\sin(kt)}{k}$$

Part a. What is the average value of $f_1(t)$? Your answer should be a number or numeric expression that can include common constants (such as π). Your answer should not include $f_1(t)$ or any integrals or infinite sums. Briefly explain.

0.
The general form for a Fourier series in trig form is

$$f_1(t) = c_0 + \sum_{k=1}^{\infty} c_k \cos(2\pi kt/T) + \sum_{k=1}^{\infty} d_k \sin(2\pi kt/T)$$

where c_0 is the average value. Since $c_0 = 0$, the average value of $f_1(t)$ is 0.

Part b. Determine the numerical value of the following integral:

$$\int_0^{2\pi} f_1(t)\cos(3t)dt$$

Your answer should be a number or numeric expression that can include common constants (such as π). Your answer should not include $f_1(t)$ or any integrals or infinite sums. Briefly explain.

0.
The sum that defines $f_1(t)$ contains trigonometric basis functions that are proportional to 1 (for c_0), $\cos(kt)$ (for c_k) and $\sin(kt)$ (for d_k) where the period T is 2π. Since the basis functions are orthogonal to each other and there are no $\cos(3t)$ terms in $f_1(t)$,

$$\int_0^{2\pi} f_1(t)\cos(3t)dt = 0$$

Part c. Determine the numerical value of the following integral:

$$\int_0^{2\pi} f_1(t)\sin(5t)dt$$

Your answer should be a number or numeric expression that can include common constants (such as π). Your answer should not include $f_1(t)$ or any integrals or infinite sums. Briefly explain.

$\pi/5$.

$$\int_{2\pi} f_1(t)\sin(5t)dt = \int_{2\pi} \sum_{k=1}^{\infty} \frac{1}{k}\sin(kt)\sin(5t)dt$$

Since the basis functions are orthogonal, this integral "sifts out" the $\sin(5t)$ component of $f_1(t)$:

$$\int_{2\pi} \sum_{k=1}^{\infty} \frac{1}{k}\sin(kt)\sin(5t)dt = \int_{2\pi} \frac{1}{5}\sin^2(5t)dt = \int_{2\pi} \frac{1}{5}\left(\frac{1}{2} - \frac{1}{2}\cos(10t)\right)dt = \int_{2\pi} \frac{1}{5}\frac{1}{2}dt = \frac{\pi}{5}$$

Part d. The function $f_1(t)$ can also be expressed as a complex exponential Fourier series as follows:

$$f_1(t) = \sum_{k=-\infty}^{\infty} a_k e^{jk\omega_o t}$$

Determine the numerical values of ω_o, a_{-2}, a_{-1}, a_0, a_1, a_2. Each answer should be a number or numeric expression that can include common constants (such as π). Your answers should not include $f_1(t)$ or any integrals or infinite sums. Briefly explain.

$\omega_o = 1$
$a_{-2} = j/4$
$a_{-1} = j/2$
$a_0 = 0$
$a_1 = -j/2$
$a_2 = -j/4$

$$f_1(t) = \sum_{k=1}^{\infty} \frac{\sin(kt)}{k} = \sum_{k=1}^{\infty} \frac{e^{jkt} - e^{-jkt}}{j2k} = \sum_{k=-\infty}^{-1} \frac{-j}{2k} e^{jkt} + \sum_{k=1}^{\infty} \frac{-j}{2k} e^{jkt} = \sum_{k=-\infty}^{\infty} a_k e^{jk\omega_o t}$$

where $\omega_o = 1$ and

$$a_k = \begin{cases} -j/2k & k \neq 0 \\ 0 & k = 0 \end{cases}$$