Find All

Consider the following signals:

$$f_1[n] = \frac{1}{2} + 6\cos(\pi n/2) + 4\cos(\pi n/5 - \pi/2)$$

$$f_2[n] = \cos(1.8\pi n) + 2\sin(2.7\pi n)$$

$$f_3[n] = \Big|\sin(\pi n/10)\Big| \qquad \text{; where } \big|x\big| \text{ represents the magnitude of } x$$

$$f_4[n] = \text{Im}\Big\{e^{j\left(2\pi n/20+\pi/2\right)}\Big\} \qquad \text{; where Im}\{x\} \text{ represents the imaginary part } x$$

Part a. Determine which (if any) of signals $f_1[n]$ through $f_4[n]$ are **symmetric** about n=0. Briefly explain.

$f_3[n]$ and $f_4[n]$.
$f_1[n]$ is not symmetric about n=0 because of the non-zero phase of the last term.
$f_2[n]$ is not symmetric about n=0 because sin is antisymmetric.
$f_3[n]$ is symmetric about n=0 because sin is purely antisymmetric, and the magnitude of a purely antisymmetric function is purely symmetric.
$f_4[n]$ is symmetric about n=0 because $f_4[n] = \text{Im}\big\{je^{j2\pi n/10}\big\} = \cos(2\pi n/10)$

Part b. Determine which (if any) of signals $f_1[n]$ through $f_4[n]$ are **periodic** with a fundamental (smallest) period N=20. Briefly explain.

$f_1[n]$, $f_2[n]$, and $f_4[n]$.
$f_1[n]$: $\cos(\pi n/2)$ is periodic with $N = 4 = 2 \times 2$. $\sin(\pi n/5 - \pi/2)$ is periodic with $N = 10 = 2 \times 5$. Their sum is periodic with $N = 2 \times 2 \times 5 = 20$.
$f_2[n]$: $\cos(1.8\pi n)$ is periodic with N=10. $\sin(2.7\pi n)$ is periodic with N=20. Their sum is periodic with N=20.
$\sin(\pi n/10)$ is periodic in N=20, and the magnitude halves the period. So $f_3[n]$ is periodic with N=10.
$f_4[n]$ is periodic with N=20 because the complex exponential is periodic with N=20.

Part c. Determine which (if any) of signals $f_1[n]$ through $f_4[n]$ can be represented by Fourier series with **purely imaginary** coefficients. Briefly explain.

None.
All of the time functions $f_1[n]$ through $f_4[n]$ are real-valued. Therefore the Fourier series coefficients

$$F[k] = \frac{1}{N}\sum_{n=\langle N\rangle} f[n]e^{-j\frac{2\pi}{N}kn} = \frac{1}{N}\sum_{n=\langle N\rangle} f[n]\Big(\cos(2\pi kn/N) + j\sin(2\pi kn/N)\Big)$$

will have purely imaginary values iff $f[n]$ is antisymmetric about n=0, so that the cosine terms sum to zero and the sine terms do not.

$f_1[n]$ is not antisymmetric about n=0 because both the constant term and the cosine term are symmetric. Similarly, $f_2[n]$ is not antisymmetric about n=0 because the cosine term is symmetric.
$f_3[n]$ is symmetric about n=0 because sin is purely antisymmetric, and the magnitude of a purely antisymmetric function is purely symmetric.
$f_4[n]$ is symmetric about n=0 because $f_4[n] = \text{Im}\big\{je^{j2\pi n/10}\big\} = \cos(2\pi n/10)$

Part d. Determine which (if any) of signals $f_i[n]$ can be represented by Fourier series coefficients $F_i[k]$ that are symmetric functions of k (i.e., $F_i[k] = F_i[-k]$). Briefly explain.

$f_3[n]$ and $f_4[n]$.
All of the time functions $f_i[n]$ are real-valued. Therefore the Fourier series coefficients

$$X[k] = \frac{1}{N}\sum_{n=\langle N\rangle} x[n]e^{-j\frac{2\pi}{N}kn} = \frac{1}{N}\sum_{n=\langle N\rangle} x[n]\Big(\cos(2\pi kn/N) + j\sin(2\pi kn/N)\Big)$$

will equal

$$X[-k] = \frac{1}{N}\sum_{n=\langle N\rangle} x[n]e^{j\frac{2\pi}{N}kn} = \frac{1}{N}\sum_{n=\langle N\rangle} x[n]\Big(\cos(2\pi kn/N) - j\sin(2\pi kn/N)\Big)$$

iff $x[n]$ is a symmetric function of n, so that the sine terms integrate to zero.

$f_1[n]$ is not symmetric about n=0 because of the phase shift in the its term.
$f_2[n]$ is not symmetric about n=0 because of the sine term.
$f_3[n]$ is symmetric about n=0 because sin is purely antisymmetric, and the magnitude of a purely antisymmetric function is purely symmetric.
$f_4[n]$ is symmetric about n=0 because $f_4[n] = \mathrm{Im}\big\{je^{j2\pi n/10}\big\} = \cos(2\pi n/10)$